1772

Theoria motuum lunae

Leonhard Euler

THEORIA MOTVVM LVNAE,

NOVA METHODO PERTRACTATA

VNA CVM

TABVLIS ASTRONOMICIS,

VNDE

AD QVODVIS TEMPVS

LOCA LVNAE

EXPEDITE COMPVTARI POSSVNT,

INCREDIBILI STVDIO ATQVE INDEFESSO LABORE TRIVM ACADEMICORVM:

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PRAEFATIO.

Quoties iam quadraginta abhinc annis theoriam Lunae euoluere eiusque motum ex principiis grauitationis receptis definire sum conatus; tot semper ac tantae difficultates se obtulerunt, vt labores meos et vlteriores inuestigationes abrumpere sim coactus. A principiis enim mechanicis tota quaestio statim ad ternas aequationes differentiales secundi gradus reducitur, quas non solum nullo modo integrare licet, sed etiam adproximationes, quibus vtique in hoc genere est acquiescendum, maximis obstaculis

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impediebantur, ita, vt nullo modo perſpicerem, quemadmodum haec inueſtigatio ex ſola theoria non tam abſolui, quam tantum aliquatenus ad vſum accommodari poſſet. Principio quidem plurimum deſudaui, vt memoratas illas aequationes differentiales ad integrationem perducerem; continuo autem magis magisque intellexi, omnes labores huius generis inutiliter inſumtum iri; neque etiam huiusmodi integrationes admodum ſunt deſiderandae; facile enim intelligitur, formulas integrales maxime futuras eſſe prolixas et intricatas, ita, vt inde nullus plane fructus in vſum aſtronomiae exſpectari poſſet. Quanquam autem iam ante triginta circiter annos tabulas Lunares condiderim atque deinceps etiam theoriam Lunae ediderim, tamen nullo modo hanc inueſtigationem penitus exhauſi, quin potius plures adhuc Lunae inaequalitates nimium erant abſconditae, quam vt eas in lucem protrahere valuiſſem. Atque hoc idem incommodum etiam vſu venit deinceps Clariſſimis Viris, Maiero et Clairaltio, qui tabulis ſuis lunaribus maximam gloriam ſunt adepti; vterque enim plures Lunae aequationes, quas ex ſola theoria haurire oportuerat, per ſolas obſeruationes quaſi diuinando produxerunt.

Hae

Hae autem difficultates maximam partem inde oriebantur, quod Lunae orbita peculiaris tribuebatur ſuper plano eclipticae mobilis et ſub angulo variabili inclinata, ita, vt ad quoduis tempus primum interſectio huius orbitae cum ecliptica, ſiue linea nodorum; tum vero etiam inclinationis quantitas per plures aequationes definiri deberet; quo facto locus Lunae verus in hac orbita per plurimas quoque aequationes determinabatur; ex quo denique longitudinem et latitudinem elici oportebat.

His igitur omnibus difficultatibus probe perpenſis cum nuper eandem quaeſtionem de nouo eſſem aggreſſus, intellexi, totum negotium longe alio modo ſuſcipi debere, vt feliciori ſucceſſu ſcopum attingamus. Pro quouis ſcilicet tempore propoſito ſtatim conſidero Lunae longitudinem mediam in ipſo plano eclipticae ſumtam, quae ſit recta ♈M, et vbicunque tunc fuerit Luna in ☾, inde ad planum eclipticae demittatur perpendiculum ☾l et ex l ad rectam ♈M agatur normalis lL, ita, vt more apud Geometras recepto locus Lunae iam determinetur per ternas coordinatas ♈L, Ll, l☾; quibus inuentis perſpicuum eſt, verum Lunae locum ☾ facillime determinari;

nari; quippe ex angulo M♁l, cuius tangens $\frac{Ll}{♁L}$, ſtatim innoteſcit vera Lunae longitudo; tum vero angulus l♁☾, cuius tangens eſt $\frac{☾l}{♁l}$, praebet latitudinem Lunae. Totum ergo negotium huc redit, vt ad quoduis tempus propoſitum quantitas harum trium coordinatarum definiatur; hanc ob rem aequationes illas differentiales ſecundi gradus, quas principia motus immediate ſuppeditant, ad iſtas ternas coordinatas reuocaui; vbi quidem ad aequationes admodum complicatas ſum deuolutus; quibus tamen hoc inſigne lucrum ſum adeptus, vt cum recta ♁M referat longitudinem mediam, ſi ſuper ea ſumatur portio ♁O diſtantiae Lunae mediae a Terra aequalis, ternae rectae OL, Ll, l☾, ſemper futurae ſint ſatis exiguae ideoque altiores earum poteſtates ſeries maxime conuergentes conſtituant. His igitur in genere obſeruatis, circa ternas iſtas quantitates incognitas, quibus locus Lunae determinatur, inprimis conſiderari debent quantitates cognitae, ex quibus ipſas incognitas definire liceat. Hae autem quantitates cognitae duplicis ſunt generis; aliae conſtantes, aliae variabiles; quatuor autem hic occurrunt quantitates conſtantes, quarum valores deinceps ex ipſis phaenomenis concludi neceſſe eſt; prima, quam littera K deſigno, con-

tinet

tinet excentricitatem orbitae lunaris, cuius quantitas vtique a motu, qui Lunae ab initio fuerit impressus, pendet, ideoque vel maior vel minor esse potuisset, quam nunc quidem reperitur; eius autem valorem ex pluribus obseruationibus conclusi $K = 0,05445$; qui ergo tam est exiguus, vt eius superiores potestates mox pro euanescentibus haberi queant; secunda quantitas constans, littera i expressa, inuoluit declinationem motus lunaris a plano eclipticae, cuius valor ideo maior minorue esset futurus, si inclinatio maior esset vel minor; plures autem obseruationes valorem huius litterae praebuerunt $i = 0,08964$ cuius altiores potestates pariter mox ad nihilum fere rediguntur: tertia vero constans, quam littera $\varkappa$ in sequentibus exhibeo, est ipsa excentricitas orbitae terrae, quae ergo nulli plane dubio est obnoxia, cum sit, vti constat, $\varkappa = 0,01679$; quarta denique constans in Lunae determinationem ingrediens pendet a parallaxi Solis, dum inuoluit rationem mediae distantiae terrae a Sole ad mediam distantiam Lunae a Terra, quam rationem in sequentibus $= 1 : a$ statuo; vnde cum nunc quidem haec ratio satis exacte cognita sit censenda, conclusi fore $a = \frac{1}{390}$, cuius ergo iam secunda potestas negligi tuto poterit. Praeter has vero constan-

tes

tes ad quoduis tempus, quo locus Lunae deſideratur, noſſe oportet quatuor angulos tempori proportionales, quos ergo expedite ex tabulis mediorum motuum excerpere licebit. Horum angulorum primus, quem littera p deſigno, exhibet elongationem mediam Lunae a Sole, dum ſcilicet a longitudine media Lunae ſubtrahitur longitudo media Solis; ſecundus autem angulus littera q expreſſus eſt anomalia media Lunae, quam tabulae ad omnia temporis momenta exhibere ſolent, quae cum inter ſe non penitus conueniant, facile euenire poſſet, vt hi anguli q aliquanto maiores vel minores ſint ſtatuendi, quam in his vel illis tabulis exhibentur; tertius angulus littera r deſignatus conuenit cum eo, qui in tabulis argumentum latitudinis Lunae medium appellatur ac reperriri ſolet, dum longitudo Nodi adſcendentis media a longitudine Lunae media ſubtrahatur; qui prouti in tabulis exhibetur, vtique leui correctione indigere poſſet. Quartus denique angulus cognitus littera t expreſſus ipſam anomaliam mediam Solis deſignat ideoque nulla plane correctione egere eſt cenſendus. Tota igitur analyſis eo dirigi debet, vt ad quoduis tempus valores ternarum illarum coordinatarum, quas poſui ♁L = $1+x$ ſumta ſcilicet ♁O = 1, denotante vnitate diſtanftantiam mediam Lunae a Terra; $Ll = r$ et $l☾ = z$, per quantitates illas cognitas tam constantes, quam variabiles determinentur. Atque hinc iam notiones admodum adaequatas adipiscimur omnium inaequalitatum, quibus motus Lunae perturbatur, inter quas alias vix vllus ordo intercedere videri posset. Praecipuum scilicet hic auxilium mihi in hoc negotio suppeditauit idonea distributio omnium inaequalitatum in certas classes. Ad primam scilicet classem refero eas inaequalitates, quae a solo *angulo* p siue elongatione media Solis a *Luna* pendent, et quas Astronomi sub nomine variationis complecti consueuere. Secunda classis eas continet inaequalitates, quae a sola excentricitate lunari K pendent; quas omnino subdiuidi conuenit in ordines, prouti scilicet vel ipsa quantitas K vel eius quadratum K^2 vel cubus K^3 inaequalitates illas adficit; facile enim intelligitur, vltra tertiam potestatem ipsius K adscendi non opus esse; has ergo simpliciter excentricas adpellare liceat. Tertia classis refertur ad excentricitatem Solis κ, quae propterea simpliciter solaris adpellari potest. Quarta classis adfecta est littera α, ac propterea inaequalitates eo pertinentes commode parallacticae vocabuntur; ad quintam denique classem eas referimus inaequalitates, quae a constanti i seu inclinatione orbitae lunaris pendent, quae quatenus ſolam longitudinem Lunae adficiunt, vulgo ſub nomine reductionis comprehendi ſolent; latitudo enim manifeſto potiſſimum in hac claſſe definitur. Claſſibus autem his conſtitutis, eae vario modo inſuper inter ſe permiſceri poſſunt; vnde inaequalitates quaſi mixtae naſcuntur, quas pluribus ordinibus ſum complexus. Hos autem ordines ſtatim diſtingui oportet prouti vel ad longitudinem Lunae ſeu quod redit eodem, ad priores binas coordinatas x et y referuntur; latitudo vero ſiue tertia coordinata $l☾ = z$ peculiaribus ordinibus continetur. Pro longitudine autem ſiue binis coordinatis x et y ordines ſequenti modo conſtitui:

$$x = \mathfrak{O} + K\mathfrak{P} + K^2\mathfrak{Q} + K^3.\mathfrak{R} + a\mathfrak{S} + aK\mathfrak{T} + \varkappa\mathfrak{U} + \varkappa K\mathfrak{V} + \varkappa K^2\mathfrak{W} + a\varkappa\mathfrak{w} + ii\mathfrak{X} + iiK\mathfrak{Y} + ii\varkappa\mathfrak{Z}$$

$$y = O + KP + K^2Q + K^3R + aS + aKT + \varkappa U + \varkappa KV + \varkappa K^2W + a\varkappa w + iiX + iiKY + ii\varkappa Z.$$

pro latitudine autem ſeu tertia coordinata z ordines ita ſe habent:

$$z = i\mathfrak{p} + iK\mathfrak{q} + iK^2\mathfrak{r} + i\varkappa\mathfrak{s} + i^3.\mathfrak{u} + ia.\mathfrak{t}.$$

Ipſa autem haec ordinum diſtributio maximum adiumentum attulit ad calculum feliciori ſucceſſu expediendum; cum enim ipſae formulae generales pro ternis coordinatis tan-

tope-

topere essent prolixae et complicatae vt quaelibet integram paginam adimpleuerit, atque adeo in infinitum expansae; postquam eas ad definitos ordines traduximus, quasi praeter exspectationem in formas satis concinnas se contrahi sunt passae, ita, vt pro quolibet ordine inuestigationem seorsim instituere licuerit; quem laborem in ipsis aequationibus principalibus nemo exantlare valuisset.

Ratio autem illius contractionis in eo est sita, quod ambae litterae O et Ɔ vix vnam partem centesimam vnitatis superent; vnde cum earum cubi ne millionesimae quidem parti aequentur, eos et multo magis altiores harum litterarum potestates negligere licuit, quandoquidem vna pars centies millesima vnitatis in loco Lunae tantum duo minuta secunda valet. Quocirca sufficere visum est, singulos terminos, qui valores litterarum Ɔ et O constituunt, vsque ad sextam figuram decimalem exigere.

Praeterea pro euolutione singulorum horum ordinum regulam facilem inueni, quae vniuersam integrationis rationem in se complectitur, ita, vt eius beneficio omnes terminos satis expedite definire licuerit, cuiusmodi artificium in aequationibus generalibus nullo modo adplicari potuisset. Hoc igitur modo singulos ordines successiue expediuimus et cum termini primi ordinis vsque ad sextam figuram decimalem fuerint exacti, secundum ordinem charactere K insignitum ad quinque figuras euoluere licuit, id quod etiam abunde sufficit, cum termini huius ordinis insuper in fractionem K sint ducendi. In litteris porro 𝔔 et Q per quadratum K K multiplicandis gradus praecisionis vix ad quartam figuram vsque extenditur; vnde tamen ob multiplicatorem K K error non ad minutum secundum vnum ascendit; hic enim probe tenendum est, quo magis ordines complicantur, eo pauciores figuras in fractionibus decimalibus iustas esse prodituras; atque adeo ibi occurrunt certi termini, in quibus ob singularem complicationem praecisio adhuc minor euadit. Ob quam caussam in ordine litterarum 𝔑 et R quidam termini iam aliquot minutis secundis a vero aberrare possunt; atque hoc ipsum incommodum etiam se exserit in litteris 𝔙 et V, vbi vix ad vnum minutum secundum certi esse possumus, quod quidem in se parui esset momenti, sed quia eaedem litterae ingrediuntur in sequentes 𝔚 et W, error plurium minutorum secundorum vtique esset metuendus, quam ob caussam euolutionem harum litterarum ne suscipere quidem sumus ausi. Tum vero etiam ob similem caussam ordinem charactere ii K K signandum penitus praetermit-

tere

tere sumus coacti. Quae omissio instituto nostro eo minus visa est noxia, quod inaequalitates horum ordinum per se sint valde parvae ac fortasse non ad quadrantem vnius minuti assurgunt neque idcirco necesse iudicauimus totum hunc calculum tantopere prolixum de nouo instituere et ad maiorem praecisionis gradum exsequi; praecipue cum duae illae vel tres inaequalitates huic incommodo obnoxiae multo facilius immediate ex obseruationibus concludi possint. Interim tamen in gratiam theoriae maxime esset optandum, vt exercitati calculatores hunc laborem in se susciperent atque omnia momenta ad maiorem adcurationis gradum determinarent. Quem in finem vtique necesse foret, in simplicioribus saltem ordinibus cubos litterarum $\mathfrak{D}$ et $\mathfrak{O}$ nondum negligere, sed potius iis admissis fractiones decimales vsque ad octauam figuram prosequi; hoc enim modo cum etiam in sequentibus ordinibus omnes determinationes centies adcuratiores obtineantur, neutiquam amplius erit verendum, ne omnes plane inaequalitates exactissime definitae sint proditurae. Talis autem labor sine dubio multo magis erit molestus et operosus, ac fortasse vix intra anni spatium absolui posset, praecipue cum in hoc genere singulos calculos bis vel ter repetere omnino sit necesse; atque haec etiam est caussa, quod nos his, quos in hoc opere expediuimus, calculis iam tantopere defatigati tam immensum laborem suscipere non sumus ausi, quilibet enim, qui omnes calculos hic expositos vel leuiter perpendere voluerit, facile agnoscet, vix vllam adhuc quaestionem analyticam etiamnunc esse pertractatam, quae tam intricatas calculi discussiones et tam prolixos calculos postulauerit.

His autem, quae ad theoriam pertinent, expeditis plurimum adhuc difficilis deprehenditur ipsa formularum inuentarum adplicatio ad verum motum Lunae definiendum; primum enim ex ipsis obseruationibus concludi debent veri valores litterarum nostrarum constantium K et i, tum vero etiam vera loca media tam apogaei Lunae, quam nodorum, ad quoduis tempus propositum, quandoquidem in his elementis error vnius minuti primi inesse posset. Hae autem determinationes haud difficulter expediri possent, si modo sufficiens multitudo adcuratissimarum Lunae obseruationum praesto esset; verum pace Astronomorum dixerim, fere omnes obseruationes lunares, quae vulgo sunt prolatae, ita esse comparatas, vt vltra integrum minutum primum a veritate aberrare sint censendae; id quod inprimis de iis est tenendum, quae ex culminationibus Lunae sunt deductae, vbi primum

mum altitudo limbi lunaris ſiue ſuperioris ſiue inferioris, tum vero appulſus limbi ſiue antecedentis ſiue conſequentis ad meridianum obſeruari ſolet; in altitudine autem tam ipſa, quam per refractionem corrigenda vix errorem decem ſecundorum euitare licebit; deinde in appulſu ad meridianum facile certe vno minuto ſecundo temporis aberrari poteſt; vnde in loco error iam quindecim ſecundorum gignitur. Praeterea diametrum Lunae adparentem exactiſſime noſſe oporteret, in quo pariter vix omnes errores euitari licet; praecipue autem locus Lunae geocentricus, quo in hoc negotio egemus, exactiſſimam parallaxeos cognitionem poſtulat, quae cum potiſſimum a theoria pendeat, hic plerumque ſine dubio pluribus minutis ſecundis aberrari ſolet. His igitur erroribus colligendis vix vnquam ſperare licebit, Lunae loca obſeruata intra vnum minutum primum cum veritate conſentire; ſponte autem intelligitur, hanc ipſam obſeruationum incertitudinem in cauſſa eſſe, quod ſupra memorata elementa vix ae ne vix quidem ex aequationibus definiri queant, niſi quis ingentem obſeruationum numerum in ſubſidium vocare voluerit. Quocirca eas horum elementorum determinationes, quas in hoc opere ex variis obſeruationibus concluſimus, neutiquam pro certis venditamus, neque diffitemur, eas in-

insigni adhuc emendatione indigere, dum enim certis obseruationibus, quibus vsi sumus, nimis tribuimus, facile euenire potest, vt nostrae tabulae ab aliis iusto nimis aberrent, idquod nequaquam theoriae vitio est vertendum; deinde etiam loca apogaei et nodorum ita assumsimus prouti in tabulis Majerianis exhibentur, vbi iterum fortasse non leui correctione foret opus. Interim tamen his ipsas tabulas, quas hic subiunximus, raro vltra minutum primum ab obseruationibus discrepare comperimus; ita, vt Astronomi his tabulis pari fere successu, ac Majerianis vel Clairaltianis, tuto vti possent; idque eo magis, quod istae tabulae multo faciliorem calculum postulent, quam aliae; dum omnia plane momenta ex quatuor tantum angulis tempori proportionalibus definiantur atque adeo ipsa Lunae latitudo immediate ex his angulis deriuetur, quae alioquin satis molestam nodorum correctionem ipsumque Lunae locum in sua orbita requirere solet. Neque etiam adeo difficile videtur, si quis has tabulas cum plurimis obseruationibus conferre voluerit, aliquot leuibus correctionibus adhibendis has tabulas ad multo maiorem perfectionis gradum euehere; vnde certe iam maximus vsus in Astronomiam esset redundaturus.

NOVAE

NOVAE THEORIAE MOTVVM LVNAE LIBER PRIMVS

CONTINENS IPSAM LVNAE THEORIAM.

PARS PRIMA.

INVESTIGATIO AEQVATIONVM DIFFERENTIALIVM, MOTVM LVNAE CONTINENTIVM.

CAPVT I.

PRAENOTANDA CIRCA MOTVM LVNAE.

§. 1.

Adcurata et perfecta cognitio motuum Lunae, vnde tabulae astronomicae veritati exactissime congruentes condi queant, tantopere abscondita summisque difficultatibus inuoluta deprehenditur, vt vires humani ingenii longe superare videatur.

§. 2.

Maxima ſine dubio difficultas ad ſolutionem famoſi illius problematis, de motu trium corporum ſe mutuo attrahentium, reuoluitur, cuius enodatio completa omnes analyſeos vires tranſcendere videtur; quantumuis enim Geometrae in hoc problemate adhuc deſudarint; omnia, quae praeſtiterunt, ad euolutionem caſuum maxime particularium reſtringuntur, quos adeo neutiquam generaliter reſoluere, ſed tantum ad adproximationes reuocare licuit; tantum abeſt, vt quisquam gloriari poſſit, ſe huius quaeſtionis ſolutione eſſe potitum.

§. 3.

Quamuis autem hoc problema perfecte expedire liceret; tamen adhuc longiſſime a completa motuum Lunae cognitione eſſemus remoti; primum enim in hoc problemate vires, quibus terna corpora ſe mutuo attrahunt, rationem inuerſam duplicatam diſtantiarum ſequi aſſumuntur; id quod ſecundum vera attractionis principia non euenit, niſi ipſa corpora fuerint perfecte ſphaerica. Verum nunc quidem extra dubium eſt poſitum, cuncta corpora coeleſtia ab hac figura magis minusue diſcrepare; inprimis autem phaenomena nutationis Lunae manifeſto docuerunt, eius figuram ſatis notabiliter a ſphaerica recedere; ex quo vires, quibus ſiue ad Solem ſiue ad Terram

Terram vrgetur, aliquantillum a ratione memorata aberrent necesse est; accedente insuper ratione biquadrato distantiae reciproce proportionali. Verum quidem est, hanc aberrationem esse quam minimam; sed quoniam hic de accuratissima motuum Lunae cognitione sermo est, istam circumstantiam neutiquam praetermittere licuit.

§. 4.

Praeterea vero Luna non solum a viribus Terrae et Solis sollicitatur, sed etiam actioni reliquorum planetarum, praesertim Iouis et Veneris, subiecta deprehenditur; quo fit, vt ista inuestigatio non amplius in problemate de tribus corporibus contineatur; quin etiam si eueniat, vt quispiam Cometa non longe a Luna praeterlabatur, insignes certe perturbationes in eius motu oriri possent; cuiusmodi effectus in tabulas referre nullo modo liceret.

§. 5.

Ob has igitur rationes nunquam certe omnibus numeris absolutam motuum Lunae cognitionem sperare licet; quae scilicet omnes etiam quam minimas aberrationes adcurate definiret; ex quo nos contentos esse oportebit, si modo ad cognitionem vero proximam nobis pertingere licuerit. Hic autem felicissime vsu venit, vt omnes istae aberrationes tam sint exiguae, vt in loco Lunae vix ac ne vix quidem ad inte-

integrum minutum assurgere soleant; omnes autem Astronomi lubentissime tantillum errorem condonabunt, et is vtique plurimum praestitisse censendus est, qui tales tabulas lunares exhibere valuerit, quae nunquam a veritate vltra vnum minutum sint aberraturae; ac fortasse hos ipsos errores infra semi-minutum primum deprimere licebit.

§. 6.

His rationibus adducti hic tantum tria corpora, Solem scilicet et Terram cum Luna, sumus contemplaturi, quae quidem se mutuo secundum rationem inuersam quadratorum distantiae attrahant; atque hic etiam commodissime vsu venit, vt non solum massa Solis maxime superet vtramque massam Terrae et Lunae; sed etiam distantia Solis quasi infinities superet distantiam Lunae a Terra; quae binae circumstantiae nisi locum haberent, omnes certe conatus nostri in hac inuestigatione plane forent irriti.

§. 7.

Interim tamen massa Lunae, quae prae Terra neutiquam vt euanescens spectari potest, nostram inuestigationem non mediocriter esset perturbatura, nisi aliunde huic incommodo remedium adferri posset; at vero hanc quaestionem ita tractando immutare licet, vt massa Lunae penitus ex calculo remoueatur.

§. 8.

§. 8.

Quo hoc fieri possit, considerandum est, quod, si duo corpora se mutuo attrahentia circa commune centrum grauitatis, vtrumque in sua orbita, reuoluantur; eorum motus relatiuus perinde se sit habiturus, ac si vtriusque massa in centro vnius collecta quiescat, alterum vero omni inertia destitutum quasi simplex punctum circa illud reuolueretur. Ita si habeantur duo corpora in A et B, quorum massae iisdem litteris designentur, et quae circa commune centrum grauitatis O vtcunque moueantur; hunc motum facillime ita calculo determinabimus, vt in puncto *C* ambas massas A + B collectas concipiamus et nunc in motum puncti *b* inertia carentis inquiramus; huius quippe motus respectu C plane conueniet cum motu, quo corpus B circa alterum A ferri spectabitur, si modo distantia C *b* aequalis fuerit distantiae A B, atque initio punctum *b* parem motum acceperit, quo corpus B circa A suum motum inceperit. Namque si ad tempus quodcunque locus puncti *b* fuerit cognitus; posita distantia C *b* = *z* ducitur per centrum grauitatis O recta A O B ipsi C *b* parallela capianturque partes Fig. I.

$$OA = \frac{Bz}{A+B} \text{ et } OB = \frac{Az}{A+B}$$

quo facto erunt puncta A et B loca duorum nostrorum corporum pro eodem temporis momento.

§. 9.

§. 9.

Quo istam considerationem ad institutum nostrum transferamus, ante omnia notari conuenit, motum Lunae cum motu Terrae ita esse comparatum, vt commune centrum grauitatis in sectione conica circa Solem secundum regulas Keplerianas promoueatur; quanquam enim hoc non omni rigore veritati est consentaneum; tamen quoniam distantia Solis tantopere excedit distantiam Lunae a Terra, eiusque massa est quam maxima; ista suppositio tam parum a veritate recedet, vt errores inde oriundi pro nihilo haberi possent; praecipue cum scopus noster non ad summum praecisionis gradum dirigatur.

§. 10.

Fig. II. Hinc ergo repetamus inuestigationes nostras circa motum lunae, sitque curua A Θ sectio illa conica, quam commune centrum grauitatis Terrae et Lunae circa Solem in S positum describit; vocemus massam Solis $= S$; Terrae $= T$; Lunae $= L$ et in Θ concipiamus corpus, cuius massa $= T + L$ et cuius motus circa Solem sequatur regulas Keplerianas; deinde loco Lunae statuamus in A punctum seu corpusculum inertia destitutum, quod tam ad Solem, quam ad massam illam in Θ attrahatur in ratione reciproca duplicata distantiarum. Quo posito si valuerimus motum huius puncti A definire eiusque locum ad quoduis tempus

pus

pus assignare; facillime inde ipsum motum Lunae ex centro Terrae spectatum definire poterimus. Hunc in finem tantum opus est, rectam $A\Theta$ produci in T, ita, vt sit $\Theta T = \frac{L \cdot \Theta A}{T+L}$; deinde capiatur in eadem directione $LA = \Theta T$, siue $\Theta L = \frac{T \cdot \Theta A}{T+L}$; hocque modo T repraesentabit verum locum centri Terrae; L autem verum locum centri Lunae; manifestum autem est, distantiam TL aequalem fore distantiae ΘA et in eadem directione versari.

§. 11.

Hoc igitur modo non solum eum motum definiemus, quo Luna circa Terram ferri cernitur, sed etiam ipsum motum Terrae, quatenus a Luna attracta ab orbita Kepleriana deflectit, poterimus assignare, quandoquidem Terra non in puncto Θ secundum tabulas vulgares, sed in T reuera reperitur; atque in hoc ipso consistit effectus vis acceleratricis Lunae, qua motus Terrae perturbatur. Hinc autem facile intelligitur, istam motus Terrae perturbationem esse tam exiguam, vt fere penitus negligi possit, praecipue vbi quaestio de Luna instituitur. Vulgo massa Lunae 70ies minor aestimari solet, quam massa Terrae, vt sit $L = \frac{1}{70} \cdot T$; tum vero ex parallaxi Solis distantia $S\Theta$ 400cies superare inuenta est distantiam mediam Lunae a Terra; hinc interuallum $\Theta T = \frac{1}{71} \Theta \Lambda$. Ob hanc caussam longitudo Terrae maxime afficietur, dum Luna in quadraturis verſatur; tum enim ob angulum S Θ T rectum error in longitudine Solis aequalis erit angulo, cuius tangens $= \frac{\Theta T}{S \Theta} = \frac{1}{71.400} = \frac{1}{28400}$, qui angulus valet $7''. 3$. quem ergo errorem eo tutius in Luna negligi licet. Praeterea etiam hinc euidens eſt, ob actionem Lunae Terram aliquantillum de plano eclipticae depelli poſſe, quandoquidem eius diſtantia ab hoc plano circiter 70^{ies}. minor erit, quam diſtantia Lunae ab eodem. Quare ſi latitudo Lunae fuerit circiter $5° \frac{1}{4}$ ſiue eius diſtantia ab ecliptica $= \frac{1}{10}. \Theta \Lambda$; tum Terra indidem diſtabit interuallo $= \frac{1}{700} \Theta \Lambda$, quod per diſtantiam S Θ diuiſum dabit latitudinem, ſub qua Terra ex Sole videbitur; quae ergo erit $= \frac{1}{700.400}$; cumque ea ſit decies minor, quam ſuperior angulus $7''. 3$; ne ad integrum quidem minutum ſecundum aſcendere poterit.

§. 12.

Inprimis autem hic notari oportet, hoc modo ſemper locum Lunae geocentricum inueniri, ſeu coeli punctum, vbi ſpectator in centro terrae poſitus Lunam eſſet viſurus; tum vero inſuper diſtantia Lunae a centro Terrae ſemper aequalis erit illi $\Theta \Lambda$. Quocirca ſi nobis licuerit, motum puncti illius Λ ad puncta S et Θ attracti determinare; ſimul ſcopo nobis propoſito perfecte ſatisfecerimus, ſiquidem hinc ad quoduis tempus non ſolum locum Lunae geocentricum,

ſed

ſed etiam eius veram a centro Terrae diſtantiam aſſignare poterimus.

§. 13.

His omnibus probe perpenſis manifeſtum eſt, totum negotium, quod hic tractandum ſuſcepimus, ad ſolutionem ſequentis problematis reuocari.

Problema.

Si corpus in Θ, cuius maſſa $= T + L$, circa Solem in S, cuius maſſa $= S$, deſcribat eam ipſam ellipſem, quam Sol circa Terram percurrere videtur; tum vero ſi corpusculum quam minimum in Λ motum acceperit quemcunque et iugiter attrahatur ad corpora in Θ et S viribus notiſſimis $\frac{T+L}{\Theta\Lambda^2}$ et $\frac{S}{S\Lambda^2}$; quibus poſitis, quaeritur motus corpuſculi iſtius Λ, ita, vt inde ad quoduis tempus non ſolum eius locus Λ in coelo, ſed etiam diſtantia ΘΛ aſſignari queat.

Ad ſolutionem igitur huius problematis omnes vires intendamus, quemadmodum ex ſequentibus capitibus elucebit.

CAPUT II.

FORMULAE FUNDAMENTALES PRO MOTU LUNAE.

§. 14.

Constituto centro Solis in S, sit A ☉ orbita illa elliptica, quam commune centrum grauitatis Terrae et Lunae circa Solem esset descripturum; hic scilicet sumimus, planum tabulae referre planum eclipticae, extra quod vbicunque in Z reperiatur nunc quidem centrum Lunae, vel potius illius puncti, quod in locum Lunae substituimus. Ex puncto isto Z, in planum eclipticae cadat perpendiculum Z Y, atque ex Y ad axem positione fixum S A ducatur perpendicularis Y X, ita vt locus Lunae Z nobis determinetur per ternas coordinatas S X $= x$; X Y $= y$ et Y Z $= z$; vbi per se patet, rectam fixam S A ad punctum aequinoctiale vernum dirigi solere.

Fig. III.

§. 15.

Ducantur praeterea rectae S ☉; S Y; ☉ Y; ☉ Z et S Z; tum vero ex ☉ agatur ☉ a parallela axi S A, quae proinde etiam ad initium eclipticae ♈ tendit,

tendit, vnde longitudines computari solent. His positis patet, angulum $A S \Theta$ exhibere longitudinem Terrae ex Sole visae, seu potius centri illius communis grauitatis, quod loco Terrae substituimus; vicissim ergo longitudo Solis habebitur, si ad illum angulum $A S \Theta$ sex signa addantur. Porro autem angulus $Y \Theta Z$ dabit latitudinem Lunae et angulus $a \Theta Y$ longitudinem eius. Deinde vero vocemus distantiam $S \Theta = u$; angulum $A S \Theta = \Phi$; distantiam $S Z = v$ et denique distantiam $\Theta Z = w$. Demittatur nunc in axem $S A$ ex Θ perpendiculum ΘP; eritque

$$\Theta P = u . \sin. \Phi \text{ et } S P = u . \cos. \Phi$$

hincque, ob

$$P X = x - u \cos. \Phi, \text{ erit}$$

$$\Theta Y^2 = (x - u . \cos. \Phi)^2 + (y - u . \sin. \Phi)^2$$

et hinc

$$\Theta Z^2 = w^2 = z^2 + (x - u . \cos. \Phi)^2 + (y - u . \sin. \Phi)^2$$

Manifestum vero est, esse

$$S Y^2 = x^2 + y^2 \text{ et } S Z^2 = v^2 = z^2 + x^2 + y^2 .$$

§. 16.

Nunc igitur vt huc accommodemus principia mechanica, denotent, vt supra, S massam Solis, T massam Terrae et L massam Lunae; quoniam autem in Θ consideramus corpus ex massa Solis et Lunae compositum $= T + L$; statuamus breuitatis gratia $T + L = \Theta$, dum massa corporis in Z spectatur vt nulla. Iam perspicuum est, punctum Z immediate vrgeri a duabus viribus acceleratricibus, altera ad Solem secundum ZS, quae est $\frac{S}{v^2}$; altera vero ad Θ secundum $Z\Theta$, quae est $\frac{\Theta}{w^2}$. Quare si has vires resoluamus secundum directiones ternarum nostrarum coordinatarum, prior dat

pro directione XS vim $= \frac{Sx}{v^3}$

pro directione YX vim $= \frac{S.y}{v^3}$

pro directione ZY vim $= \frac{Sz}{v^3}$

Simili modo posterior vis praebet

pro directione XS vim $= \frac{\Theta(x - u.cos.\Phi)}{w^3}$

pro directione YX vim $= \frac{\Theta(y - u.sin.\Phi)}{w^3}$

pro directione ZY vim $= \frac{\Theta z}{w^3}$.

§. 17.

Deinde quanquam massa Solis est quasi infinita respectu massae Θ, ita, vt Sol a vi, qua ad Θ attrahitur, nullum sensibilem motum accipiat; tamen secundum praecepta mechanica vim, qua S ad Θ attrahitur, quae est $= \frac{\Theta}{u^2}$; contrario modo in punctum Z transferamus, ita, vt hoc punctum praeter vires iam memoratas insuper sollicitari sit censendum secuncundum PS vi $= \frac{\Theta \cos.\Phi}{u^2}$ et fecundum ΘP vi $= \frac{\Theta.\sin.\Phi}{u^2}$; ficque omnino punctum Z follicitabitur tribus viribus fequentibus

I°. fecundum XS vi $= \frac{Sx}{v^3} + \frac{\Theta(x - u.\cos.\Phi)}{w^3} + \frac{\Theta.\cos.\Phi}{u^2}$

II°. fecundum YX vi $= \frac{Sy}{v^3} + \frac{\Theta(y - u.\sin.\Phi)}{w^3} + \frac{\Theta.\sin.\Phi}{u^2}$

III°. fecundum ZY vi $= \frac{Sz}{v^3} + \frac{\Theta z}{w^3}$.

§. 18.

Defignet nunc $d\tau$ elementum temporis, in formulis differentio-differentialibus pro conftanti habendum, atque vires ternae modo memoratae fecundum nota principia motus tres fequentes fuppeditabunt aequationes:

I°. $\frac{\alpha ddx}{d\tau^2} + \frac{Sx}{v^3} + \frac{\Theta(x - u.\cos.\Phi)}{w^3} + \frac{\Theta.\cos.\Phi}{u^2} = 0.$

II°. $\frac{\alpha ddy}{d\tau^2} + \frac{Sy}{v^3} + \frac{\Theta(y - u.\sin.\Phi)}{w^3} + \frac{\Theta.\sin.\Phi}{u^2} = 0.$

III°. $\frac{\alpha ddz}{d\tau^2} + \frac{Sz}{v^3} + \frac{\Theta z}{w^3} = 0.$

atque his tribus aequationibus totus Lunae motus determinatur, vbi α certam quantitatem conftantem denotat, a ratione, qua tempus τ definire libuerit, pendentem, vti mox declarabitur. Totum ergo negotium iam huc eft reductum, vt tres iftae aequationes integrentur; verum ne minima quidem fpes nobis affulget, hoc vnquam praeftandi; quare earum integrationem ne quidem tentabimus.

§. 19.

§. 19.

Quin etiamsi vnquam eo vsque perueniremus, vt harum aequationum integralia assignare liceret, aut si quis genius ea nobis reuelaret; inde profecto ne minimum quidem fructum essemus percepturi; tantopere enim sine dubio ternae istae incognitae x, y et z inter se forent permixtae, vt nulla plane via excogitari posset, qua valores singularum exhibere liceret.

CAPVT III.

CAPVT III.

CONSIDERATIO ADCVRATIOR MOTVS TERRAE SEV CORPORIS IN ⊖ CONCEPTI.

§. 20.

Plurimum interest, istam considerationem de motu terrae praemitti, antequam ipsum Lunae motum adgrediamur. Non solum enim necesse est, valores litterarum u et Φ adcuratius definiri, sed etiam inde idoneam et ad nostrum institutum adcommodatam temporis rationem consequemur, vnde simul constans illa a determinetur.

§. 21.

Formulas autem ante inuentas facile ad hunc casum adplicare poterimus, remouendo corpusculum in Z positum, ita, vt tantum corpus ⊖ ad S attrahatur atque vt punctum S in quiete conseruetur, vtraque massa $S + \Theta$ in puncto S collecta intelligatur. Hinc positis pro hoc casu binis coordinatis $SP = X$ et $P\Theta = Y$, statim habebimus has duas aequationes

$$\text{I}^\circ.\ \frac{\alpha\, ddX}{d\tau^2} + \frac{(S+\Theta)X}{u^3} = 0.$$

$$\text{II}^\circ.\ \frac{\alpha\, ddY}{d\tau^2} + \frac{(S+\Theta)Y}{u^3} = 0.$$

Verum cum sit $X = u \operatorname{cof.} \Phi$ et $Y = u \operatorname{fin.} \Phi$; habebimus

$$X . \operatorname{cof.} \Phi + Y . \operatorname{fin.} \Phi = u \text{ et}$$
$$X . \operatorname{fin.} \Phi - Y . \operatorname{cof.} \Phi = 0.$$

Hae formulae differentiatae dant

$$dX . \operatorname{cof.} \Phi + dY . \operatorname{fin.} \Phi - d\Phi (X \operatorname{fin} \Phi - Y . \operatorname{cof.} \Phi) = du$$
$$\text{fiue } dX . \operatorname{cof} \Phi + dY . \operatorname{fin.} \Phi = du$$

tum vero

$$dX . \operatorname{fin.} \Phi - dY . \operatorname{cof.} \Phi + d\Phi (X \operatorname{cof.} \Phi + Y . \operatorname{fin.} \Phi) = 0$$
$$\text{fiue } dX . \operatorname{fin.} \Phi - dY \operatorname{cof.} \Phi + u\, d\Phi = 0.$$

Hae iam formulae denuo differentiatae dant

$$ddX \operatorname{cof.} \Phi + ddY \operatorname{fin.} \Phi + d\Phi (dY \operatorname{cof.} \Phi - dX \operatorname{fin.} \Phi) = ddu$$
$$\text{fiue } ddX \operatorname{cof.} \Phi + ddY \operatorname{fin.} \Phi + u\, d\Phi^2 = ddu$$

et

$$ddX \operatorname{fin.} \Phi - ddY \operatorname{cof.} \Phi + d\Phi (dX \operatorname{cof.} \Phi + dY \operatorname{fin.} \Phi)$$
$$+ du\, d\Phi + u\, dd\Phi = 0.$$

fiue

$$ddX \operatorname{fin} \Phi - ddY \operatorname{cof.} \Phi + 2\, du\, d\Phi + u\, dd\Phi = 0.$$

§. 22.

Cum autem ex nostris aequationibus differentialibus fit

ddX

$$ddX = \frac{-(S+\Theta)X d\tau^2}{\alpha u^3} \text{ et } ddY = \frac{-(S+\Theta)Y d\tau^2}{\alpha u^3}$$

substituantur hi valores in formulis modo inuentis ac prior quidem dabit

$$\frac{-(S+\Theta)d\tau^2}{\alpha u^3}(X\cos.\Phi + Y\sin.\Phi) + ud\Phi^2 = ddu$$

siue

$$\frac{-(S+\Theta)d\tau^2}{\alpha u^2} + ud\Phi^2 = ddu$$

posterior vero

$$\frac{-(S+\Theta)d\tau^2}{\alpha u^3}(X\sin.\Phi - Y\cos.\Phi) + 2dud\Phi + udd\Phi = 0$$

siue

$$2dud\Phi + udd\Phi = 0.$$

Quocirca binae nostrae aequationes problema soluentes erunt

$$\text{I}^\circ.\ \frac{\alpha(ddu - ud\Phi^2)}{d\tau^2} = \frac{-(S+\Theta)}{u^2}.$$

$$\text{II}^\circ.\ \frac{\alpha(udd\Phi + 2dud\Phi)}{d\tau^2} = 0.$$

§. 23.

Quamuis non difficile foret, has aequationes integrare; tamen ad nostrum institutum respicientes huic negotio non immorabimur, sed id sequenti modo adgrediemur. Quia iam constat, orbitam esse ellipsin parum excentricam, sufficiet nobis ea tantum membra proferre, quae per simplicem excentricitatem sunt affecta, neglectis iis membris, in quibus quadratum altiorue potestas excentricitatis occurreret. Hunc in finem quoniam ratio temporis τ adhuc est indefinita, denotet tam hic, quam in posterum perpetuo, littera t anomaliam mediam solis proposito tempori conuenientem, quippe quae tempori est proportionalis, ideoque vti sumsimus, $ddt = 0$. Deinde statuatur distantia media Terrae a Sole $= 1$; ipsa vero excentricitas $= \varkappa$, vbi constat ex obseruationibus esse $\varkappa = 0,01678$; quo posito ex cognitis regulis Kepleri nouimus esse $u = 1 + \varkappa.\, \text{cos}.\, t$. Deinde porro ponendo longitudinem terrae mediam $= \zeta$; nouimus pariter esse $\Phi = \zeta - 2\varkappa\, \text{sin}.\, t$; quam proxime autem erit $d\zeta = dt$; vnde fit

$$\frac{du}{dt} = -\varkappa.\, \text{sin}.\, t \quad \text{et} \quad \frac{ddu}{dt^2} = -\varkappa\, \text{cos}.\, t.$$

Simili quoque modo erit

$$\frac{d\Phi}{dt} = 1 - 2\varkappa\, \text{cos}.\, t \quad \text{et} \quad \frac{dd\Phi}{dt^2} = 2\varkappa.\, \text{sin}.\, t.$$

§. 24.

Substituamus hos valores in nostris aequationibus inuentis atque non solum consensum perspiciemus, sed etiam constantem α eiusque rationem ad massam $S + \Theta$ cognoscemus, in quo ipso cardo rei versatur. Tum autem prior aequatio sequentem induet formam, reiectis terminis quadratum excentricitatis inuoluentibus,

$$\alpha(2\varkappa\, \text{cos}.\, t - 1) = \frac{-(S+\Theta)}{2\varkappa\, \text{cos}.\, t + 1} = (S+\Theta)(2\varkappa\, \text{cos}.\, t - 1)$$

adeoque $\alpha = S + \Theta$.

Altera

Altera vero aequatio sponte fit identica. Tantum igitur hic sumus lucrati, vt, cum deinceps perpetuo tempus τ per anomaliam mediam Solis exprimamus, vbique loco constantis α valorem determinatum $S + \Theta$ scribere possimus, dum scilicet distantia media Solis a Terra vnitate designatur. His ergo positis ambae aequationes superiores pro motu Terrae inuentae erunt

$$\text{I}^{\circ}.\ \frac{ddu - ud\Phi^2}{dt^2} = \frac{-1}{u^2};$$

$$\text{II}^{\circ}.\ \frac{udd\Phi + 2dud\Phi}{dt^2} = 0.$$

quae aequationes adeo in genere locum habent, quantacunque fuerit excentricitas $\varkappa$. Quando autem excentricitas $\varkappa$ est valde parua, cum fit

$$u = 1 + \varkappa \operatorname{cof.} t; \text{ erit } u^2 = 1 + 2\varkappa \operatorname{cof.} t;$$

$$\frac{1}{u} = 1 - \varkappa \operatorname{cof.} t; \ \frac{1}{u^2} = 1 - 2\varkappa \operatorname{cof.} t.$$

$$\frac{du}{dt} = -\varkappa. \operatorname{fin.} t; \ \frac{ddu}{dt^2} = -\varkappa. \operatorname{cof.} t.$$

Porro

$$\Phi = \zeta - 2\varkappa \operatorname{fin.} t; \ \frac{d\Phi}{dt} = 1 - 2\varkappa \operatorname{cof.} t \text{ et}$$

$$\frac{dd\Phi}{dt^2} = 2\varkappa. \operatorname{fin.} t.$$

Vnde concludimus, fore

$$\frac{u^2 d\Phi}{dt} = 1 - 4\varkappa^2. \operatorname{cof.} t^2 = 1.$$

Vtique autem in genere valor huius formulae debet esse constans; cum enim posterior aequatio fit

$$\frac{udd\Phi + 2dud\Phi}{dt^2} = 0$$

si ea per u multiplicata integretur, prodit

$$\frac{u^2 d\Phi}{dt} = \text{Const.}$$

§. 25.

Reuertamur nunc ad Lunam et seruata temporis mensura ex anomalia media Solis depromta, dum distantia media Solis a Terra vnitate exprimitur, habebimus iterum $a = S + \Theta$; vnde si nostras aequationes per $a = S + \Theta$ diuidamus et breuitatis gratia ponamus $\frac{S}{S+\Theta} = \mu$; et $\frac{\Theta}{S+\Theta} = \nu$. ita, vt sit $\mu + \nu = 1$. tres nostrae aequationes motum Lunae continentes sequenti modo se habebunt:

$$\text{I}^\circ.\ \frac{ddx}{dt^2} + \mu . \frac{x}{v^3} + \nu . \frac{(x - u\,\text{cos}.\Phi)}{w^3} + \nu . \frac{\text{cos}.\Phi}{u^2} = 0.$$

$$\text{II}^\circ.\ \frac{ddy}{dt^2} + \mu . \frac{y}{v^3} + \nu . \frac{(y - u.\,\text{sin}.\Phi)}{w^3} + \nu . \frac{\text{sin}.\Phi}{u^2} = 0.$$

$$\text{III}^\circ.\ \frac{ddz}{dt^2} + \mu . \frac{z}{v^3} + \nu . \frac{z}{w^3} = 0.$$

vbi cum massa Solis sit quam maxima prae Θ; euidens est, litteram μ quam minime ab vnitate deficere; litteram vero ν fractionem fore vehementer exiguam.

CAPVT IV.

CAPVT IV.

TRANSFORMATIO GENERALIS FORMVLARVM INVENTARVM.

§. 26.

Quoniam in hoc calculo binae coordinatae x et y per totam orbis magni extensionem variari possunt, eae ad adproximationes minus aptae sunt censendae; ex quo necesse est, vt nostras aequationes ad alias coordinatas reuocemus. Quod quo facilius fieri possit, ducamus ex puncto Θ, quippe ad quod motum Lunae referre conuenit, in plano eclipticae rectam quamcunque Θb et vtcunque variabilem, quam loco axis accipiamus et statuamus angulum $a\Theta b = \omega$. Iam ex puncto Y ad hunc axem Θb ducamus normalem Yx vocemusque has nouas coordinatas $\Theta x = X$; $xY = Y$ et tertiam $YZ = Z$; ita, vt sit $z = Z$ atque perspicuum est, fore

$$\Theta Z = w = \sqrt{(X^2 + Y^2 + Z^2)}.$$

§ 27.

Videamus nunc etiam, quomodo hae nouae coordinatae ad praecedentes referantur, vbi quidem attendenti mox patebit, fore

$x =$

$$x = u.\cos.\phi + X.\cos.\omega - Y.\sin.\omega$$

$$y = u.\sin.\phi + X.\sin.\omega + Y.\cos.\omega$$

existente $z = Z$.

Cum igitur ante fuisset $SZ^2 = v^2 = x^2 + y^2 + z^2$ fiet nunc per nouas nostras coordinatas

$$v^2 = u^2 + 2uX(\cos.\phi\cos.\omega + \sin.\phi.\sin.\omega) - 2uY(\cos.\phi\sin.\omega - \sin.\phi.\cos.\omega) + X^2 + Y^2 + Z^2$$

ex quo, si breuitatis gratia ponatur $\omega - \phi = \psi$; obtinebitur

$$v^2 = u^2 + 2uX\cos.\psi - 2uY\sin.\psi + X^2 + Y^2 + Z^2.$$

§. 28.

Quo vero facilius has nouas coordinatas in aequationes nostras differentiales introducamus, notemus esse

$$x\cos.\omega + y.\sin.\omega = u.\cos.\psi + X. \text{ et}$$

$$x\sin.\omega - y.\cos.\omega = u.\sin.\psi - Y,$$

quarum formularum si differentietur prima, prodit

$$dx.\cos.\omega + dy.\sin.\omega = d\omega(x\sin.\omega - y.\cos.\omega) + du.\cos.\psi - ud\psi\sin.\psi + dX.$$

Siue

$$dx.\cos.\omega + dy.\sin.\omega = d\omega(u.\sin.\psi - Y) + du.\cos.\psi - ud\psi\sin.\psi + dX.$$

Cum

Cum vero sit $d\psi = d\omega - d\Phi$, orit

$$dx.\cos.\omega + dy.\sin.\omega = du.\cos.\psi + ud\Phi.\sin.\psi + dX - Yd\omega.$$

Simili modo altera formula dabit

$$dx.\sin.\omega - dy.\cos.\omega = -d\omega(x.\cos.\omega + y.\sin.\omega) + du.\sin.\psi + ud\psi\cos.\psi - dY$$

siue

$$dx.\sin.\omega - dy.\cos.\omega = du.\sin.\psi - ud\Phi\cos.\psi - Xd\omega - dY.$$

§. 29.

Nunc porro formulas modo inuentas denuo differentiemus, ac prior quidem dabit

$$ddx\cos.\omega + ddy.\sin.\omega = ddu.\cos.\psi + du.\sin.\psi(d\Phi - d\psi) + udd\Phi.\sin.\psi + ud\Phi d\psi\cos.\psi - d\omega(dy.\cos.\omega - dx.\sin.\omega + dY) - Ydd\omega + ddX.$$

in qua formula si ponatur $d\psi = d\omega - d\Phi$, obtinetur

$$ddx.\cos.\omega + ddy.\sin.\omega = (ddu - ud\Phi^2)\cos.\psi + (udd\Phi + 2dud\Phi)\sin.\psi - d\omega\left\{\begin{matrix} du.\sin.\psi - ud\Phi\cos.\psi \\ + dy.\cos.\omega - dx.\sin.\omega \\ + dY \end{matrix}\right\} - Ydd\omega + ddX.$$

In praecedentibus autem inuenimus, esse

$$X d\omega + dY = du.\text{sin.}\psi - u d\Phi \text{cos.}\psi + dy.\text{cos.}\omega - dx.\text{sin.}\omega$$

et §. 24. demonstrauimus, esse

$$ddu - u d\Phi^2 = -\frac{dt^2}{u^2} \text{ et}$$

$$udd\Phi + 2 du d\Phi = 0.$$

quibus valoribus substitutis prodit

$$ddx.\text{cos.}\omega + ddy.\text{sin.}\omega = \frac{-dt^2.\text{cos.}\psi}{u^2} + ddX - 2dY d\omega - Y dd\omega - X d\omega^2.$$

Simili modo altera illa formula denuo differentiata praebet

$$ddx.\text{sin.}\omega - ddy.\text{cos.}\omega = (ddu - ud\Phi^2)\text{sin.}\psi - (udd\Phi + 2dud\Phi)\text{cos.}\psi - ddY - 2dXd\omega - Xdd\omega + Yd\omega^2$$

siue

$$ddx.\text{sin.}\omega - ddy.\text{cos.}\omega = \frac{-dt^2.\text{sin.}\psi}{u^2} - ddY - 2dXd\omega - Xdd\omega + Yd\omega^2$$

§. 30.

Nunc ipsas nostras aequationes differentiales §. 25. allatas simili modo combinemus, ac primo quidem haec combinatio.

I. cos.

$$\text{I.}\cos\omega + \text{II.}\sin\omega$$

producet

$$\frac{ddx.\cos.\omega + ddy.\sin.\omega}{dt^2} + \mu\frac{(u\cos.\psi + X)}{v^3} + \frac{\nu X}{w^3} + \frac{\nu.\cos.\psi}{u^2} = 0.$$

Deinde haec combinatio

$$\text{I.}\sin.\omega - \text{II.}\cos.\omega$$

dat

$$\frac{ddx.\sin.\omega - ddy.\cos.\omega}{dt^2} + \mu\frac{(u.\sin.\psi - Y)}{v^3} - \frac{\nu Y}{w^3} + \frac{\nu.\sin.\psi}{u^2} = 0.$$

Hic igitur in membris prioribus formulas modo inventas substituamus, vt pro nouis nostris coordinatis obtineamus sequentes aequationes:

$$\text{I}^{\circ}.\ \frac{-\cos.\psi}{u^2} + \frac{ddX}{dt^2} - \frac{2dYd\omega}{dt^2} - \frac{Ydd\omega}{dt^2} - \frac{Xd\omega^2}{dt^2} + \frac{\mu(u.\cos.\psi + X)}{v^3} + \frac{\nu X}{w^3} + \frac{\nu.\cos.\psi}{u^2} = 0.$$

$$\text{II}^{\circ}.\ \frac{-\sin.\psi}{u^2} - \frac{ddY}{dt^2} - \frac{2dXd\omega}{dt^2} - \frac{Xdd\omega}{dt^2} + \frac{Yd\omega^2}{dt^2} + \frac{\mu(u.\sin.\psi - Y)}{v^3} - \frac{\nu Y}{w^3} + \frac{\nu.\sin.\psi}{u^2} = 0.$$

Tertia vero aequatio ob $z = Z$ nullam alterationem patitur eritque

$$\text{III}^{\circ}.\ \frac{ddZ}{dt^2} + \frac{\mu.Z}{v^3} + \frac{\nu Z}{w^3} = 0.$$

§. 31.

Mutemus nunc signa in aequatione secunda, et singulos terminos sequenti modo disponamus, vt pro motu Lunae inter nouas nostras coordinatas X, Y et Z sequentes ternas obtineamus aequationes:

$$\text{I}^{\circ}.\quad \frac{ddX}{dt^2} - \frac{2dYd\omega}{dt^2} - \frac{Xd\omega^2}{dt^2} - \frac{Ydd\omega}{dt^2}$$
$$- \frac{\mu . \cos . \psi}{u^2} + \frac{\mu (u . \cos . \psi + X)}{v^3} + \frac{\nu X}{w^3} = 0.$$

$$\text{II}^{\circ}.\quad \frac{ddY}{dt^2} + \frac{2dXd\omega}{dt^2} - \frac{Yd\omega^2}{dt^2} + \frac{Xdd\omega}{dt^2}$$
$$+ \frac{\mu . \sin . \psi}{u^2} - \frac{\mu (u . \sin . \psi - Y)}{v^3} + \frac{\nu Y}{w^3} = 0.$$

$$\text{III}^{\circ}.\quad \frac{ddZ}{dt^2} + \frac{\mu Z}{v^3} + \frac{\nu Z}{w^3} = 0.$$

hic scilicet loco $1 - \nu$ scripsimus μ, siquidem, vti iam est notatum, $\nu + \mu = 1$.

CAPVT V.

REDVCTIO COORDINATARVM PRAECEDENTIVM AD LONGITVDINEM LVNAE MEDIAM.

§. 32.

Accipiamus nunc axem $\Theta\, b$ ita, vt ad longitudinem Lunae mediam dirigatur et cum recta $\Theta\, a$ ad initium eclipticae sit directa, angulus $a\,\Theta\, b = \omega$ designabit ipsam longitudinem mediam Lunae, quae quia tempori t est proportionalis, habebimus statim $\frac{dd\omega}{dt^2} = 0$; vnde ternae nostrae aequationes pro coordinatis X, Y et Z erunt sequentes:

$$\text{I}^\circ.\ \frac{ddX}{dt^2} - \frac{2dYd\omega}{dt^2} - \frac{Xd\omega^2}{dt^2} + \frac{\mu\cos.\psi}{u^2}$$
$$+ \frac{\mu(u\cos.\psi + X)}{v^3} + \frac{\nu X}{w^3} = 0.$$

$$\text{II}^\circ.\ \frac{ddY}{dt^2} + \frac{2dXd\omega}{dt^2} - \frac{Yd\omega^2}{dt^2} + \frac{\mu\sin.\psi}{u^2}$$
$$- \frac{\mu(u\sin.\psi - Y)}{v^3} + \frac{\nu Y}{w^3} = 0.$$

$$\text{III}^\circ.\ \frac{ddZ}{dt^2} + \frac{\mu Z}{v^3} + \frac{\nu Z}{w^3} = 0.$$

§. 33.

Hic autem ante omnia notemus, esse $w^2 = X^2 + Y^2 + Z^2$ et $v^2 = u^2 + 2u(X \cos. \psi + Y \sin \psi) + X^2 + Y^2 + Z^2$. Deinde vero ob motum Terrae cognitum, cuius distantiam mediam a Sole statuimus $= 1$, et excentricitatem orbitae $= \varkappa$, neglectis potestatibus ipsius $\varkappa$, si anomalia media Terrae vel Solis ponatur $= t$, vidimus esse $u = 1 + \varkappa \cos. t$, hincque $u^2 = 1 + 2\varkappa \cos. t$ et $\frac{1}{u^2} = 1 - 2\varkappa \cos. t$; ac $\frac{1}{u^3} = 1 - 3\varkappa \cos. t$. Praeterea vero posuimus longitudinem mediam Terrae $= \zeta$; veram vero $= \Phi$, atque ostendimus esse $\Phi = \zeta - 2\varkappa. \sin. t$; vnde angulus ψ ita definietur, vt sit $\psi = \omega - \zeta + 2\varkappa \sin. t$ vbi cum terminus $\varkappa. \sin. t$ sit tam paruus, vt quadratum ipsius $\varkappa$ cum altioribus potestatibus negligere liceat, adipiscimur sequentes valores

$$\sin. \psi = \sin. (\omega - \zeta) + 2\varkappa. \sin\, t. \cos. (\omega - \zeta)$$
$$\cos. \psi = \cos. (\omega - \zeta) - 2\varkappa. \sin. t. \sin. (\omega - \zeta).$$

§. 34.

Angulus autem iste $\omega - \zeta$ exprimit longitudinem mediam Lunae, demta longitudine media Terrae ex Sole visae; in computo autem astronomico solemus considerare longitudinem mediam Solis ex Terra visi, quae si ponatur $= \vartheta$; erit $\zeta = \vartheta \pm 180°$; ideoque angulus $\omega - \zeta = \omega - \vartheta \mp 180°$; ex quo sequitur $\sin. (\omega - \zeta) = - \sin. (\omega - \vartheta)$; et cosin

$(\omega - \zeta)$

$(\omega - \zeta) = -\cos.(\omega - \vartheta)$; quocirca postremae nostrae formulae euadent

$$\sin.\psi = -\sin.(\omega - \vartheta) - 2\varkappa \sin. t \cos.(\omega - \vartheta)$$

$$\cos.\psi = -\cos.(\omega - \vartheta) + 2\varkappa \sin. t \sin.(\omega - \vartheta).$$

§. 35.

Ponamus nunc angulum $\omega - \vartheta = p$, qui ergo inuenitur, si a longitudine media Lunae ω subtrahatur longitudo media Solis ϑ; qui angulus cum sit tempori proportionalis, statuamus $\frac{dp}{dt} = m$; vnde ob $d\vartheta = dt$, siquidem hic lentissimum motum apogaei Solis negligere licet, sequitur, fore

$$\frac{d\omega}{dt} = \frac{dp}{dt} + \frac{dt}{dt} = m + 1.$$

Sic ergo eliso differentiali $d\omega$ ternae nostrae aequationes erunt

$$\text{I}^\circ.\ \frac{ddX}{dt^2} - \frac{2(m+1)dY}{dt} - (m+1)^2 X - \frac{\mu\cos.\psi}{u^2} + \frac{\mu(u\cos.\psi + X)}{v^3} + \frac{\nu X}{w^3} = 0.$$

$$\text{II}^\circ.\ \frac{ddY}{dt^2} + \frac{2(m+1)dX}{dt} - (m+1)^2 Y + \frac{\mu\sin.\psi}{u^2} - \frac{\mu(u\sin.\psi - Y)}{v^3} + \frac{\nu Y}{w^3} = 0.$$

$$\text{III}^\circ.\ \frac{ddZ}{dt^2} + \frac{\mu Z}{v^3} + \frac{\nu Z}{w^3} = 0.$$

vbi

vbi ex obſeruationibus conſtat, eſſe $m = 12,3689530$. Porro vero introducto hoc angulo $p = \omega - \vartheta$ habebimus

$$\text{ſin.}\, \psi = -\,\text{ſin.}\, p - 2\,\varkappa.\, \text{ſin.}\, t.\, \text{coſ.}\, p$$

$$\text{coſ.}\, \psi = -\,\text{coſ.}\, p + 2\,\varkappa\, \text{ſin.}\, t.\, \text{ſin.}\, p$$

ita, vt iam in noſtris formulis anguli ζ vel ϑ et ω amplius plane non occurrant, ſed coordinatae noſtrae per ſolos angulos tempori proportionales p et t determinentur.

CAPVT VI.

EVOLVTIO TERMINORVM PER v DIVISORVM.

§. 36.

Quoniam axis noster ⊖ b cum longitudine media Lunae congruit, euidens eſt, adplicatas Y nunquam vltra certos ſatisque arctos limites extravagari poſſe; tum vero etiam abſciſſa X non vltra certos terminos digredietur; tertia vero ordinata Z per ſe intra ſatis arctos limites continetur. Inter has quidem maxima ſemper erit abſciſſa X, quae tamen nihilominus ſemper vehementer exigua erit reſpectu diſtantiae S ⊖ $= u$. Quamobrem cum inuenerimus $v^2 = u^2 + 2u(X \cos\psi - Y \sin\psi) + X^2 + Y^2 + Z^2$, vbi quidem angulum ψ relinquimus, quoniam ſufficit, valores ante aſſignatos pro ſin. ψ et coſ. ψ noſſe; manifeſtum eſt, in hac expreſſione primum terminum u^2 maxime ſuperare ſecundum membrum

$$2u(X \cos\psi - Y. \sin\psi)$$

hocque ipſum multum ſuperare tertium $X^2 + Y^2 + Z^2$.

§. 37.

Ob has rationes formulam $\frac{1}{v^3}$ commodissime in seriem maxime conuergentem euoluere licebit, quod quo clarius perspiciatur, ponamus breuitatis gratia

$$v^2 = u^2 + 2\,P\,u + Q, \text{ ita, vt sit}$$

$$P = X\,\text{cos.}\,\psi - Y.\,\text{sin.}\,\psi \text{ et } Q = X^2 + Y^2 + Z^2$$

Pro euolutione ergo cum sit

$$\frac{1}{v^3} = (u^2 + 2\,P\,u + Q)^{-\frac{3}{2}};$$

regula notissima praebet

$$\frac{1}{v^3} = \frac{1}{u^3} - \frac{3(2Pu + Q)}{2.u^5} + \frac{3.5(2Pu + Q)^2}{2.4.u^7},$$

neque enim vlterius progredi necesse est.

Hinc ergo singula membra euoluendo consequimur

$$\frac{1}{v^3} = \frac{1}{u^3} - \frac{3P}{u^4} + \frac{15P^2 - 3Q}{2u^5}$$

vbi si loco P et Q valores assumti substituantur, colligimus, fore

$$\frac{1}{v^3} = \frac{1}{u^3} - \frac{3(X\,\text{cos.}\,\psi - Y.\,\text{sin.}\,\psi)}{u^4} + \frac{3X^2(5.\text{cos.}\,\psi^2 - 1)}{2.u^5}$$
$$- \frac{15.XY\,\text{sin.}\,\psi.\,\text{cos.}\,\psi}{u^5} + \frac{3Y^2(5.\text{sin.}\,\psi^2 - 1)}{2.u^5} - \frac{3Z^2}{2.u^5}$$

vbi quidem in secunda dimensione ipsarum X, Y et Z subsistimus.

§ 38.

Subſtituamus ergo hanc expreſſionem in ſingulis noſtris aequationibus et quoniam in prima aequatione occurrit membrum $\frac{\mu(u.\cos.\psi+X)}{v^3}$, ex duabus partibus conſtans, vtramque ſeorſim euoluamus; eritque

$$\frac{\mu u.\cos.\psi}{v^3} = \frac{\mu.\cos.\psi}{u^2} - \frac{3\mu X\cos.\psi^2}{u^3} + \frac{3\mu Y\sin.\psi.\cos.\psi}{u^3}$$
$$+ \frac{3\mu X^2.\cos.\psi(5.\cos.\psi^2-1)}{2.u^4} - \frac{15.\mu XY.\sin.\psi.\cos.\psi^2}{u^4}$$
$$+ \frac{3\mu Y^2.\cos.\psi(5.\sin.\psi^2-1)}{2.u^4} - \frac{3\mu Z^2.\cos.\psi}{2u^4}$$

ſimili modo erit

$$\frac{\mu X}{v^3} = \frac{\mu X}{u^3} - \frac{3\mu X^2\cos.\psi}{u^4} + \frac{3\mu XY.\sin.\psi}{u^4}.$$

adeoque

$$\frac{\mu(u.\cos.\psi+X)}{v^3} = \frac{\mu.\cos.\psi}{u^2} - \frac{\mu X(3.\cos.\psi^2-1)}{u^3}$$
$$+ \frac{3\mu.Y.\sin.\psi.\cos.\psi}{u^3} + \frac{3\mu X^2\cos.\psi(5.\cos.\psi^2-3)}{2u^4}$$
$$- \frac{3\mu.XY.\sin.\psi(5\cos.\psi^2-1)}{u^4}$$
$$+ \frac{3\mu.Y^2\cos.\psi(5.\sin.\psi^2-1)}{2.u^4} - \frac{3.\mu.Z^2.\cos.\psi}{2.u^4}$$

quo valore in prima aequatione ſubſtituto, prodit

Prima Aequatio.

$$\frac{ddX}{dt^2} - \frac{2(m+1)dY}{dt} - (m+1)^2 X + \frac{\nu X}{w^3}$$
$$- \frac{\mu X(3.\cos.\psi^2 - 1)}{u^3} + \frac{3.\mu Y.\sin.\psi.\cos.\psi}{u^3}$$
$$+ \frac{3.\mu X^2.\cos.\psi(5.\cos.\psi^2 - 3)}{2.u^4} - \frac{3\mu.XY.\sin.\psi(5.\cos.\psi^2 - 1)}{u^4}$$
$$+ \frac{3\mu Y^2.\cos.\psi(5.\sin.\psi^2 - 1)}{2.u^4} - \frac{3\mu Z^2.\cos.\psi}{2u^4} = 0.$$

§. 39.

Simili modo hanc substitutionem in secunda aequatione faciamus, in qua occurrit hoc membrum $-\frac{\mu(u\sin.\psi - Y)}{v^3}$, quod per partes euoluamus.

$$-\frac{\mu.u.\sin.\psi}{v^3} = -\frac{\mu.\sin.\psi}{u^2} + \frac{3\mu X\sin.\psi.\cos.\psi}{u^3}$$
$$- \frac{3\mu Y\sin.\psi^2}{u^3} - \frac{3\mu X^2\sin.\psi(5.\cos.\psi^2 - 1)}{2u^4}$$
$$+ \frac{15.\mu XY\sin.\psi^2.\cos.\psi}{u^4} - \frac{3\mu Y^2\sin.\psi(5.\sin.\psi^2 - 1)}{2.u^4}$$
$$+ \frac{3\mu Z^2\sin.\psi}{2u^4}$$

atque simili modo

$$\frac{\mu Y}{v^3} = \frac{\mu Y}{u^3} - \frac{3\mu XY.\cos.\psi}{u^4} + \frac{3\mu Y^2\sin.\psi}{u^4}$$

adeoque totum membrum

$$-\frac{\mu(u\sin.\psi - Y)}{v^3} = -\frac{\mu.\sin.\psi}{u^2} + \frac{3\mu X\sin.\psi\cos.\psi}{u^3}$$
$$+ \frac{\mu Y(3.\sin.\psi^2 - 1)}{u^3} - \frac{3\mu X^2\sin.\psi(5.\cos.\psi^2 - 1)}{2.u^4}$$
$$+ \frac{3\mu XY\cos.\psi(5.\sin.\psi^2 - 1)}{u^4} - \frac{3\mu Y^2.\sin.\psi(5.\sin.\psi^2 - 3)}{2.u^4}$$
$$+ \frac{3.\mu.Z^2\sin.\psi}{2.u^4}$$

atque

atque hinc habebitur

Secunda Aequatio.

$$\frac{ddY}{dt^2} + \frac{2(m+1)\,dX}{dt} - (m+1)^2 Y + \frac{\nu Y}{w^3}$$
$$+ \frac{3\mu X \sin.\psi.\cos.\psi}{u^3} - \frac{\mu Y(3.\sin.\psi^2 - 1)}{u^3}$$
$$- \frac{3\mu X^2.\sin.\psi(5.\cos.\psi^2 - 1)}{2.u^4} + \frac{3.\mu XY\cos.\psi(5.\sin.\psi^2 - 1)}{u^4}$$
$$- \frac{3\mu Y^2\sin.\psi(5.\sin.\psi^2 - 3)}{2.u^4} + \frac{3\mu Z^2.\sin.\psi}{2.u^4} = 0.$$

§. 40.

Tertia aequatio nulla prorsus laborat difficultate, cum terminus per v^3 diuisus fit $\frac{\mu Z}{v^3}$; ex quo ita se habet

Tertia Aequatio.

$$\frac{ddZ}{dt^2} + \frac{\nu.Z}{w^3} + \frac{\mu Z}{u^3} - \frac{3\mu Z(X\cos.\psi - Y.\sin.\psi)}{u^4} = 0.$$

CAPVT VII.

ELIMINATIO QVANTITATVM u ET ψ EX AEQVATIONIBVS PRAECEDENTIBVS.

§. 41.

Cum sit $\frac{1}{u^3} = 1 - 3\,\varkappa.\cos.t$. et $\frac{1}{u^4} = 1 - 4\,\varkappa.\cos.t$; tum vero $\sin.\psi = -\sin.p - 2\,\varkappa.\sin.t.\cos.p$ et $\cos.\psi = -\cos.p + 2\,\varkappa.\sin.t.\sin.p$; hos valores in singulis terminis nostrarum aequationum, vbi quidem occurrunt, seorsim substituamus atque in prima quidem aequatione pro termino $-\frac{\mu X(3.\cos.\psi^2 - 1)}{u^3}$ habebimus

$$3.\cos.\psi^2 - 1 = 3.\cos.p^2 - 12.\varkappa.\sin.t.\sin.p.\cos.p - 1$$

adeoque $\frac{-\mu X(3.\cos.\psi^2 - 1)}{u^3}$

$$= -X(3.\cos.p^2 - 1) + 12\,\varkappa X\sin.t.\sin.p.\cos.p$$
$$+ 3\,\varkappa X\cos.t(3.\cos.p^2 - 1)$$

Pro membro $\frac{3\mu Y.\sin.\psi.\cos.\psi}{u^3}$ ob

$$\sin.\psi.\cos.\psi = \sin.p.\cos.p + 2\,\varkappa\sin.t(\cos.p^2 - \sin.p^2)$$

erit

erit

$$\frac{\sin.\psi.\cos.\psi}{u^3} = \sin.p.\cos.p + 2x\sin.t(\cos.p^2 - \sin.p^2)$$
$$-3x\cos.t.\sin.p.\cos.p$$

hincque illud membrum fiet

$$+\frac{3\mu Y\sin.\psi.\cos.\psi}{u^3} = 3Y.\sin.p.\cos.p + 6xY.\sin.t(\cos.p^2 - \sin.p^2)$$
$$-9xY\cos.t.\sin.p.\cos.p.$$

Simili modo pro membro $+\frac{3\mu X^2.\cos.\psi(5.\cos.\psi^2-3)}{2u^4}$

habetur

$$5.\cos.\psi^2 - 3 = 5.\cos.p^2 - 3 - 10x.\sin.t.\cos.p\sin.p \text{ et}$$
$$\cos.\psi(5.\cos.\psi^2 - 3) = -\cos.p(5.\cos.p^2 - 3)$$
$$+6x.\sin.t\sin.p(5\cos.p^2 - 1)$$

adeoque

$$+\frac{3\mu X^2\cos.\psi(5.\cos.\psi^2-3)}{2u^4} = -\frac{3.X^2.\cos.p(5.\cos.p^2-3)}{2}$$
$$+9xX^2\sin.t.\sin.p(5.\cos.p^2 - 1)$$
$$+6xX^2.\cos.t.\cos.p(5.\cos.p^2 - 3).$$

Pro termino sequente $-\frac{3\mu XY.\sin.\psi(5.\cos.\psi^2-1)}{u^4}$ habetur

$$(5.\cos.\psi^2 - 1)\sin.\psi = -\sin.p(5.\cos.p^2 - 1)$$
$$-2x.\sin.t.\cos.p(5\cos.p^2 - 10.\sin.p^2 - 1)$$

quod ductum in

$$-\frac{3\mu XY}{u^4} = -3XY + 12xXY\cos.t \text{ dabit}$$
$$-\frac{3\mu XY\sin.\psi(5.\cos.\psi^2-1)}{u^4} = 3XY\sin.p(5\cos.p^2 - 1)$$
$$-12.xXY.\cos.t\sin.p(5.\cos.p^2 - 1)$$
$$+6xXY\sin.t.\cos.p(5.\cos.p^2 - 10.\sin.p^2 - 1)$$

Sequi-

Sequitur nunc terminus $\frac{+3\mu Y^2 . cos.\psi(5.sin.\psi^2-1)}{2.u^4}$, pro quo habetur

$$(5.\sin.\psi^2-1)\cos.\psi = -\cos.p(5.\sin.p^2-1)$$
$$-2x.\sin.t.\sin.p(10.\cos.p^2-5.\sin.p^2+1)$$

quod ductum in $\frac{3\mu Y^2}{2u^4}$ producit

$$+\frac{3\mu Y^2.\cos.\psi(5.\sin.\psi^2-1)}{2.u^4} = -\frac{3Y^2\cos.p(5.\sin.p^2-1)}{2}$$
$$-3xY^2\sin.t.\sin.p(10.\cos.p^2-5.\sin.p^2+1)$$
$$+6xY^2\cos.t.\cos.p(5.\sin.p^2-1)$$

Vltimum denique membrum erit

$$\frac{-3\mu Z^2\cos.\psi}{2u^2} = \frac{3}{2}Z^2\cos.p - 3xZ^2\sin.t.\sin.p$$
$$-6xZ^2\cos.t.\cos.p$$

ob $\frac{\cos.\psi}{u^4} = -\cos.p + 2x.\sin.t\sin.p + 4x\cos.t.\cos.p.$

§. 42.

Terminos primae aequationis hoc modo euolutos ita in quatuor membra partiamur, vt primum contineat terminos, vbi X et Y vnicam tenent dimensionem, littera x non adfectos; quod membrum designemus per $\mathfrak{A}$; secundum membrum contineat terminos, vbi X, Y et Z ad secundam dimensionem assurgunt neque littera x adficiuntur, quod membrum designemus per $\mathfrak{B}$; ad tertium membrum littera $\mathfrak{C}$ signandum referamus terminos primae dimensionis littera x adfectos; quartum denique membrum $\mathfrak{D}$ complectitur terminos duarum dimensionum per x adfectos; sicque habebimus

$\mathfrak{A} =$

$$\mathfrak{A} = -X(3.\cos. p^2 - 1) + 3Y.\sin. p.\cos. p$$

$$\mathfrak{B} = -\tfrac{3}{2}X^2 \cos. p(5.\cos. p^2 - 3) + 3XY \sin. p(5.\cos. p^2 - 1) - \tfrac{3}{2}Y^2.\cos. p(5.\sin. p^2 - 1) + \tfrac{3}{2}Z^2 \cos. p.$$

$$\mathfrak{C} = +3\varkappa X(4.\sin. t.\sin. p.\cos. p + 3.\cos. t.\cos. p^2 - \cos. t) + 3\varkappa Y(2.\sin. t(\cos. p^2 - \sin. p^2) - 3\cos. t \sin. p.\cos. p)$$

$$\mathfrak{D} = +3\varkappa X^2(3.\sin. t \sin. p(5.\cos. p^2 - 1) + 2.\cos. t.\cos. p(5.\cos. p^2 - 3)) + 6\varkappa XY(\sin. t.\cos. p(5.\cos. p^2 - 10.\sin. p^2 - 1) - 2.\cos. t.\sin. p(5.\cos. p^2 - 1)) - 3\varkappa Y^2(\sin. t.\sin. p(10.\cos. p^2 - 5.\sin. p^2 + 1) - 2.\cos. t.\cos. p(5.\sin. p^2 - 1)) - 3\varkappa Z^2(\sin. t.\sin. p + 2.\cos. t.\cos. p)$$

quibus valoribus notatis ita se habebit

Aequatio Prima.

$$\frac{ddX}{dt^2} - \frac{2(m+1)dY}{dt} - (m+1)^2 X + \frac{\nu X}{u^3} + \mathfrak{A} + \mathfrak{B} + \mathfrak{C} + \mathfrak{D} = 0.$$

§. 43.

Simili modo tractemus secundam aequationem, vbi primum occurrit terminus $+\frac{3\mu X \sin. \psi.\cos. \psi}{u^3}$ vnde ob

$$\frac{\sin. \psi \cos. \psi}{u^3} = \sin. p.\cos. p + 2\varkappa.\sin. t(\cos. p^2 - \sin. p^2) - 3\varkappa \cos. t.\sin. p.\cos. p$$

erit hic primus terminus

$$3X.\sin. p.\cos. p + 6\varkappa X \sin. t(\cos. p^2 - \sin. p^2) - 9\varkappa X \cos. t.\sin. p.\cos. p$$

Pro secundo termino $\frac{-\mu Y.(3.\sin.\psi^2-1)}{u^3}$, erit

$$3.\sin.\psi^2-1=3.\sin.p^2-1+12.\varkappa.\sin.t.\sin.p.\cos.p$$

quod per

$$-\frac{\mu Y}{u^3}=-\mu Y+3\mu.\varkappa Y.\cos.t$$

multiplicatum, praebet secundum terminum

$$-Y(3.\sin.p^2-1)-12\varkappa Y.\sin.t.\sin.p.\cos.p$$
$$+3\varkappa Y\cos.t(3.\sin.p^2-1)$$

Pro tertio termino $\frac{-3\mu X^2\sin.\psi(5.\cos.\psi^2-1)}{2u^4}$ erit

$$\sin.\psi(5.\cos.\psi^2-1)=-\sin.p(5.\cos.p^2-1)$$
$$-2\varkappa.\sin.t.\cos.p(5.\cos.p^2-10.\sin.p^2-1)$$

quod ductum in

$$-\frac{3\mu X^2}{2u^4}=-\tfrac{3}{2}X^2(1-4\varkappa.\cos.t)$$

praebet tertium terminum

$$+\tfrac{3}{2}X^2\sin.p(5.\cos.p^2-1)$$
$$+3\varkappa X^2\sin.t.\cos.p(5.\cos.p^2-10.\sin.p^2-1)$$
$$-6\varkappa X^2\cos.t.\sin.p(5.\cos.p^2+1)$$

Pro quarto termino $\frac{+3\mu XY\cos.\psi(5.\sin.\psi^2-1)}{u^4}$ habetur

$$\cos.\psi(5.\sin.\psi^2-1)=-\cos.p(5.\sin.p^2-1)$$
$$-2\varkappa\sin.t.\sin.p(10.\cos.p^2-5.\sin.p^2+1)$$

quod ductum in

$$\frac{3\mu XY}{u^4}=3XY(1-4\varkappa.\cos.t)$$

praebet

praebet quartum terminum

$$-3\,XY.\,\text{cof.}\,p\,(5.\,\text{fin.}\,p^2-1)$$
$$-6\varkappa\,XY\,\text{fin.}\,t.\,\text{fin.}\,p\,(10.\,\text{cof.}\,p^2-5.\,\text{fin.}\,p^2+1)$$
$$+12.\varkappa\,XY\,\text{cof.}\,t.\,\text{cof.}\,p\,(5.\,\text{fin.}\,p^2-1)$$

Pro quinto termino $\frac{-3\mu Y^2\,\text{fin.}\,\psi\,(5.\,\text{fin.}\,\psi^2-3)}{2u^4}$ habetur

$$\text{fin.}\,\psi\,(5.\,\text{fin.}\,\psi^2-3)=-\text{fin.}\,p\,(5.\,\text{fin.}\,p^2-3)$$
$$-6\varkappa\,\text{fin.}\,t.\,\text{cof.}\,p\,(5\,\text{fin.}\,p^2-1)$$

quod ductum in

$$\frac{-3\mu Y^2}{2u^4}=-\tfrac{3}{8}Y^2(1-4\varkappa\,\text{cof.}\,t)$$

praebet quintum terminum

$$+\tfrac{3}{8}Y^2\,\text{fin.}\,p\,(5.\,\text{fin.}\,p^2-3)$$
$$+9\varkappa.\,Y^2\,\text{fin.}\,t\,\text{cof.}\,p\,(5.\,\text{fin.}\,p^2-1)$$
$$-6\varkappa\,Y^2.\,\text{cof.}\,t.\,\text{fin.}\,p\,(5.\,\text{fin.}\,p^2-3)$$

Pro vltimo denique termino $+\frac{3\mu Z^2.\,\text{fin.}\,\psi}{2u^4}$ eft

$$\text{fin.}\,\psi=-\text{fin.}\,p-2\varkappa.\,\text{fin.}\,t.\,\text{cof.}\,p$$

quo ducto in

$$\frac{3\mu Z^2}{2u^4}=\tfrac{3}{8}Z^2(1-4\varkappa.\,\text{cof.}\,t)$$

prodit fextus terminus

$$-\tfrac{3}{8}Z^2.\,\text{fin.}\,p-3\varkappa\,Z^2.\,\text{fin.}\,t.\,\text{cof.}\,p+6\varkappa\,Z^2\,\text{cof.}\,t.\,\text{fin.}\,p.$$

§. 44.

Iftos terminos euolutos fimili modo in quatuor membra diftribuamus, quae litteris A, B, C, D de-

 figne-

fignemus, ita, vt fit

$$A = 3\,X.\,\text{fin.}\,p.\,\text{cof.}\,p - Y\,(3.\,\text{fin.}\,p^2 - 1)$$

$$B = \tfrac{3}{2}\,X^2.\,\text{fin.}\,p\,(5.\,\text{cof.}\,p^2 - 1) - 3\,XY\,\text{cof.}\,p\,(5.\,\text{fin.}\,p^2 - 1)$$
$$+ \tfrac{3}{2}\,Y^2\,\text{fin.}\,p\,(5.\,\text{fin.}\,p^2 - 3) - \tfrac{3}{2}\,Z^2.\,\text{fin.}\,p$$

$$C = 3\,\varkappa X\,(2.\,\text{fin.}\,t\,(\text{cof.}\,p^2 - \text{fin.}\,p^2) - 3.\,\text{cof.}\,t.\,\text{fin.}\,p.\,\text{cof.}\,p)$$
$$+ 3\,\varkappa Y\,(\text{cof.}\,t\,(3.\,\text{fin.}\,p^2 - 1) - 4.\,\text{fin.}\,t.\,\text{fin.}\,p.\,\text{cof.}\,p).$$

$$D = 3\,\varkappa X^2 \left\{ \begin{array}{l} \text{fin.}\,t.\,\text{cof.}\,p\,(5.\,\text{cof.}\,p^2 - 10.\,\text{fin.}\,p^2 - 1) \\ \quad - 2.\,\text{cof.}\,t.\,\text{fin.}\,p\,(5.\,\text{cof.}\,p^2 - 1) \end{array} \right.$$
$$- 6\,\varkappa XY \left\{ \begin{array}{l} \text{fin.}\,t.\,\text{fin.}\,p\,(10.\,\text{cof.}\,p^2 - 5.\,\text{fin.}\,p^2 + 1) \\ \quad - 2.\,\text{cof.}\,t.\,\text{cof.}\,p\,(5.\,\text{fin.}\,p^2 - 1). \end{array} \right.$$
$$+ 3\,\varkappa Y^2 \left\{ \begin{array}{l} 3.\,\text{fin.}\,t.\,\text{cof.}\,p\,(5.\,\text{fin.}\,p^2 - 1) \\ \quad - 2.\,\text{cof.}\,t.\,\text{fin.}\,p\,(5.\,\text{fin.}\,p^2 - 3) \end{array} \right.$$
$$+ 3\,\varkappa Z^2\,(2.\,\text{cof.}\,t.\,\text{fin.}\,p - \text{fin.}\,t.\,\text{cof.}\,p)$$

His valoribus notatis ita fe habet

Aequatio Secunda.

$$\frac{ddY}{dt^2} + \frac{2(m+1)dX}{dt} - (m+1)^2\,Y + \frac{\mu Y}{u^3}$$
$$+ A + B + C + D = 0.$$

§. 45.

Pro tertia noftra aequatione primus terminus tali reductione indigens eft $+\frac{\mu Z}{u^3}$, qui ftatim praebet $Z - 3\,\varkappa\,Z.\,\text{cof.}\,t.$

Pro fecundo termino $\frac{-3\mu XZ\,\text{cof.}\,\psi}{u^4}$ ob

$$\text{cof.}\,\psi = -\,\text{cof.}\,p + 2\,\varkappa.\,\text{fin.}\,t.\,\text{fin.}\,p$$

mul-

multiplicandum per

$$-\frac{3\mu XZ}{u^4} = -3\,XZ\,(1-4\varkappa.\,\mathrm{cof.}\,t)$$

fiet hic terminus

$$3\,XZ.\,\mathrm{cof.}\,p - 6\varkappa\,XZ.\,\mathrm{fin.}\,t.\,\mathrm{fin.}\,p - 12\varkappa\,XZ.\,\mathrm{cof.}\,t.\,\mathrm{cof.}\,p.$$

Tertius denique terminus $+\frac{3\mu YZ\,\mathrm{fin.}\,\psi}{u^4}$ ob

$$\mathrm{fin.}\,\psi = -\mathrm{fin.}\,p - 2\varkappa.\,\mathrm{fin.}\,t.\,\mathrm{cof.}\,p$$

ducendum in

$$+\frac{3\mu YZ}{u^4} = 3\,YZ\,(1-4\varkappa.\,\mathrm{cof.}\,t)$$

euadet

$$-3\,YZ.\,\mathrm{fin.}\,p - 6\varkappa.\,YZ.\,\mathrm{fin.}\,t.\,\mathrm{cof.}\,p + 12\varkappa.\,YZ.\,\mathrm{cof.}\,t.\,\mathrm{fin.}\,p.$$

§. 46.

Distribuamus hos terminos iterum in quatuor membra, litteris $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$ designanda, ita, vt sit

$$\mathfrak{a} = Z$$

$$\mathfrak{b} = 3\,XZ.\,\mathrm{cof.}\,p - 3\,YZ.\,\mathrm{fin.}\,p$$

$$\mathfrak{c} = -3\varkappa\,Z.\,\mathrm{cof.}\,t$$

$$\mathfrak{d} = -6\varkappa\,XZ\,(\mathrm{fin.}\,t.\,\mathrm{fin.}\,p + 2.\,\mathrm{cof.}\,t.\,\mathrm{cof.}\,p)$$
$$-6\varkappa\,YZ\,(\mathrm{fin.}\,t.\,\mathrm{cof.}\,p - 2.\,\mathrm{cof.}\,t.\,\mathrm{fin.}\,p)$$

quibus valoribus notatis ita se habebit

Aequatio Tertia.

$$\frac{ddZ}{dt^2} + \frac{vZ}{w^3} + \mathfrak{a} + \mathfrak{b} + \mathfrak{c} + \mathfrak{d} = 0.$$

CAPVT VIII.

VLTERIOR REDVCTIO HARVM FORMVLARVM AD SIMPLICES SINVS ET COSINVS ANGVLORVM.

§. 47.

Quo has formulas porro ad simplices sinus et cosinus angulorum reducamus, sequentes reductiones notissimas notasse iuuabit

$$\text{sin.}\, p^2 = \tfrac{1}{2} - \tfrac{1}{2}\,\text{cos.}\, 2p$$
$$\text{sin.}\, p.\, \text{cos.}\, p = \tfrac{1}{2}.\, \text{sin.}\, 2p$$
$$\text{cos.}\, p^2 = \tfrac{1}{2} + \tfrac{1}{2}\,\text{cos.}\, 2p$$
$$\text{sin.}\, p^3 = \tfrac{3}{4}\,\text{sin.}\, p - \tfrac{1}{4}.\, \text{sin.}\, 3p$$
$$\text{sin.}\, p^2\, \text{cos.}\, p = \tfrac{1}{4}\,\text{cos.}\, p - \tfrac{1}{4}\,\text{cos.}\, 3p$$
$$\text{sin.}\, p.\, \text{cos.}\, p^2 = \tfrac{1}{4}\,\text{sin.}\, p + \tfrac{1}{4}\,\text{sin.}\, 3p$$
$$\text{cos.}\, p^3 = \tfrac{3}{4}\,\text{cos.}\, p + \tfrac{1}{4}.\, \text{cos.}\, 3p.$$

Praeterea vero etiam notari oportet sequentes reductiones

$$\text{fin.}\,t.\,\text{fin.}\,P = \tfrac{1}{2}\,\text{cof.}(P-t) - \tfrac{1}{2}\,\text{cof.}(P+t)$$

$$\text{fin.}\,t.\,\text{cof.}\,P = -\tfrac{1}{2}\,\text{fin.}(P-t) + \tfrac{1}{2}\,\text{fin.}(P+t)$$

$$\text{cof.}\,t.\,\text{fin.}\,P = \tfrac{1}{2}.\,\text{fin.}(P-t) + \tfrac{1}{2}\,\text{fin.}(P+t)$$

$$\text{cof.}\,t.\,\text{cof.}\,P = \tfrac{1}{2}\,\text{cof.}(P-t) + \tfrac{1}{2}\,\text{cof.}(P+t)$$

§. 48.

Secundum has ergo reductiones euoluamus pro prima nostra aequatione formulas 𝔄, 𝔅, ℭ, 𝔇 ac primo quidem cum 𝔄 constet duabus partibus, pars prior $-X(3.\,\text{cof.}\,p^2 - 1)$ fiet $-\tfrac{1}{2}X(1 + 3.\,\text{cof.}\,2p)$; altera vero pars

$$+ 3\,Y.\,\text{fin.}\,p.\,\text{cof.}\,p = \tfrac{3}{2}\,Y.\,\text{fin.}\,2p;$$

ita, vt fit

$$\mathfrak{A} = -\tfrac{1}{2}X(1 + 3.\,\text{cof.}\,2p) + \tfrac{3}{2}\,Y.\,\text{fin.}\,2p.$$

§. 49.

Formula 𝔅 consistit quatuor partibus, quarum prima $-\tfrac{3}{2}X^2(5.\,\text{cof.}\,p^3 - 3.\,\text{cof.}\,p)$ reducitur ad hanc formam $-\tfrac{3}{8}X^2(3.\,\text{cof.}\,p + 5.\,\text{cof.}\,3p)$.

Secunda $+3\,X\,Y(5.\,\text{cof.}\,p^2.\,\text{fin.}\,p - \text{fin.}\,p)$ reducitur ad $+\tfrac{3}{4}X\,Y(\text{fin.}\,p + 5.\,\text{fin.}\,3p)$.

Tertia pars $-\tfrac{3}{2}Y^2(5.\,\text{fin.}\,p^2.\,\text{cof.}\,p - \text{cof.}\,p)$ reducitur ad $-\tfrac{3}{8}Y^2(\text{cof.}\,p - 5.\,\text{cof.}\,3p)$.

At quarta pars $+\tfrac{3}{2}Z^2\,\text{cof.}\,p$ nulla reductione indiget; quocirca habebimus

𝔅 =

$$\mathfrak{B} = -\tfrac{3}{8}X^2(3.\cos.p + 5.\cos.3p) + \tfrac{3}{4}XY(\sin.p + 5.\sin.3p)$$
$$-\tfrac{3}{8}Y^2(\cos.p - 5.\cos.3.p) + \tfrac{3}{2}Z^2.\cos.p.$$

§. 50.

Formula $\mathfrak{C}$ duabus partibus consistit, quarum prima

$$3\kappa X(4.\sin.t.\sin.p.\cos.p + 3.\cos.t.\cos.p^2 - \cos.t)$$

statim reducitur ad hanc formam:

$$\tfrac{3}{2}\kappa X(4.\sin.t.\sin.2p + \cos.t + 3.\cos.t.\cos.2p)$$

deinde vero ad hanc:

$$\tfrac{3}{4}\kappa X(2.\cos.t + 7.\cos.(2p-t) - \cos.(2p+t)).$$

Secunda pars

$$3\kappa Y(2.\sin.t(\cos.p^2 - \sin.p^2) - 3.\cos.t.\sin.p.\cos.p)$$

statim redit ad hanc formam:

$$\tfrac{3}{2}\kappa Y(4.\sin.t.\cos.2p - 3.\cos.t.\sin.2p)$$

deinde vero ad hanc

$$-\tfrac{3}{4}\kappa Y(7.\sin.(2p-t) - \sin.(2p+t)).$$

vnde fit

$$\mathfrak{C} = \tfrac{3}{4}\kappa X(2.\cos.t + 7.\cos(2p-t) - \cos.(2p+t))$$
$$-\tfrac{3}{4}\kappa Y(7.\sin.(2p-t) - \sin.(2p+t)).$$

§. 51.

Formula $\mathfrak{D}$ quatuor constat partibus, quarum prima

$$+3\kappa X^2(3.\sin.t(5.\sin.p.\cos.p^2 - \sin.p) + 2.\cos.t(5.\cos.p^3 - 3.\cos.p)),$$

statim

statim reducitur ad hanc formam

$$+\tfrac{3}{4}\varkappa X^2 \left\{\begin{array}{l} 3.\,\text{sin.}\,t\,(\text{sin.}\,p + 5.\,\text{sin.}\,3p) \\ + 2.\,\text{cos.}\,t.\,(3.\,\text{cos.}\,p + 5.\,\text{cos.}\,3p) \end{array}\right.$$

deinde vero ad hanc:

$$\tfrac{3}{8}\varkappa X^2 \left\{\begin{array}{l} 9.\,\text{cos.}\,(p-t) + 25.\,\text{cos.}\,(3p-t) \\ + 3.\,\text{cos.}\,(p+t) - 5.\,\text{cos.}\,(3p+t) \end{array}\right.$$

Secunda pars

$$+ 6\varkappa XY \left\{\begin{array}{l} \text{sin.}\,t\,(5.\,\text{cos.}\,p^3 - 10.\,\text{sin.}\,p^2\,\text{cos.}\,p - \text{cos.}\,p) \\ - 2.\,\text{cos.}\,t\,(5.\,\text{sin.}\,p.\,\text{cos.}\,p^2 - \text{sin.}\,p) \end{array}\right.$$

statim reducitur ad hanc formam:

$$\tfrac{3}{2}\varkappa XY \left\{\begin{array}{l} \text{sin.}\,t\,\text{cos.}\,p + 15.\,\text{sin.}\,t.\,\text{cos.}\,3p \\ - 2.\,\text{cos.}\,t.\,\text{sin.}\,p - 10.\,\text{cos.}\,t.\,\text{sin.}\,3p \end{array}\right.$$

deinde vero ad hanc:

$$-\tfrac{3}{4}\varkappa XY \left\{\begin{array}{l} 3.\,\text{sin.}\,(p-t) + 25.\,\text{sin.}\,(3p-t) \\ + \text{sin.}\,(p+t) - 5.\,\text{sin.}\,(3p+t) \end{array}\right.$$

Tertia pars

$$- 3\varkappa Y^2 \left\{\begin{array}{l} \text{sin.}\,t\,(10.\,\text{cos.}\,p^2.\,\text{sin.}\,p - 5.\,\text{sin.}\,p^3 + \text{sin.}\,p) \\ - 2.\,\text{cos.}\,t\,(5.\,\text{sin.}\,p^2.\,\text{cos.}\,p - \text{cos.}\,p) \end{array}\right.$$

statim reducitur ad hanc formam:

$$-\tfrac{3}{4}\varkappa Y^2 \left\{\begin{array}{l} - \text{sin.}\,t.\,\text{sin.}\,p + 15.\,\text{sin.}\,t.\,\text{sin.}\,3p \\ - 2\,\text{cos.}\,t.\,\text{cos.}\,p + 10.\,\text{cos.}\,t.\,\text{cos.}\,3p \end{array}\right.$$

deinde vero ad hanc

$$+\tfrac{3}{8}\varkappa Y^2 \left\{ \begin{array}{l} 3.\cos.(p-t)-25.\cos.(3p-t) \\ +\cos.(p+t)+5.\cos.(3p+t) \end{array} \right.$$

Quarta denique pars

$$-3\varkappa Z^2 (\sin. t.\sin. p + 2.\cos. t.\cos. p)$$

statim praebet

$$-\tfrac{3}{2}\varkappa Z^2 (3.\cos.(p-t) + \cos.(p+t))$$

quibus partibus collectis habemus

$$\mathfrak{D} = \tfrac{3}{8}\varkappa X^2 \left\{ \begin{array}{l} 9.\cos.(p-t) + 25.\cos.(3p-t) \\ +3.\cos.(p+t) - 5.\cos.(3p+t) \end{array} \right.$$

$$-\tfrac{3}{4}\varkappa XY \left\{ \begin{array}{l} 3.\sin.(p-t) + 25.\sin.(3p-t) \\ +\sin.(p+t) - 5.\sin.(3p+t) \end{array} \right.$$

$$+\tfrac{3}{8}\varkappa Y^2 \left\{ \begin{array}{l} 3.\cos.(p-t) - 25.\cos.(3p-t) \\ +\cos.(p+t) + 5.\cos.(3p+t) \end{array} \right.$$

$$-\tfrac{3}{2}\varkappa Z^2 (3.\cos.(p-t) + \cos.(p+t)).$$

§. 52.

Eodem modo pro secunda aequatione euoluamus formulas A, B, C, D ac primo quidem formulae A prima pars $+3X.\sin.p.\cos.p$ statim praebet $+\tfrac{3}{2}X.\sin.2p$. Altera pars vero $-Y(3.\sin.p^2-1)$ reducitur ad hanc formam $-\tfrac{1}{2}Y(1-3.\cos.2p)$ ita, vt sit

$$A = +\tfrac{3}{2}X\sin.2p - \tfrac{1}{2}Y(1-3.\cos.2p).$$

§. 53.

§. 53.

Pro formula B pars prima

$$+\tfrac{3}{2}X^2 \sin. p\,(5.\cos. p^2 - 1)$$

reducitur ad hanc formam

$$+\tfrac{3}{8}X^2 (\sin. p + 5.\sin. 3p).$$

Secunda pars $-3XY \cos. p\,(5.\sin. p^2 - 1)$ redit ad hanc formam: $-\tfrac{3}{4}XY(\cos. p - 5.\cos. 3p)$.

Tertia pars $+\tfrac{3}{2}Y^2 \sin. p\,(5.\sin. p^2 - 3)$ praebet

$$+\tfrac{3}{8}Y^2 (3.\sin. p - 5.\sin. 3p).$$

Quarta autem pars $-\tfrac{3}{2}Z^2.\sin. p$ nulla reductione indiget; ita, vt sit

$$B = \tfrac{3}{8}X^2(\sin. p + 5.\sin. 3p) - \tfrac{3}{4}XY(\cos. p - 5.\cos. 3p)$$
$$+\tfrac{3}{8}Y^2(3.\sin. p - 5.\sin. 3p) - \tfrac{3}{2}Z^2.\sin. p.$$

§. 54.

Pro formula C prior pars

$$+3\varkappa X(2.\sin. t(\cos. p^2 - \sin. p^2) - 3.\cos. t.\sin. p.\cos. p)$$

statim reducitur ad hanc formam:

$$+\tfrac{3}{2}\varkappa X(4.\sin. t.\cos. 2p - 3.\cos. t.\sin. 2p)$$

tum vero ad hanc:

$$-\tfrac{3}{4}\varkappa X(7.\sin.(2p - t) - \sin.(2p + t))$$

Pars vero altera

$$+3\varkappa Y(\cos. t\,(3.\sin. p^2 - 1) - 4.\sin. t.\sin. p.\cos. p)$$

 primo

primo reducitur ad hanc formam

$$-\tfrac{3}{8}\varkappa Y(4.\sin.t.\sin.2p+3.\cos.t.\cos.2p-\cos.t)$$

deinde vero ad hanc:

$$+\tfrac{3}{4}\varkappa Y(2.\cos.t-7.\cos.(2p-t)+\cos.(2p+t))$$

Quocirca erit

$$C=-\tfrac{3}{4}\varkappa X(7.\sin.(2p-t)-\sin.(2p+t))$$
$$+\tfrac{3}{4}\varkappa Y(2.\cos.t-7.\cos.(2p-t)+\cos.(2p+t)).$$

§. 55.

Simili modo formulae D pars prima

$$3\varkappa X^2\begin{cases}\sin.t.\cos.p(5.\cos.p^2-10.\sin.p^2+1)\\ -2.\cos.t.\sin.p(5.\cos.p^2-1)\end{cases}$$

statim redit ad hanc formam:

$$+\tfrac{3}{4}\varkappa X^2\begin{cases}\sin.t(\cos.p+15.\cos.3p)\\ -2.\cos.t(\sin.p+5.\sin.3p)\end{cases}$$

porro vero ad hanc:

$$-\tfrac{3}{8}\varkappa X^2\begin{cases}3.\sin.(p-t)+25.\sin.(3p-t)\\ +\sin.(p+t)-5.\sin.(3p+t)\end{cases}$$

Secunda pars

$$-6\varkappa XY\begin{cases}\sin.t(10.\cos.p^2\sin.p-5.\sin.p^3+\sin.p)\\ +2.\cos.t(5.\sin.p^2\cos.p-\cos.p)\end{cases}$$

statim reducitur ad hanc formam:

$$+\tfrac{3}{2}\varkappa XY\begin{cases}\sin.t.\sin.p-15.\sin.t.\sin.3p\\ +2.\cos.t.\cos.p-10.\cos.t.\cos.3p\end{cases}$$

deinde

deinde vero ad hanc:

$$+\tfrac{3}{4}\,n\,X\,Y\left\{\begin{array}{l}3.\,\text{cos.}\,(p-t)-25.\,\text{cos.}\,(3p-t)\\+\,\text{cos.}\,(p+t)+5.\,\text{cos.}\,(3p+t)\end{array}\right.$$

Tertia pars

$$+3\,n\,Y^2\left\{\begin{array}{l}3.\,\text{sin.}\,t\,(5.\,\text{sin.}\,p^2.\,\text{cos.}\,p-\text{cos.}\,p)\\-\,2.\,\text{cos.}\,t\,(5.\,\text{sin.}\,p^3-3.\,\text{sin.}\,p)\end{array}\right.$$

statim reducitur ad hanc formam:

$$+\tfrac{3}{4}\,n\,Y^2\left\{\begin{array}{l}3.\,\text{sin.}\,t.\,\text{cos.}\,p-15.\,\text{sin.}\,t.\,\text{cos.}\,3p\\-\,6.\,\text{cos.}\,t.\,\text{sin.}\,p+10.\,\text{cos.}\,t.\,\text{sin.}\,3p\end{array}\right.$$

tum vero ad hanc:

$$-\tfrac{3}{8}\,n\,Y^2\left\{\begin{array}{l}9.\,\text{sin.}\,(p-t)-25.\,\text{sin.}\,(3p-t)\\+\,3.\,\text{sin.}\,(p+t)+5.\,\text{sin.}\,(3p+t)\end{array}\right.$$

Quarta pars

$$+3.\,n\,Z^2\,(2.\,\text{cos.}\,t.\,\text{sin.}\,p-\text{sin.}\,t.\,\text{cos.}\,p)$$

statim redit ad hanc formam

$$+\tfrac{3}{2}\,n\,Z^2\,(3.\,\text{sin.}\,(p-t)+\text{sin.}\,(p+t))$$

sicque habebimus

$$D=-\tfrac{3}{8}\,n\,X^2\left\{\begin{array}{l}3.\,\text{sin.}\,(p-t)+25.\,\text{sin.}\,(3p-t)\\+\,\text{sin.}\,(p+t)-5.\,\text{sin.}\,(3p+t)\end{array}\right.$$

$$+\tfrac{3}{4}\,n\,X\,Y\left\{\begin{array}{l}3.\,\text{cos.}\,(p-t)-25.\,\text{cos.}\,(3p-t)\\+\,\text{cos.}\,(p+t)+5.\,\text{cos.}\,(3p+t)\end{array}\right.$$

$$-\tfrac{3}{8}\,n\,Y^2\left\{\begin{array}{l}9.\,\text{sin.}\,(p-t)-25.\,\text{sin.}\,(3p-t)\\+\,3.\,\text{sin.}\,(p+t)+5.\,\text{sin.}\,(3p+t)\end{array}\right.$$

$$+\tfrac{3}{2}\,n\,Z^2\,(3.\,\text{sin.}\,(p-t)+\text{sin.}\,(p+t))$$

§. 56.

Restat denique tertia aequatio, eiusque formulae a, b, c, d; at $a = Z$ nulla indiget reductione, neque etiam $b = 3\,X\,Z.\,\mathrm{cos}.\,p - 3\,Y\,Z.\,\mathrm{sin}.\,p$; quin et manet $c = -3\,\varkappa\,Z.\,\mathrm{cos}.\,t$; at pro d pars prior

$$-6\,\varkappa\,X\,Z\,(\mathrm{sin}.\,t.\,\mathrm{sin}.\,p + 2.\,\mathrm{cos}.\,t.\,\mathrm{cos}.\,p)$$

abit in hanc:

$$-3\,\varkappa\,X\,Z\,(3.\,\mathrm{cos}.\,(p-t) + \mathrm{cos}.\,(p+t))$$

pars vero posterior

$$-6\,\varkappa\,Y\,Z\,(\mathrm{sin}.\,t.\,\mathrm{cos}.\,p - 2.\,\mathrm{cos}.\,t.\,\mathrm{sin}.\,p)$$

redit ad formam:

$$+3\,\varkappa\,Y\,Z\,(3.\,\mathrm{sin}.\,(p-t) + \mathrm{sin}.\,(p+t))$$

sicque pro hac aequatione omnino habebimus

$$a = Z$$

$$b = 3\,X\,Z.\,\mathrm{cos}.\,p - 3\,Y\,Z.\,\mathrm{sin}.\,p$$

$$c = -3\,\varkappa\,Z.\,\mathrm{cos}.\,t$$

$$d = -3\,\varkappa\,X\,Z\,(3.\,\mathrm{cos}.\,(p-t) + \mathrm{cos}.\,(p+t)) + 3\,\varkappa\,Y\,Z\,(3.\,\mathrm{sin}.\,(p-t) + \mathrm{sin}.\,(p+t)).$$

§. 57.

Ipsas autem tres nostras aequationes iam supra exhibuimus §§. 42, 44 et 46. vbi ergo tantum loco litterarum 𝔄, 𝔅, ℭ, 𝔇; A, B, C, D; et a, b, c, d, valores in hoc capite inuenti substitui sunt intelligendi.

CAPVT IX.

CAPVT IX.

REDVCTIO TERNARVM NOSTRARVM AEQVATIONVM AD ALIAS COMMODIORES TERNAS COORDINATAS.

§. 58.

Quoniam distantiam mediam Terrae a Sole per vnitatem expressimus, statuamus distantiam mediam Lunae a Terra $= a$; ita, vt sit $1 : a$, vti parallaxis media Lunae ad parallaxin Solis; vnde proxime colligitur $a = \frac{1}{390}$; quo posito facile intelligitur, ternas coordinatas, quibus locus Lunae definitur, ad hanc ipsam distantiam mediam a referri conuenire.

§. 59.

Quod autem ad primam harum coordinatarum $\Theta x = X$ attinet; ea ipsa valorem habebit medium ipsi a circiter aequalem; quippe quo modo erit maior, modo minor; binae vero reliquae Y et Z valores modo positiuos, modo negatiuos sortientur; quocirca statuamus

X =

$$X = a(1+x);\ Y = ay;\ Z = az;$$

ita, vt in posterum has quantitates, x, y, z tanquam coordinatas pro loco Lunae simus spectaturi; quippe quae non solum respectu distantiae Solis a Terra $= 1$, sed etiam respectu distantiae mediae Lunae $= a$ satis erunt exiguae; quae circumstantia ad adproximationes sequentes inprimis est necessaria. Ceterum hic monendum, ne istae nouae litterae x, y, z cum istis, quibus supra sumus vsi, confundantur.

§. 60.

Factis autem his substitutionibus ob

$$dX = a\,dx;\ \text{et}\ ddX = a\,ddx;$$

omnes partes priores nostrarum aequationum, vbi scilicet litterae X, Y, Z vnicam habent dimensionem, nunc post hanc substitutionem per quantitatem a erunt multiplicatae; id, quod etiam tenendum est de iis formulis, quae litteris 𝔄, A, et $\mathfrak{a}$; vel ℭ, C, et $\mathfrak{c}$ adficiuntur. At vero formulae litteris 𝔅, B, et $\mathfrak{b}$ vel 𝔇, D et $\mathfrak{d}$ adfectae omnes factorem habebunt quadratum a^2.

§. 61.

His autem factis substitutionibus fiet

$$w^2 = X^2 + Y^2 + Z^2 = a^2((1+x)^2 + y^2 + z^2)$$

hincque

$$w^3 = a^3((1+x)^2 + y^2 + z^2)^{\frac{3}{2}}$$

Hanc

Hanc ob rem, cum ν sit fractio quam minima, statuamus in posterum $\frac{\nu}{a^3} = \lambda$; atque deinceps ostendetur, fore propemodum $\lambda = 180$. Hoc observato pro prima nostra aequatione habebimus

$$\frac{\nu X}{w^2} = \frac{\lambda . a (1 + x)}{((1 + x)^2 + y^2 + z^2)^{\frac{3}{2}}}$$

pro secunda autem aequatione erit

$$\frac{\nu . Y}{w^2} = \frac{\lambda . a . y}{((1 + x)^2 + y^2 + z^2)^{\frac{3}{2}}}$$

at pro tertia

$$\frac{\nu . Z}{w^2} = \frac{\lambda . a . z}{((1 + x)^2 + y^2 + z^2)^{\frac{3}{2}}}$$

§. 62.

His constitutis, si singulas nostras aequationes per a dividamus, eae sequenti modo prodibunt expressae:

Aequatio Prima.

$$\frac{ddx}{dt^2} - \frac{2(m+1)dy}{dt} - (m+1)^2 . (1+x)$$
$$+ \frac{\lambda (1+x)}{((1+x)^2 + y^2 + z^2)^{\frac{3}{2}}} + \frac{\mathfrak{A}}{a} + \frac{\mathfrak{B}}{a} + \frac{\mathfrak{C}}{a} + \frac{\mathfrak{D}}{a} = 0.$$

Aequatio Secunda.

$$\frac{ddy}{dt^2}+\frac{2(m+1)\,dx}{dt}+(m+1)^2.y$$

$$+\frac{\lambda.y}{((1+x)^2+y^2+z^2)^{\frac{3}{2}}}+\frac{A}{a}+\frac{B}{a}+\frac{C}{a}+\frac{D}{a}=0.$$

Aequatio Tertia.

$$\frac{ddz}{dt^2}+\frac{\lambda.z}{((1+x)^2+y^2+z^2)^{\frac{3}{2}}}+\frac{\mathfrak{A}}{a}+\frac{\mathfrak{B}}{a}+\frac{\mathfrak{C}}{a}+\frac{\mathfrak{D}}{a}=0.$$

quae ergo erunt tres noſtrae aequationes fundamentales, vnde motum Lunae determinari oportebit.

§. 63.

Faciamus easdem ſubſtitutiones in formulis annexis ac pro prima quidem noſtra aequatione habebimus

$$\frac{\mathfrak{A}}{a}=-\tfrac{1}{2}(1+x)(1+3.\text{cof.}\,2p)+\tfrac{3}{2}y.\,\text{ſin.}\,2p$$

$$\frac{\mathfrak{B}}{a}=-\tfrac{3}{8}a(1+x)^2(3.\text{cof.}\,p+5.\text{cof.}\,3p)$$
$$+\tfrac{3}{4}a(1+x)y(\text{ſin.}\,p+5.\text{ſin.}\,3p)$$
$$-\tfrac{3}{8}ay^2(\text{cof.}\,p-5.\text{cof.}\,3p)+\tfrac{3}{2}a\,z^2.\,\text{cof.}\,p$$

$$\frac{\mathfrak{C}}{a}=\tfrac{3}{4}\kappa(1+x)(2.\text{cof.}\,t+7.\text{cof.}(2p-t)-\text{cof.}(2p+t))$$
$$-\tfrac{3}{4}\kappa.y(7.\text{ſin.}(2p-t)-\text{ſin.}(2p+t))$$

$$\frac{\mathfrak{D}}{c} = \tfrac{3}{8} an(1+x)^2 \left\{ \begin{array}{l} 9.\cos.(p-t) + 25.\cos.(3p-t) \\ +3.\cos.(p+t) + 5.\cos.(3p+t) \end{array} \right.$$

$$-\tfrac{3}{4} an(1+x)y \left\{ \begin{array}{l} 3.\sin.(p-t) + 25.\sin.(3p-t) \\ +\sin.(p+t) - 5.\sin.(3p+t) \end{array} \right.$$

$$+\tfrac{3}{8} any^2 \left\{ \begin{array}{l} 3.\cos.(p-t) + 25.\cos.(3p-t) \\ +\cos.(p+t) + 5.\cos.(3p+t) \end{array} \right.$$

$$-\tfrac{3}{8} anz^2 (3.\cos.(p-t) + \cos.(p+t))$$

quae formulae secundum dimensiones nouarum coordinatarum x, y et z dispositae sequentes formas induent

$$\frac{\mathfrak{A}}{c} = -\tfrac{1}{4} - \tfrac{3}{4}\cos.2p - \tfrac{1}{2}x - \tfrac{3}{2}x\cos.2p + \tfrac{3}{2}y\sin.2p$$

$$\frac{\mathfrak{B}}{c} = -\tfrac{1}{8} a(3.\cos.p + 5.\cos.3p)$$

$$-\tfrac{3}{4} ax(3.\cos.p + 5.\cos.3p) + \tfrac{3}{4} ay(\sin.p + 5\sin.3p)$$

$$-\tfrac{3}{8} ax^2(3.\cos.p + 5.\cos.3p) + \tfrac{3}{4} axy(\sin.p + 5.\sin.3p)$$

$$-\tfrac{3}{8} ay^2(\cos.p - 5.\cos.3p) + \tfrac{3}{8} az^2.\cos.p$$

$$\frac{\mathfrak{C}}{c} = +\tfrac{3}{4} n(2.\cos.t + 7.\cos.(2p-t) - \cos.(2p+t))$$

$$+\tfrac{3}{4} nx(2.\cos.t + 7.\cos.(2p-t) - \cos.(2p+t))$$

$$-\tfrac{3}{4} ny(7.\sin.(2p-t) - \sin.(2p+t))$$

$\frac{\mathfrak{D}}{c} =$

$$\frac{D}{a} = +\tfrac{3}{8}an \left\{ \begin{array}{l} 9.\,\mathrm{cos.}(p-t) + 25.\,\mathrm{cos.}(3p-t) \\ +3.\,\mathrm{cos.}(p+t) - 5.\,\mathrm{cos.}(3p+t) \end{array} \right.$$

$$+\tfrac{3}{4}anx \left\{ \begin{array}{l} 9.\,\mathrm{cos.}(p-t) + 25.\,\mathrm{cos.}(3p-t) \\ +3.\,\mathrm{cos.}(p+t) - 5.\,\mathrm{cos.}(3p+t) \end{array} \right.$$

$$-\tfrac{3}{4}any \left\{ \begin{array}{l} 3.\,\mathrm{sin.}(p-t) + 25.\,\mathrm{sin.}(3p-t) \\ +\mathrm{sin.}(p+t) - 5.\,\mathrm{sin.}(3p+t) \end{array} \right.$$

$$+\tfrac{3}{8}anx^2 \left\{ \begin{array}{l} 9.\,\mathrm{cos.}(p-t) + 25.\,\mathrm{cos.}(3p-t) \\ +3.\,\mathrm{cos.}(p+t) - 5.\,\mathrm{cos.}(3p+t) \end{array} \right.$$

$$-\tfrac{3}{4}anxy \left\{ \begin{array}{l} 3.\,\mathrm{sin.}(p-t) + 25.\,\mathrm{sin.}(3p-t) \\ +\mathrm{sin.}(p+t) - 5.\,\mathrm{sin.}(3p+t) \end{array} \right.$$

$$+\tfrac{3}{8}any^2 \left\{ \begin{array}{l} 3.\,\mathrm{cos.}(p-t) - 25.\,\mathrm{cos.}(3p-t) \\ +\mathrm{cos.}(p+t) + 5.\,\mathrm{cos.}(3p+t) \end{array} \right.$$

$$-\tfrac{3}{8}anz^2\,(3.\,\mathrm{cos.}(p-t) + \mathrm{cos.}(p+t))$$

§. 64.

Eodem modo pro secunda aequatione, terminis secundum dimensionum numerum dispositis, habebimus

$$\frac{A}{a} = +\tfrac{3}{2}\,\mathrm{sin.}\,2p, +\tfrac{3}{2}x\,\mathrm{sin.}\,2p - \tfrac{3}{2}y + \tfrac{3}{2}y.\,\mathrm{cos.}\,2p.$$

$$\frac{B}{a} = \tfrac{3}{8}a\,(\mathrm{sin.}\,p + 5.\,\mathrm{sin.}\,3p),$$

$$+\tfrac{3}{4}ax(\mathrm{sin.}\,p + 5.\,\mathrm{sin.}\,3p) - \tfrac{3}{4}ay(\mathrm{cos.}\,p - 5.\,\mathrm{cos.}\,3p),$$

$$+\tfrac{3}{8}ax^2(\mathrm{sin.}\,p + 5.\,\mathrm{sin.}\,3p) - \tfrac{3}{4}axy(\mathrm{cos.}\,p - 5.\,\mathrm{cos.}\,3p)$$

$$+\tfrac{3}{8}ay^2(3.\,\mathrm{sin.}\,p - 5.\,\mathrm{sin.}\,3p) - \tfrac{3}{8}az^2.\,\mathrm{sin.}\,p.$$

$\frac{C}{a} =$

$$\frac{C}{a} = -\tfrac{3}{4} n \left(7.\,\sin.(2p - t) - \sin.(2p + t) \right),$$
$$-\tfrac{3}{4} n x \left(7.\,\sin.(2p - t) - \sin.(2p + t) \right)$$
$$+\tfrac{3}{4} n y \left(2.\,\cos.\, t - 7.\,\cos.(2p - t) + \cos.(2p + t) \right)$$

$$\frac{D}{a} = -\tfrac{3}{8} a n \left\{ \begin{array}{l} 3.\,\sin.(p-t) + 25.\,\sin.(3p-t) \\ + \sin.(p+t) - 5.\,\sin.(3p+t), \end{array} \right.$$
$$-\tfrac{3}{4} a n x \left\{ \begin{array}{l} 3.\,\sin.(p-t) + 25.\,\sin.(3p-t) \\ + \sin.(p+t) - 5.\,\sin.(3p+t) \end{array} \right.$$
$$+\tfrac{3}{4} a n y \left\{ \begin{array}{l} 3.\,\cos.(p-t) - 25.\,\cos.(3p-t) \\ + \cos.(p+t) + 5.\,\cos.(3p+t), \end{array} \right.$$
$$-\tfrac{3}{8} a n x^2 \left\{ \begin{array}{l} 3.\,\sin.(p-t) + 25.\,\sin.(3p-t) \\ + \sin.(p+t) - 5.\,\sin.(3p+t) \end{array} \right.$$
$$+\tfrac{3}{4} a n x y \left\{ \begin{array}{l} 3.\,\cos.(p-t) - 25.\,\cos.(3p-t) \\ + \cos.(p+t) + 5.\,\cos.(3p+t) \end{array} \right.$$
$$-\tfrac{3}{8} a n y^2 \left\{ \begin{array}{l} 9.\,\sin.(p-t) - 25.\,\sin.(3p-t) \\ + 3.\,\sin.(p+t) + 5.\,\sin.(3p+t) \end{array} \right.$$
$$+\tfrac{3}{2} a n z^2 \left(3.\,\sin.(p-t) + \sin.(p+t) \right)$$

§. 65.

In tertia denique aequatione habebimus simili modo

$$\frac{A}{a} = z$$
$$\frac{B}{a} = 3\,a\,z.\,\cos.\,p, + 3\,a\,x\,z\,\cos.\,p - 3\,a\,y\,z\,\sin.\,p$$
$$\frac{C}{a} = -\,3\,n\,z.\,\cos.\,t.$$

$$\frac{b}{a} = -3\,a\,\varkappa\,z\,(3.\,\mathrm{cos}.\,(p-t) + \mathrm{cos}.\,(p+t))$$
$$-\,3\,a\,\varkappa\,x\,z\,(3.\,\mathrm{cos}.\,(p-t) + \mathrm{cos}.\,(p+t))$$
$$+\,3\,a\,\varkappa\,y\,z\,(3.\,\mathrm{sin}.\,(p-t) + \mathrm{sin}.\,(p+t)).$$

§. 66.

Plurimum intererat, in ſingulis his membris partes ſecundum dimenſiones coordinatarum noſtrarum x, y et z diſponi; quoniam enim hae coordinatae ſunt admodum paruae prae vnitate, eo clarius hinc adparet, quaenam partes reſpectu reliquarum ſint valde exiguae. Deinde ratione ipſorum horum membrorum manifeſtum eſt, prima membra, $\frac{M}{a}$; $\frac{A}{a}$, et $\frac{a}{a}$ reliquis cenſenda eſſe maiora, propterea quod litterae a et $\varkappa$ ſunt fractiones ſatis paruae; vnde ſequitur, vltima membra $\frac{\mathfrak{D}}{a}$, $\frac{D}{a}$ et $\frac{b}{a}$ praecedentibus multo eſſe minora atque in ſequentibus patebit, haec postrema membra ſine vllo errore negligi poſſe.

CAPVT X.

CAPVT X.

EVOLVTIO MEMBRORVM PER w^3 DIVISORVM SEV EORVM, QVAE LITTERA λ SVNT ADFECTA.

§. 67.

Quoniam coordinatae x, y et z prae vnitate sunt valde paruae, formulam irrationalem

$$((1+x)^2+y^2+z^2)^{\frac{3}{2}}$$

commode in seriem vehementer conuergentem euoluere licebit; quod quo facilius praestari possit, partem $(1+x)^2$ primum non euoluamus, sed tanquam simplicem terminum spectemus, cuius respectu pars altera y^2+z^2 vtique erit exigua, ex quo secundum notissimam euolutionis regulam habebimus

$$((1+x)^2+y^2+z^2)^{-\frac{3}{2}} = \frac{1}{(1+x)^3} - \frac{3(y^2+z^2)}{2(1+x)^5} + \frac{3.5(y^2+z^2)^2}{2.4(1+x)^7} - \frac{3.5.7(y^2+z^2)^3}{2.4.6(1+x)^9} \text{ etc.}$$

Abunde autem sufficit, hanc seriem vsque ad sextam potestatem produxisse, quandoquidem, vti in sequentibus patebit, ne sexta quidem dimensione opus est.

§. 68.

§. 68.

Hac facta euolutione, leui negotio denominatores tollentur ope sequentium reductionum:

$$\frac{1}{(1+x)^2}=1-2x+3x^2-4x^3+5x^4-6x^5+7x^6.$$
$$\frac{1}{(1+x)^3}=1-3x+6x^2-10.x^3+15.x^4-21.x^5.$$
$$\frac{1}{(1+x)^4}=1-4x+10.x^2-20.x^3+35.x^4.$$
$$\frac{1}{(1+x)^5}=1-5x+15.x^2-35.x^3.$$
$$\frac{1}{(1+x)^6}=1-6x+21.x^2.$$
$$\frac{1}{(1+x)^7}=1-7.x.$$
$$\frac{1}{(1+x)^8}=1.$$

§. 69.

Cum igitur in prima nostra aequatione occurrat membrum

$$\frac{\lambda(1+x)}{((1+x)^2+y^2+z^2)^{\frac{3}{2}}},$$

habebimus,

$$\frac{1+x}{((1+x)^2+y^2+z^2)^{\frac{3}{2}}}=\frac{1}{(1+x)^2}-\frac{3(y^2+z^2)}{2(1+x)^4}$$
$$+\frac{3.5(y^2+z^2)^2}{2.4.(1+x)^6}-\frac{3.5.7(y^2+z^2)^3}{2.4.6.(1+x)^8}$$

qui

qui termini fecundum dimenfionum numerum euoluantur hoc modo:

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + 5.x^4 - 6.x^5 + 7.x^6$$

$$\frac{-3(y^2+z^2)}{2(1+x)^4} = -\tfrac{3}{2}(y^2+z^2) + 6x(y^2+z^2) - 15x^2(y^2+z^2) + 30x^3(y^2+z^2) - \tfrac{105}{2}x^4(y^2+z^2)$$

$$\frac{+15(y^2+z^2)^2}{8(1+x)^6} = +\tfrac{15}{8}(y^2+z^2)^2 - \tfrac{45}{4}x(y^2+z^2)^2 + \tfrac{315}{8}x^2(y^2+z^2)^2$$

$$\frac{-35(y^2+z^2)^3}{16(1+x)^8} = -\tfrac{35}{16}(y^2+z^2)^3$$

quocirca iftud membrum primae aequationis

$$\frac{\lambda(1+x)}{((1+x)^2+y^2+z^2)^{\frac{3}{2}}}$$

euoluitur in fequentes terminos:

$\lambda, -2\lambda x, +3\lambda x^2 - \tfrac{3}{2}\lambda(y^2+z^2), -4\lambda x^3 + 6\lambda x(y^2+z^2),$
$+5\lambda x^4 - 15\lambda x^2(y^2+z^2) + \tfrac{15}{8}\lambda(y^2+z^2)^2,$
$-6\lambda x^5 + 30\lambda x^3(y^2+z^2) - \tfrac{45}{4}\lambda x(y^2+z^2)^2,$
$+7\lambda x^6 - \tfrac{105}{2}\lambda x^4(y^2+z^2) + \tfrac{315}{8}\lambda x^2(y^2+z^2)^2 - \tfrac{35}{16}(y^2+z^2)^3$

§. 70.

Simili modo pro fecunda aequatione inftituantur fequentes euolutiones:

$$\frac{y}{(1+x)^3} = y - 3xy + 6x^2y - 10.x^3y + 15.x^4y - 21.x^5y$$

$$\frac{-3y(y^2+z^2)}{2(1+x)^5} = -\tfrac{3}{2}y(y^2+z^2) + \tfrac{15}{2}xy(y^2+z^2) - \tfrac{45}{2}x^2y(y^2+z^2) + \tfrac{105}{2}x^3y(y^2+z^2)$$

$$\frac{+15.y(y^2+z^2)^2}{8(1+x)^7} = +\tfrac{15}{8}y(y^2+z^2)^2 - \tfrac{105}{8}xy(y^2+z^2)^2$$

vnde pro noſtra ſecunda aequatione membrum

$$\frac{\lambda y}{((1+x)^2+y^2+z^2)^{\frac{5}{2}}}$$

reſoluitur in terminos ſequentes ſecundum dimenſiones diſpoſitos:

$\lambda y, -3\lambda xy, +6\lambda x^2y-\frac{3}{2}\lambda y(y^2+z^2),$

$-10.\lambda x^3y+\frac{15}{2}\lambda xy(y^2+z^2),$

$+15.\lambda x^4y-\frac{45}{2}\lambda x^2y(y^2+z^2)+\frac{15}{8}\lambda y(y^2+z^2)^2,$

$-21.\lambda x^5y+\frac{105}{2}\lambda x^3y(y^2+z^2)-\frac{105}{8}\lambda xy(y^2+z^2)^2.$

§. 71.

Pro tertia denique aequatione fiant iſtae euolutiones:

$$\frac{z}{(1+x)^3}=z-3xz+6x^2z-10.x^3z+15.x^4z-21.x^5z$$

$$\frac{-3z(y^2+z^2)}{2(1+x)^5}=-\frac{3}{2}z(y^2+z^2)+\frac{15}{2}xz(y^2+z^2)$$

$$-\frac{45}{2}x^2z(y^2+z^2)+\frac{105}{2}x^3z(y^2+z^2)$$

$$\frac{+15.z(y^2+z^2)^2}{8(1+x)^7}=+\frac{15}{8}z(y^2+z^2)^2-\frac{105}{8}xz(y^2+z^2)^2$$

vnde pro tertia aequatione membrum

$$\frac{\lambda z}{((1+x)^2+y^2+z^2)^{\frac{5}{2}}}$$

euoluitur in ſequentes terminos

$\lambda z, - 3\lambda x z, + 6\lambda x^2 z - \frac{3}{2}\lambda z(y^2 + z^2),$

$- 10.\lambda x^3 z + \frac{15}{2}\lambda x z(y^2 + z^2),$

$+ 15.\lambda x^4 z - \frac{45}{2}\lambda x^2 z(y^2 + z^2) + \frac{15}{8}\lambda z(y^2 + z^2)^2,$

$- 21.\lambda x^5 z + \frac{105}{2}\lambda x^3 z(y^2 + z^2) - \frac{105}{8}\lambda x z(y^2 + z^2)^2.$

§. 72.

Operae iam pretium erit, iſtos euolutos valores in noſtris tribus aequationibus actu ſubſtitui; at vero conuenict, cum his modo euolutis terminis etiam eos, qui in formulis $\frac{\mathfrak{A}}{a}$, $\frac{A}{a}$ et $\frac{a}{a}$ continentur, ſimul coniungi et ſecundum dimenſionum numerum diſponi; reliqua vero membra per $a_{,}, x$ et $a\,x$ adfecta ſeorſim, vt hactenus, characteribus aſſumtis repraeſententur, quandoquidem in calculo haec poſteriora membra peculiari modo tractari oportet. Hoc obſeruato ſequenti modo repraeſentabitur

Aequatio Prima.

$$\frac{ddx}{dt^2} - \frac{2(m+1)dy}{dt}, + \lambda - m^2 - 2m - \tfrac{3}{2} - \tfrac{3}{2}\,\text{coſ.}\,2p,$$

$$-(2\lambda + m^2 + 2m + \tfrac{3}{2})x - \tfrac{3}{2}x.\text{coſ.}\,2p + \tfrac{3}{2}y.\,\text{ſin.}\,2p,$$

$$+ 3\lambda x^2 - \tfrac{3}{2}\lambda(y^2 + z^2), - 4\lambda x^3 + 6\lambda x(y^2 + z^2),$$

$$+ 5\lambda x^4 - 15.\lambda x^2(y^2 + z^2) + \tfrac{15}{8}\lambda(y^2 + z^2)^2,$$

$$- 6\lambda x^5 + 30.\lambda x^3(y^2 + z^2) - \tfrac{45}{4}\lambda x(y^2 + z^2)^2,$$

$$+ 7\lambda x^6 - \tfrac{105}{2}\lambda x^4(y^2 + z^2) + \tfrac{315}{8}\lambda x^2(y^2 + z^2)^2$$

$$- \tfrac{35}{16}(y^2 + z^2)^3 + \frac{\mathfrak{B}}{a} + \frac{\mathfrak{C}}{a} + \frac{\mathfrak{D}}{a} = 0.$$

Aequatio Secunda.

$$\frac{ddy}{dt^2} + \frac{2(m+1)dx}{dt}, + \tfrac{3}{2}\,\text{fin.}\,2p, + (\lambda - m^2 - 2m - \tfrac{3}{4})y$$
$$+ \tfrac{3}{2}x.\text{fin.}\,2p + \tfrac{3}{2}y.\text{cof.}\,2p, - 3\lambda xy, + 6\lambda x^2y - \tfrac{3}{2}\lambda y(y^2+z^2),$$
$$- 10.\lambda x^3y + \tfrac{15}{2}\lambda xy(y^2+z^2),$$
$$+ 15.\lambda.x^4y - \tfrac{45}{2}\lambda x^2y(y^2+z^2) + \tfrac{15}{8}\lambda y(y^2+z^2)^2,$$
$$- 21.\lambda x^5y + \tfrac{105}{2}\lambda x^3y(y^2+z^2) - \tfrac{105}{8}\lambda xy(y^2+z^2)^2$$
$$+ \frac{B}{a} + \frac{C}{a} + \frac{D}{a} = 0.$$

Aequatio Tertia.

$$\frac{ddz}{dt^2}, + (\lambda+1)z, - 3\lambda xz, + 6\lambda x^2z - \tfrac{3}{2}\lambda z(y^2+z^2),$$
$$- 10.\lambda.x^3z + \tfrac{15}{2}\lambda xz(y^2+z^2), + 15.\lambda x^4z - \tfrac{45}{2}\lambda x^2z(y^2+z^2)$$
$$+ \tfrac{15}{8}\lambda z(y^2+z^2)^2, - 21.\lambda x^5z + \tfrac{105}{2}\lambda x^3z(y^2+z^2)$$
$$- \tfrac{105}{8}\lambda xz(y^2+z^2)^2 + \frac{b}{a} + \frac{c}{a} + \frac{d}{a} = 0.$$

CAPVT XI.

DETERMINATIO LITTERAE λ IN NOSTRAS AEQVATIONES INTRODVCTAE.

§. 73.

Cum sit $\lambda = \frac{\nu}{a^3}$ et $\nu = \frac{\Theta}{S+\Theta}$ (§. 25.); haec postrema fractio valorem quidem habet determinatum, sed quem nobis aliter, nisi per ipsa phaenomena, cognoscere non licet, cum potius ex cognito aliunde valore litterae ν ratio massarum Θ et S concludi debeat. Hic quidem meminisse iuuabit, litteram S denotare massam Solis, litteram vero Θ aggregatum massarum Terrae et Lunae coniunctim.

§. 74.

Quod autem ad quantitatem a attinet, quam spectamus tanquam distantiam mediam Lunae a Terra, dum posuimus $X = a(1 + x)$, facile perspicitur, hanc quantitatem valorem per se determinatum non habere, sed plurimum a nostro arbitrio pendere; nulla enim necessitas vrget, vt huic litterae a valor praecise intermedius inter maximum et minimum valorem, quem quantitas X vnquam accipere potest, tribuatur, sed dummodo valor litterae a non notabiliter discrepet a medio illo arithmetico inter minimos et maximos valores X, neutiquam calculus noster turbabitur, cum semper, vtcunque littera a assumeretur, valor ipsius x peculiares inde determinationes acciperet. Ex quo nobis omnino liberum relinquitur, quantitatem hanc a siue aliquantillum maiorem siue minorem accipere, quam illud medium arithmeticum exigeret. Ex quo manifestum est, etiam ipsam litteram λ arbitrio nostro permitti, dumne extra certos limites extrauagetur.

§. 75.

Mox autem patebit, quantitatem x per seriem cosinuum certorum angulorum exprimi, dum altera coordinata y per similem seriem sinuum exhibetur; cum enim huiusmodi cosinus et sinus mox positiuos, mox negatiuos valores sortiantur, hac ratione commodissime valores litterarum x et y repraesentabuntur. Sin autem quantitas x praeter huiusmodi cosinus quantitatem quampiam constantem inuolueret; facile patet, eius valores positiuos vel maiores vel minores esse futuros negatiuis sicque omnino esset optandum, vt talis quantitas constans vel plane euanesceret vel saltim admodum exigua redderetur.

§. 76.

§. 76.

Quodsi ergo sumamus, quantitatem x per seriem certorum cosinuum, y vero per similem seriem sinuum exprimi, dum scilicet ipsi anguli tempori sunt proportionales, euidens est, in prima nostra aequatione terminum $\frac{ddx}{dt^2}$ per meros cosinus expressum iri, atque idem quoque euenire in altero termino $\frac{dy}{dt}$; ex terminis autem sequentibus, vbi quantitates x et y ad duas pluresue dimensiones assurgunt, fieri quidem posset, vt per eorum euolutionem quantitates constantes nascerentur, quae quidem vehementer paruae essent *futurae*, *propterea* quod ipsi hi termini per se sunt vehementer parui. Cum igitur in prima nostra aequatione insit terminus constans $\lambda - m^2 - 2m - \frac{3}{2}$; perspicuum est, nisi is sit vehementer paruus, aequationem nostram subsistere non posse; quare cum valor ipsius λ propemodum arbitrio nostro relinquatur, commodissime statuere licebit

$$\lambda = m^2 + 2m + \frac{3}{2} = (m+1)^2 + \frac{1}{2}.$$

Hoc enim modo certi erimus, in valorem ipsius x vel nullam plane quantitatem constantem absolutam vel saltim vehementer exiguam esse ingressuram.

§. 77.

Perpetuo ergo in posterum sit $\lambda = (m+1)^2 + \frac{1}{2}$ atque idcirco ante omnia necesse est, in eius valorem verum

verum numericum follicite inquiri; supra autem posuimus $m = \frac{dp}{dt}$, ita, vt sit $1 : m$ vti motus anomaliae mediae Solis t ad motum medium Lunae a Sole eodem tempore peractum. Sumamus ergo hoc tempus esse annum communem $365^{dier.}$ quo tempore motus anomaliae mediae Solis ex tabulis colligitur $11^s\ 29^\circ\ 44'\ 39''$, ad quem si motus apogei, qui interim est $1'\ 1''$ addatur, habebitur motus medius Solis pro eodem tempore $11^s\ 29^\circ\ 45'\ 40''$. Verum pro eodem tempore est motus Lunae medius $13^{rev.}\ 4^s\ 9^\circ\ 23'\ 3''$; vnde concluditur motus Lunae a Sole $= 12^{rev.}\ 4^s\ 9^\circ\ 37'\ 23''$; consequenter

$$m = \frac{12^{rev.}\ 4^s\ 9^\circ\ 37'\ 23''}{11^s\ 29^\circ 44'\ 39''}$$

$$\text{siue } m = \frac{16018643''}{1295079''} = 12,368854;$$

ac propterea $m + 1 = 13,36885$; cuius quadratum fiet $(m + 1)^2 = 178,7263$; ex quo, addito $\frac{1}{2} = 0,5$ prodit $\lambda = 179,2263$.

§ 78.

Verum circa hanc determinationem probe obseruari oportet, formulam $m + 1$, a qua λ potissimum pendet, hic stare pro $\frac{d\omega}{dt}$, dum ω longitudinem Lunae mediam referebat; quare cum interuallo vnius anni motus medius Lunae sit $13^{rev.}\ 4^s\ 9^\circ\ 23'\ 3''$;

haec

haec quantitas per motum annuum anomaliae mediae Solis 11^s 29° 44′ 39″ diuisa verum dabit valorem pro $m + 1$; qui ergo erit 13, 368903; atque hinc colligitur $\lambda = (m + 1)^2 + \frac{1}{8} = 179, 228928$; quo ergo valore in sequentibus calculis vtemur.

§. 79.

Constituto hoc valore litterae λ, poterimus nunc ternas nostras aequationes plane euolutas hic exhibere, vbi quidem in prima aequatione terminum purum x ad duo membra priora differentialia referamus, quoniam, vti mox patebit, ratio integrationis seu potius operatio aequiualens id suadet; reliqua vero membra secundum dimensiones disponamus, quibus denique reliqua per a, u et $a\,u$ adfecta subiungamus. Praeterea vero hic pulcerrime vsu venit, vt in secunda aequatione terminus absolutus purus y prorsus ex calculo euanescat.

Prima Aequatio.

$$\frac{ddx}{dt^2} - \frac{2(m+1)dy}{dt} - 3\lambda x, - \tfrac{3}{2}\,\text{cos}.\,2p, - \tfrac{3}{2}x\,\text{cos}.\,2p + \tfrac{3}{2}y.\,\text{sin}.\,2p,$$
$$+ 3\lambda x^2 - \tfrac{3}{2}\lambda(y^2+z^2), - 4\lambda x^3 + 6\lambda x(y^2+z^2),$$
$$+ 5\lambda x^4 - 15.\lambda x^2(y^2+z^2) + \tfrac{15}{8}\lambda(y^2+z^2)^2,$$
$$- 6\lambda x^5 + 30.\lambda x^3(y^2+z^2) - \tfrac{45}{4}\lambda x(y^2+z^2)^2,$$
$$+ 7\lambda x^6 - \tfrac{105}{2}\lambda x^4(y^2+z^2) + \tfrac{315}{8}\lambda x^2(y^2+z^2)^2 - \tfrac{35}{16}(y^2+z^2)^3,$$
$$- \tfrac{3}{8}a(3.\,\text{cos}.\,p + 5.\,\text{cos}.\,3p),$$
$$- \tfrac{3}{4}ax(3.\,\text{cos}.\,p + 5.\,\text{cos}.\,3p) + \tfrac{3}{4}ay(\text{sin}.\,p + 5.\,\text{sin}.\,3p),$$
$$- \tfrac{3}{8}ax^2(3.\,\text{cos}.\,p + 5.\,\text{cos}.\,3p) + \tfrac{3}{4}axy(\text{sin}.\,p + 5.\,\text{sin}.\,3p)$$
$$- \tfrac{3}{8}ay^2(\text{cos}.\,p - 5.\,\text{cos}.\,3p) + \tfrac{3}{2}az^2\,\text{cos}.\,p,$$
$$+ \tfrac{3}{4}n(2.\,\text{cos}.\,t + 7.\,\text{cos}.(2p-t) - \text{cos}.(2p+t))$$
$$+ \tfrac{3}{4}nx(2.\,\text{cos}.\,t + 7.\,\text{cos}.(2p-t) - \text{cos}.(2p+t))$$
$$- \tfrac{3}{4}ny(7.\,\text{sin}.(2p-t) - \text{sin}.(2p+t))$$
$$+ \tfrac{3}{8}an(9.\,\text{cos}.(p-t) + 3.\,\text{cos}.(p+t) + 25.\,\text{cos}.(3p-t) - 5.\,\text{cos}.(3p+t)),$$
$$+ \tfrac{3}{4}anx(9.\,\text{cos}.(p-t) + 3.\,\text{cos}.(p+t) + 25.\,\text{cos}.(3p-t) - 5.\,\text{cos}.(3p+t))$$
$$- \tfrac{3}{4}any(3.\,\text{sin}(p-t) + \text{sin}.(p+t) + 25.\,\text{sin}.(3p-t) - 5.\,\text{sin}.(3p+t)),$$
$$+ \tfrac{3}{8}anx^2(9.\,\text{cos}.(p-t) + 3.\,\text{cos}.(p+t) + 25.\,\text{cos}.(3p-t) - 5.\,\text{cos}.(3p+t))$$
$$- \tfrac{3}{4}anxy(3.\,\text{sin}.(p-t) + \text{sin}.(p+t) + 25.\,\text{sin}.(3p-t) - 5.\,\text{sin}.(3p+t))$$
$$+ \tfrac{3}{8}any^2(3.\,\text{cos}.(p-t) + \text{cos}(p+t) - 25.\,\text{cos}.(3p-t) + 5.\,\text{cos}.(3p+t))$$
$$- \tfrac{3}{8}anz^2(3.\,\text{cos}.(p-t) + \text{cos}.(p+t))$$
$$= 0.$$

Secunda

Secunda Aequatio.

$$\frac{ddy}{dt^2} + \frac{2(m+1)dx}{dt}, + \tfrac{3}{2}\sin. 2p, + \tfrac{3}{2}x \sin. 2p + \tfrac{3}{2}y \cos. 2p,$$
$$-3\lambda xy, + 6\lambda x^2y - \tfrac{3}{2}\lambda y(y^2+z^2),$$
$$-10\lambda x^3y + \tfrac{15}{2}\lambda xy(y^2+z^2),$$
$$+15\lambda x^4y - \tfrac{45}{2}\lambda x^2y(y^2+z^2) + \tfrac{15}{8}\lambda y(y^2+z^2)^2,$$
$$-21\lambda x^5y + \tfrac{105}{2}\lambda x^3y(y^2+z^2) - \tfrac{105}{8}\lambda xy(y^2+z^2)^2,$$
$$+ \tfrac{3}{8}a(\sin. p + 5.\sin. 3p),$$
$$+ \tfrac{3}{4}ax(\sin. p + 5.\sin. 3p) - \tfrac{3}{4}ay(\cos. p - 5.\cos. 3p),$$
$$+ \tfrac{3}{8}ax^2(\sin. p + 5.\sin. 3p) - \tfrac{3}{4}axy(\cos. p - 5.\cos. 3p)$$
$$+ \tfrac{3}{8}ay^2(3.\sin. p - 5.\sin. 3p) - \tfrac{3}{8}az^2.\sin. p,$$
$$- \tfrac{3}{4}\varkappa(7.\sin.(2p-t) - \sin.(2p+t))$$
$$- \tfrac{3}{4}\varkappa x(7.\sin.(2p-t) - \sin.(2p+t))$$
$$+ \tfrac{3}{4}\varkappa y(2.\cos. t - 7.\cos.(2p-t) + \cos.(2p+t)),$$
$$- \tfrac{3}{8}a\varkappa(3.\sin.(p-t) + \sin(p+t) + 25.\sin.(3p-t) - 5.\sin(3p+t)),$$
$$- \tfrac{3}{4}a\varkappa x(3.\sin.(p-t) + \sin.(p+t) + 25.\sin.(3p-t) - 5.\sin.(3p+t))$$
$$+ \tfrac{3}{4}a\varkappa y(3.\cos.(p-t) + \cos.(p+t) - 25.\cos.(3p-t) + 5.\cos.(3p+t))$$
$$- \tfrac{3}{8}a\varkappa x^2(3.\sin.(p-t) + \sin.(p+t) + 25.\sin.(3p-t) - 5.\sin.(3p+t))$$
$$+ \tfrac{3}{4}a\varkappa xy(3.\cos.(p-t) + \cos.(p+t) - 25.\cos(3p-t) + 5.\cos.(3p+t))$$
$$- \tfrac{3}{8}a\varkappa y^2(9.\sin.(p-t) + 3.\sin.(p+t) - 25.\sin.(3p-t) + 5.\sin(3p+t))$$
$$+ \tfrac{3}{8}a\varkappa z^2(3.\sin.(p-t) + \sin.(p+t))$$
$$= 0.$$

Tertia Aequatio.

$$\frac{ddz}{dt^2} + (\lambda + 1)z - 3\lambda x z + 6\lambda x^2 z - \tfrac{3}{2}\lambda z(y^2 + z^2)$$
$$- 10\lambda x^3 z + \tfrac{15}{2}\lambda x z(y^2 + z^2),$$
$$+ 15.\lambda x^4 z - \tfrac{45}{2}\lambda x^2 z(y^2 + z^2) + \tfrac{15}{8}\lambda z(y^2 + z^2)^2,$$
$$- 21.\lambda x^5 z + \tfrac{105}{2}\lambda x^3 z(y^2 + z^2) - \tfrac{105}{8}\lambda x z(y^2 + z^2)^2,$$
$$+ 3 a z.\operatorname{cof.} p, + 3 a x z \operatorname{cof.} p - 3 a y z.\operatorname{fin.} p,$$
$$- 3 n z.\operatorname{cof.} t, - 3 a n z(3.\operatorname{cof.}(p - t) + \operatorname{cof.}(p + t))$$
$$- 3 a n x z(3.\operatorname{cof.}(p - t) + \operatorname{cof.}(p + t))$$
$$+ 3 a n y z(3.\operatorname{fin.}(p - t) + \operatorname{fin.}(p + t))$$
$$= 0.$$

CAPUT XII.

CAPVT XII.

REGVLAE GENERALES PRO RESOLVTIONE NOSTRARVM AEQVATIONVM.

§. 80.

Iam obseruauimus, quantitatem x per seriem infinitam cosinuum certorum angulorum, quantitatem vero y per similem seriem sinuum eorundem angulorum exprimi. Cuius rei ratio vel inde patet, quod in prima aequatione termini absoluti, qui nullam nostrarum variabilium inuoluunt, continent cosinus angulorum $2p$; p; $3p$; t; item $2p + t$; $p + t$; $3p + t$, dum in similibus terminis secundae aequationis sinus eorundem angulorum reperiuntur; ex quo necesse est, vt quantitas x etiam eosdem cosinus, quantitas vero y eosdem sinus complectatur. Praeterea vero manifestum est, ex terminis magis complicatis vtriusque aequationis per multiplicationem sinuum et cosinuum oriri angulos vtcunque ex memoratis siue per additionem siue per subtractionem resultantes; atque in priore quidem aequatione istorum angulorum cosinus, in altera vero sinus. Quocirca etiam necesse est, vt quantitas x insuper omnium horum angulorum cosinus, quantitas y vero sinus inuoluat, vnde per vlteriorem combinationem denuo noui anguli oriuntur; quorum cosinus in valorem ipsius x, sinus vero in valorem ipsius y ingredi debebunt; ita, vt omnium horum angulorum numerus reuera in infinitum excrescere sit censendus.

§. 81.

Quo haec clarius percipiamus, in vtraque aequatione, prima scilicet et secunda, partes principales a partibus annexis distinguamus, pro prima scilicet aequatione ad partes principales referamus tres primores terminos

$$\frac{ddx}{dt^2} - \frac{2(m+1)dy}{dt} - 3\lambda x,$$

quippe in quibus binae variabiles x et y vnicam habent dimensionem neque angulum quempiam p et s sibi habent admixtum; omnes vero reliquos sequentes terminos sub nomine partium annexarum complectamur. In secunda vero aequatione pars principalis tantum continet binos terminos primores

$$\frac{ddy}{dt^2} + \frac{2(m+1)dx}{dt}$$

reliqui omnes partibus annexis tribuantur. Has quoque partes principales in aequationibus nostris ante euolutis, commatibus a partibus annexis distinximus.

§. 82.

§. 82.

Quoniam in partibus annexis, vbi coordinatas nostrae multifariam inter se complicantur, tam multi anguli oriuntur, quos denuo in ipsos valores x et y ingredi oportet; ponamus, si partes annexae primae aequationis hoc modo euoluantur, ibi occurrere terminum $\mathfrak{M}$. cos. ω; vbi coefficiens $\mathfrak{M}$ est numerus constans; ω autem denotet angulum quemcunque, qui quidem semper tempori est proportionalis et statuamus $\frac{d\omega}{dt} = \mu$; tum vero ex euolutione partium annexarum secundae aequationis occurret terminus huius formae M. sin. ω; cuius coefficiens M itidem erit quantitas constans. Quibus positis manifestum est, ipsas quantitates x et y similes terminos inuoluere debere, quia alioquin isti termini ex partibus annexis oriundi per partes principales non tollerentur, vti natura aequationum postulat.

§. 83.

Statuamus ergo hinc quantitatem x complecti terminum $\mathfrak{N}$. cos. ω; quantitatem vero y terminum N. sin. ω; atque nunc quidem exoritur haec quaestio maximi momenti, quemadmodum ex terminis cognitis $\mathfrak{M}$. cos. ω et M. sin. ω, quos scilicet partes annexae exhibent, inuestigari oporteat ipsos terminos $\mathfrak{N}$. cos. ω et N. sin. ω, qui inde ad valores ipsarum x et y accedunt. Haec autem quaestio facile sequenti modo

modo resoluetur; tantum enim opus est, valores quaesitos $\mathfrak{N}. \cos. \omega$ et $N \sin. \omega$ loco x et y in partibus principalibus vtriusque aequationis substitui; quae enim inde nascuntur, illos terminos ex partibus annexis ortos destruere debent. Posito autem $x = \mathfrak{N}. \cos. \omega$ et $y = N. \sin. \omega$ ob $d\omega = \mu\, dt$; fiet

$$\frac{dx}{dt} = -\mu\, \mathfrak{N}. \sin\, \omega; \quad \frac{ddx}{dt^2} = -\mu^2\, \mathfrak{N}. \cos. \omega; \text{ et}$$
$$\frac{dy}{dt} = \mu.\, N. \cos. \omega; \quad \frac{ddy}{dt^2} = -\mu^2.\, N. \sin. \omega.$$

vnde nanciscimur binas sequentes aequationes:

$$\text{I}^\circ.\ -\mu^2\, \mathfrak{N}. \cos. \omega - 2(m+1)\mu.\, N \cos. \omega - 3\lambda\, \mathfrak{N}. \cos. \omega + \mathfrak{M}. \cos. \omega = 0.$$

$$\text{II}^\circ.\ -\mu^2.\, N. \sin. \omega - 2(m+1)\mu.\, \mathfrak{N}. \sin. \omega + M. \sin. \omega = 0.$$

§. 84.

Hae ergo duae aequationes ad meros terminos constantes reducuntur; fientque

$$\text{I}^\circ.\ -\mu^2.\, \mathfrak{N} - 2(m+1)\mu\, N - 3\lambda\, \mathfrak{N} + \mathfrak{M} = 0.$$

$$\text{II}^\circ.\ -\mu^2\, N - 2(m+1)\mu\, \mathfrak{N} + M = 0.$$

Posterior statim dat

$$N = -\frac{2(m+1)}{\mu}\mathfrak{N} + \frac{M}{\mu^2}$$

qui valor in priore substitutus praebet

$$-\mu^2.\, \mathfrak{N} + 4(m+1)^2\, \mathfrak{N} - \frac{2(m+1)}{\mu} M - 3\lambda\, \mathfrak{N} + \mathfrak{M} = 0.$$

Nunc

Nunc autem recordemur, esse $\lambda = (m+1)^2 + \frac{1}{4}$ vnde fit $(m+1)^2 = \lambda - \frac{1}{4}$ sicque habebitur

$$-\mu^2.\mathfrak{N} + (4\lambda - 2)\mathfrak{N} - 3\lambda\mathfrak{N}$$
$$-\frac{2(m+1)}{\mu}M + \mathfrak{M} = 0$$

siue

$$-\mu^2\mathfrak{N} + (\lambda - 2)\mathfrak{N} - \frac{2(m+1)}{\mu}M + \mathfrak{M} = 0$$

vnde concluditur

$$\mathfrak{N} = \frac{\frac{2(m+1)}{\mu}.M - \mathfrak{M}}{\lambda - 2 - \mu^2}$$

Inuento autem valore $\mathfrak{N}$ erit

$$N = \frac{2(m+1)}{\mu}\mathfrak{N} + \frac{M}{\mu^2}.$$

§. 85.

Quoties ergo euolutis partibus annexis primae et secundae aequationis in priore occurrat terminus $\mathfrak{M}.\cos.\omega$; in altera vero terminus $M.\sin.\omega$; existente $d\omega = \mu\, dt$, tum similes termini accedent ad ipsos valores litterarum x et y, ac pro x quidem iungi debebit terminus $\mathfrak{N}.\cos.\omega$, ita, vt sit

$$\mathfrak{N} = \frac{\frac{2(m+1)}{\mu}.M - \mathfrak{M}}{\lambda - 2 - \mu^2}$$

tum

tum vero pro y adiungi oportebit terminum N. fin. ω, existente

$$N = \frac{M}{\mu^2} - \frac{2(m+1)}{\mu}. \mathfrak{N}.$$

§. 86.

In prima aequatione vsu venire potest, vt in euolutione partium annexarum prodeat terminus absolute constans, puta α; dum pro secunda aequatione correspondens terminus plane non occurrit, sicque pro hoc casu erit $\mathfrak{M} = \alpha$; $M = 0$; et $\mu = 0$. At si in nostris formulis isti valores surrogentur nihil inde definitur, propterea quod $\frac{M}{\mu}$ abit in $\frac{0}{0}$. Verum ipsae hae aequationes initiales §. 84. dubium statim tollunt; prima enim dat $- 3\lambda\mathfrak{N} + \alpha = 0$ ideoque $\mathfrak{N} = + \frac{\alpha}{3\lambda}$; at per se manifestum est, fore $N = 0$.

§. 87.

Hic autem obseruasse plurimum iuuabit, si eueniat, vt sit proxime siue $\mu = 0$, siue $\mu^2 = \lambda - 2$; vtroque casu valorem litterae $\mathfrak{N}$ in immensum excrescere posse, quod deinde etiam in altera littera N pariter continget. Quamobrem hi duo casus, quo vel μ habet valde exiguum valorem vel proxime aequetur ipsi $\sqrt{(\lambda - 2)}$, praecipue sunt notatu digni, quoniam inde in vtramque quantitatem x et y ingentes termini ingredi possunt, qui adeo fierent infiniti,

si

si reuera foret vel $\mu = 0$ vel $\mu = \sqrt{(\lambda + 2)}$; priore quidem casu res nulla laboraret difficultate, quoniam ad casum modo tractatum, vbi $\mu = 0$ reuoluitur. Alter vero multo magis foret notatu dignus, quoniam inde non amplius cosinus vel sinus ω, sed ipse angulus ω ad valores ipsarum x et y accederet; propterea quod haec formula $\frac{\sin.\,\delta\omega}{\delta}$ casu, quo δ euanescit, abit in ipsum angulum ω.

§. 88.

Huiusmodi autem casus in motu Lunae aliisue motibus realibus nunquam euenire posse vel inde concludere licet, quod, si quantitates x et y inuoluerent angulum quemcunque ω, labente tempore tales termini in infinitum excrescerent; praeterea vero inde in partibus annexis prodirent termini quadratum cubum ~~atque adeo omnes superiores potestates~~ eiusdem anguli inuoluentes; vnde etiam similes termini in ipsas expressiones pro x et y ingredi deberent, quod vtique maxime foret absurdum.

§. 89.

Cum igitur hanc elegantem dederimus regulam resolutioni binarum priorum aequationum inseruientem, multo facilius erit, simili modo tertiam nostram aequationem resoluere. Cum enim hic pars principalis sit censenda $\frac{ddz}{dt^2} + (\lambda + 1)z$, reliquis terminis ad partes annexas reiectis, si ipsa quantitas z per seriem sinuum certorum angulorum exprimatur, in euolutione partium annexarum itidem meri sinus occurrent, inter quos si reperiatur talis terminus $M. \sin. \omega$ existente iterum $d\omega = \mu. dt$; hinc etiam ad ipsum valorem litterae z similis terminus $N. \sin. \omega$ accedet; facta enim substitutione fit

$$-\mu^2. N. \sin. \omega + (\lambda + 1) N. \sin. \omega + M. \sin. \omega = 0$$

vnde statim colligitur

$$N = \frac{M}{\mu^2 - \lambda - 1}$$

sicque ex cognito M facillime reperitur valor N.

CAPVT XIII.

INTRODVCTIO ANOMALIAE MEDIAE LVNAE, AC PRAETEREA ARGVMENTI LATITVDINIS.

§. 90.

Anguli, quorum sinus vel cosinus hactenus in nostras aequationes ingrediuntur, sunt primo elongatio Lunae media a Sole $= p$, et secundo anomalia media Solis $= t$ et nonnulli alii ex horum combinatione orti. At si has aequationes integrare liceret; facile intelligitur per integrationem completam alium insuper angulum in valores ipsarum x et y introductum iri; quamuis ergo ipsam integrationem suscipere non liceat, tamen istum angulum designemus littera q et statuamus $dq = n.dt$; quem angulum sine integratione sequenti ratiocinio colligere poterimus.

§. 91.

Pro prima ergo nostra quantitate x sit terminus hinc in eam ingressus $= K.\cos. q$, qui cum involuat constantem ex integratione oriundam, coefficiens K denotabit quantitatem mere arbitrariam, cuius valorem demum ex ipso Lunae motu concludi oportebit. Ad has circumstantias si leuiter tantum attendamus, facili intelligemus, istum coefficientem constantem K idem exprimere, quod vulgo in astronomia excentricitas vocari solet; tum vero ipse angulus q manifesto conuenit cum anomalia media Lunae, quatenus etiam tempori est proportionalis; vnde posuimus $d q = n d t$.

§. 92.

Quodsi ergo in seriem cosinuum, qua quantitas x exprimitur, ingrediatur terminus K cos. q in seriem vero sinuum pro quantitate y terminus respondens N. sin. q; tum euolutis omnibus partibus annexis pro priore quidem aequatione plures termini eiusdem formae cos. q resultare poterunt quos omnes simul sumtos denotemus formula 𝔐. cos. q. Simili modo pro altera aequatione omnes termini formae sin. q iunctim sumti praebeant M. sin. q; atque nunc sequens ratiocinium institui conueniet.

§. 93.

Quoniam per hypothesin quantitas x complectitur terminum K. cos. q et numerus K indeterminatus relinqui debet, si huc superiorem regulam in §. 85. accommodemus ob $\mathfrak{N} = K$ et $\mu = n$, proueniet ista aequatio

K =

$$K = \frac{\frac{2(m+1)}{n}.\,M - \mathfrak{M}}{\lambda - 2 - n^2}$$

ex qua non coefficiens K ſed ipſe numerus n determinari debet; facile autem patet, ſemper n ita poſſe definiri, vt huic aequationi ſatisfiat; atque eo inuento habebimus pro quantitate y, vti ipſa regula oſtendit,

$$N = \frac{M}{n^2} - \frac{2(m+1)}{n}.\,K.$$

§. 94.

Difficillimum autem foret, hoc modo in verum valorem numeri n inquirere; quandoquidem pro $\mathfrak{M}$ et M omnes plane terminos huius formae coniunctim ſumi oporteret; alioquin enim verus valor litterae n non innoteſceret; quamobrem vtique cogimur valorem huius litterae n ex ipſis obſeruationibus colligere; quia vero anomalia media q prodit, ſi longitudo apogei ſubtrahatur a longitudine media Lunae; vnde ſi vt ſupra ex tabulis capiamus pro interuallo temporis 365^{dier}.

Mot. Lunae med. - - -	$13^{rev.}$	4^s.	9°.	$23'$.	$3''$.
Mot. apog. - - - -		1	10	39	50
Mot. anom. med. q -	$13^{rev.}$	2^s.	28°.	$43'$.	$13''$.
qui angulus diuiſus per -		11	29	44	39

quippe qui anomaliae mediae Solis t reſpondet, dabit valorem numeri

$$n = \frac{17167393}{1295079} = 13,255865.$$

§. 95.

§. 95.

Quanquam autem iste valor ex ipso coelo est petitus, tamen si rem accurate prosequi velimus, eo vti non licet; cum enim Luna non solum a viribus Solis et Terrae vrgeatur, sed insuper aliis quibusdam quam minimis viribus turbetur motus iste apogei ex tabulis desumtus pro effectu omnium harum virium iunctim sumtarum est censendus; hic autem tantum binas vires principales Solis et Terrae contemplamur, ex quo eueniet, vt valor litterae n ex supra inuenta aequatione

$$K = \frac{\frac{2(m+1)}{n} M - \mathfrak{M}}{\lambda - 2 - n^2}$$

aliquantillum ab illo valore discrepare debeat; quantum scilicet reliquae exiguae vires in hoc negotio efficere valent; atque hinc mirari non debemus, si iste valor ipsius n ex obseruationibus conclusus non perfecte isti aequationi satisfaciat.

§. 96.

Vtique autem in nostro calculo litterae n eum tantum valorem tribui oporteret, qui ipsi ratione solarum virium Terrae et Solis conuenit, id quod fieri poterit, si a motu illo annuo apogei Lunae effectus a viribus extraordinariis oriundus subtrahatur; constat enim a huiusmodi viribus omnium planetarum aphelia

aphelia aliquantillum promoueri; namque si planetae principales a sola vi Solis vrgerentur, eorum apsides inter stellas fixas plane immotae essent perseueraturae. Nullum igitur est dubium, quin vires exiguae perturbatrices motum apogei Lunae aliquantillum promoueant.

§. 97.

Cum aphelium Terrae ob actionem harum virium intra stellas fixas quotannis per 13″ circiter promoueatur, earundem virium effectus in Luna circiter tredecies maior fore est putandus, propterea quod *Luna interea* tredecim reuolutiones absoluit sicque effectus harum virium in motu annuo apogei Lunae foret quasi 170″ ratione stellarum fixarum, ideoque ratione aequinoctiorum $221'' = 3'\,41''$. Auferamus ergo hunc effectum a motu annuo apogei ex tabulis sumto; siue quod eodem redit ad motum anomaliae addamus 221″ et habebimus pro illo numeratore 1716761 4″; vnde concluditur

$$n = 13,25604.$$

§. 98.

Si tantum ad inaequalitates, quae in motu Lunae deprehenduntur, respiceremus, perinde fere foret, vtro valore pro n vteremur, cum tota differentia sit tam exigua, vt inde in locum Lunae discrimen vix aliquot minutorum secundorum redundare posset. Praeclare autem nobiscum agi censendum esse supra iam animaduertimus, dummodo in motu Lunae assignando non vltra 20 vel 30″ aberrauerimus.

§. 99.

Introductio igitur huius anguli q ad binas nostras priores aequationes pertinet; tertia vero aequatio etiam nouum quendam angulum recipiet, quem littera r designemus, atque facile intelligitur, hunc angulum conuenire cum astronomorum argumento latitudinis medio, quod reperitur, si a longitudine Lunae media subtrahamus longitudinem Nodi ascendentis. Ponamus ergo tertiam nostram quantitatem z continere istum terminum praecipuum $i. \sin. r$, atque manifestum est, coefficientem i quantitatem inclinationis orbitae lunaris ad eclipticam inuoluere, qui ergo perinde ac superior K arbitrarius natura sua esse debet.

§. 100.

Ad hunc autem angulum ex theoria determinandum ponamus $d r = l. d t$, atque, facta euolutione partium annexarum in tertia aequatione, ponamus omnes terminos formae $\sin. r$ occurrentes iunctim sumtos esse $M. \sin. r$; quare cum ipsa quantitas z contineat terminum $i. \sin. r$, regula superiori ex §. 89. huc translata fit $\omega = r$; $\mu = l$; $N = i$, sicque habebitur ista aequatio $i = \frac{M}{l^2 - \lambda - 1}$ ex qua aequatione non i, sed ipse numerus l debet determinari.

§. 101.

§. 101.

Valor autem, quem hinc pro l eliciemus, vix discrepat ab eo, quem coelum ostendit, neque ergo operae pretium erit, actionis exiguarum illarum virium hic rationem habere, vnde valorem ipsius l ex obseruationibus deductum adhibebimus

Motus annuus med. ☾	$13^{rev.} 4^{s}\ 9^{\circ}\ 23'\ 3''$
Motus Nodi annuus	$19^{\circ}\ 19'\ 43''$ retrogr.
Motus argum. latitud.	$13^{rev.} 4^{s}\ 28^{\circ}\ 42'\ 46''$

vnde concludimus,

$$l = \frac{17383366}{1295075} = 13,42263$$

quo valore in posterum vtemur.

CAPVT XIV.

DE VARIIS ORDINIBVS ERRORVM SIVE INAEQVALITATVM LVNAE.

§. 102.

Iam animaduertimus, ternas nostras coordinatas x, y, z per series infinitas certorum sinuum et cosinuum exprimi; nunc autem ostendemus, has easdem series in certos ordines seu classes commode diuidi posse, ac primo quidem quantitates x et y eiusmodi terminos inuoluent, qui neque litteras a et $\varkappa$, neque elementa modo introducta K et i contineant; hosque terminos absolutos vocabimus, qui nobis primum ordinem constituent; et manifesto ex terminis absolutis ipsarum aequationum differentialium, qui sunt $-\frac{3}{2}$ cos. $2p$ et $+\frac{3}{2}$ sin. $2p$, originem trahunt.

§. 103.

Deinde introducta excentricitate K manifestum est, in valores litterarum x et y ingressuros esse terminos, qui non solum per ipsam hanc litteram K ſint multiplicati, ſed etiam per eiusdem quadratum KK quin et per K^3, omnesque altiores eius poteſtates; quia vero infra videbimus, valorem huius litterae K eſſe quaſi $\frac{1}{18}$, ſufficiet hic ad cubum K^3 proceſſiſſe, dum altiores poteſtates ob paruitatem tuto negligere licebit; atque hinc tres ordines terminorum conſtituemus, prouti vel per ipſam litteram K, vel eius quadratum K^2 vel eius cubum K^3 fuerint multiplicati.

§. 104.

Porro vero ingredientur quoque eiusmodi termini, qui ſiue littera a ſiue littera $\varkappa$ erunt adfecti, atque inſuper earum producto $a\varkappa$, quippe talia membra in ipſis aequationibus differentialibus reperiuntur. Praeterea vero iidem coefficientes quoque per excentricitatem ipſam K eiusque poteſtates multiplicabuntur; quia vero valor litterae a eſt tantum $\frac{1}{393}$, ſufficiet addidiſſe ordinem aK; pro littera vero $\varkappa = \frac{1}{88}$ propemodum poterimus etiam factorem K^2 adiungere ſicque hinc noui ordines erunt conſtituendi, characteribus a, aK, $\varkappa$, $\varkappa$K, $\varkappa K^2$ et $a\varkappa$ deſignandi.

§. 105.

Cum tertia noſtra coordinata z per tertiam aequationem differentialem potiſſimum definiatur, eiusque omnes termini ipſam litteram z inuoluant, cuius valor praecipue ab inclinatione ante introducta i ita pendebit, vt omnes plane termini per i futuri fint multiplicati, quandoquidem fi effet $i = 0$, totus valor ipfius z euanefcere deberet, quocirca feriei infinitae, quae valorem ipfius z exhibet, omnes termini vel in folam litteram i, vel in iK, vel iKK vel in ia, vel $i\varkappa$ vel etiam i^2 erunt ducti. Facile autem patebit, ob paruitatem hinc multiplicatorem ia tuto omitti poffe, ita, vt hic fequentes ordines tantum conftitui conueniat; qui fint adfecti vel per i vel per iK vel iK^2 vel $i\varkappa$ vel denique per i^2.

§. 106.

His igitur quinque ordinibus pro quantitate z conftitutis hos ordines fequenti modo repraefentemus:

$$z = i.\mathfrak{p} + iK\mathfrak{q} + iK^2.\mathfrak{r} + i\varkappa\,\mathfrak{s} + i^2.\mathfrak{t}.$$

hinc quia in duas priores aequationes vbique tantum quadratum ipfius z ingreditur, ftatuamus breuitatis gratia

$$z^2 = i^2.♄ + i^2K\,♃ + i^2K^2\,♂ + i^2\varkappa\,☿.$$

reliquos ordines, qui hinc nafci poffent, ob paruitatem reiiciemus; erit autem hic

$$♄ = \mathfrak{p}^2$$
$$♃ = 2.\mathfrak{p}.\mathfrak{q}$$
$$♂ = 2.\mathfrak{p}\mathfrak{r} + \mathfrak{q}^2$$
$$☿ = 2\mathfrak{p}\mathfrak{s}.$$

§. 107.

Quatenus autem ista formula z^2 in ipsas quoque aequationes duas priores ingreditur; hinc quoque noui ordines ad quantitates x et y insuper accedent; quamobrem istas quantitates sequenti modo expressas assumamus:

$$x = \mathfrak{O} + K.\mathfrak{P} + K^2\mathfrak{Q} + K^3\mathfrak{R} + a.\mathfrak{S} + aK.\mathfrak{T} + \varkappa.\mathfrak{U}$$
$$+ \varkappa K.\mathfrak{V} + \varkappa K^2\mathfrak{W} + a\varkappa\,\mathfrak{w}$$
$$+ i^2\mathfrak{X} + i^2 K\mathfrak{Y} + i^2 K^2.\mathfrak{Z}.$$

$$y = O + KP + K^2Q + K^3R + a.S + aKT + \varkappa U$$
$$+ \varkappa KV + \varkappa K^2 W + a\varkappa w$$
$$+ i^2 X + i^2 KY + i^2 K^2 Z.$$

Infra autem patebit, litterarum $\mathfrak{O}$ et O valores vix vnam partem centesimam vnitatis excedere; vnde formulae, vbi hae litterae duas accipiunt dimensiones, iam tam erunt exiguae, vt plures earum dimensiones sine errore praetermittere liceat; reliquae vero litterae hic adhibitae ad vnitatem vsque atque adeo vltra excrescere possunt; quare earum altiores potestates eatenus tantum reiici possunt, quatenus earum coefficientes fiunt quam minimi.

§. 108.

His autem valoribus pro nostris coordinatis x, y, z constitutis, si eos in ipsis nostris aequationibus substi-

ſubſtitui intelligamus, ſingula membra per mutuam multiplicationem ad certos ordines reuocabuntur, atque adeo vehementer complicatos, ex quibus autem alios ordines in noſtros calculos non admittemus, niſi qui modo ante ſint conſignati. Hoc obſeruato ſingula coordinatarum producta, quae in noſtris aequationibus occurrunt, ſecundum iſtos conſtitutos ordines euoluamus, reiectis perpetuo iis terminis, in quibus litterae $\mathfrak{O}$ et O plures duabus ſortiuntur dimenſiones.

§. 109.

Ordo primus abſolutus.

$$x = \mathfrak{O};\ y = O;\ x^2 = \mathfrak{O}^2;\ xy = \mathfrak{O}O;\ y^2 = O^2;$$

producta magis compoſita pro hoc ordine nihil praebent. Inprimis autem huc referenda ſunt ipſa membra abſoluta noſtrarum aequationum.

§. 110.

Secundus ordo, K.

x dat $\mathfrak{P}$; $y \ldots P$;

$x^2 \ldots 2\,\mathfrak{O}\,\mathfrak{P}$; $xy \ldots \mathfrak{O}\,P + O\,\mathfrak{P}$; $y^2 \ldots 2\,O\,P$.

$x^3 \ldots 3\,\mathfrak{O}^2\,\mathfrak{P}$; $x^2y \ldots 2\,\mathfrak{O}\,O\,\mathfrak{P} + \mathfrak{O}^2\,P$;

$xy^2 \ldots 2\,\mathfrak{O}\,O\,P + O^2.\,\mathfrak{P}$; $y^3 \ldots 3\,O^2\,P$.

reliqua membra omnia nihil pro hoc ordine producunt.

§. 111.

§. 111.

Ordo tertius: K^3.

x praebet $\mathfrak{Q}$
y . . . Q
x^2 . . . $2\mathfrak{O}\mathfrak{Q} + \mathfrak{P}^2$
xy . . . $O\mathfrak{Q} + \mathfrak{P}.P + \mathfrak{O}Q$
y^2 . . . $2OQ + P^2$
x^3 . . . $3\mathfrak{O}^2\mathfrak{Q} + 3\mathfrak{O}\mathfrak{P}^2$
x^2y . . . $2\mathfrak{O}OQ + \mathfrak{O}P^2 + 2O\mathfrak{P}P + O^2\mathfrak{Q}$
y^3 . . . $3O^2Q + 3O.P^2$.
x^4 . . . $6.\mathfrak{O}^2\mathfrak{P}^2$
x^3y . . . $3.\mathfrak{O}.O.\mathfrak{P}^2 + 3.\mathfrak{O}^2.\mathfrak{P}.P$.
x^2y^2 . . . $O^2\mathfrak{P}^2 + 4\mathfrak{O}O\mathfrak{P}.P + \mathfrak{O}^2P^2$.
xy^3 . . . $3\mathfrak{O}OP^2 + 3O^2\mathfrak{P}.P$.
y^4 . . . $6.O^2P^2$.

§. 112.

Ordo quartus: K^4.

x praebet $\mathfrak{R}$
y . . . R
x^2 . . . $2\mathfrak{O}\mathfrak{R} + 2\mathfrak{P}.\mathfrak{Q}$
xy . . . $O\mathfrak{R} + \mathfrak{Q}P + \mathfrak{P}.Q + \mathfrak{O}R$
y^2 . . . $2OR + 2PQ$
x^3 . . . $3\mathfrak{O}^2\mathfrak{R} + 6.\mathfrak{O}\mathfrak{P}.\mathfrak{Q} + \mathfrak{P}^3$

$x^2y \ldots 2\mathfrak{O}O\mathfrak{R} + 2O\mathfrak{P}.\mathfrak{Q} + 2\mathfrak{O}P\mathfrak{Q} + P.\mathfrak{P}^2$
$+ 2\mathfrak{O}\mathfrak{P}.Q + \mathfrak{O}^2.R.$

$xy^2 \ldots 2\mathfrak{O}OR + 2\mathfrak{O}PQ + 2O\mathfrak{P}Q + \mathfrak{P}.P^2$
$+ 2OP.\mathfrak{Q} + O^2.\mathfrak{R}.$

$y^3 \ldots 3O^2R + 6.OPQ + P^3.$

$x^4 \ldots 12.\mathfrak{O}^2\mathfrak{P}.\mathfrak{Q} + 4\mathfrak{O}\mathfrak{P}^3$

$x^3y \ldots 6.\mathfrak{O}O.\mathfrak{P}.\mathfrak{Q} + O.\mathfrak{P}^3 + 3\mathfrak{O}^2P\mathfrak{Q}$
$+ 3\mathfrak{O}.P.\mathfrak{P}^2 + 3.\mathfrak{O}^2\mathfrak{P}.Q$

$x^2y^2 \ldots 2.O^2\mathfrak{P}.\mathfrak{Q} + 4\mathfrak{O}OP.\mathfrak{Q} + 2O.\mathfrak{P}^2.P$
$+ 2.\mathfrak{O}^2.PQ + 4\mathfrak{O}O.\mathfrak{P}.Q + 2\mathfrak{O}\mathfrak{P}.P^2.$

$xy^3 \ldots 6.\mathfrak{O}OPQ + \mathfrak{O}P^3 + 3O^2\mathfrak{P}.Q$
$+ 3O.\mathfrak{P}.P^2 + 3O^2P.\mathfrak{Q}$

$y^4 \ldots 12.O^2PQ + 4.OP^3.$

$x^5 \ldots 10.\mathfrak{O}^2.\mathfrak{P}^3.$

$x^4y \ldots 4.\mathfrak{O}.O.\mathfrak{P}^3 + 6.\mathfrak{O}^2\mathfrak{P}^2.P.$

$x^3y^2 \ldots O^2\mathfrak{P}^3 + 6.\mathfrak{O}O\mathfrak{P}^2P + 3.\mathfrak{O}^2\mathfrak{P}.P^2$

$x^2y^3 \ldots \mathfrak{O}^2P^3 + 6.\mathfrak{O}O.\mathfrak{P}.P^2 + 3.O^2\mathfrak{P}^2.P.$

$xy^4 \ldots 4\mathfrak{O}OP^3 + 6.O^2\mathfrak{P}.P^2$

$y^5 \ldots 10.O^2.P^3.$

§. 113.

Ordo quintus: a.

x praebet $\mathfrak{S}$; y praebet S;

x^2

$x^2 \ldots 2\mathfrak{O}\mathfrak{S}$; $xy \ldots \mathfrak{O}S + O\mathfrak{S}$; $y^2 \ldots 2OS$

$x^3 \ldots 3\mathfrak{O}^2\mathfrak{S}$; $x^2y \ldots 2\mathfrak{O}O\mathfrak{S} + \mathfrak{O}^2S$.

$xy^2 \ldots 2\mathfrak{O}OS + O^2\mathfrak{S}$; $y^3 \ldots 3O^2S$.

§. 114.

Ordo sextus: *a* K.

x praebet $\mathfrak{T}$; $y \ldots T$; $x^2 \ldots 2\mathfrak{O}\mathfrak{T} + 2\mathfrak{P}.\mathfrak{S}$.

$xy \ldots O\mathfrak{T} + \mathfrak{S}P + \mathfrak{P}.S + \mathfrak{O}T$.

$y^2 \ldots 2OT + 2PS$.; $x^3 \ldots 3\mathfrak{O}^2\mathfrak{T} + 6.\mathfrak{O}.\mathfrak{P}.\mathfrak{S}$;

$x^2.y \ldots 2\mathfrak{O}O\mathfrak{T} + 2O\mathfrak{P}.\mathfrak{S} + 2\mathfrak{O}\mathfrak{S}.P$
$+ 2\mathfrak{O}\mathfrak{P}.S + \mathfrak{O}^2T$.

$xy^2 \ldots 2\mathfrak{O}OT + 2\mathfrak{O}PS + 2O\mathfrak{P}.S$
$+ 2OP\mathfrak{S} + O^2.\mathfrak{T}$.

$y^3 \ldots 3O^2T + 6.OP.S$.

$x^4 \ldots 12.\mathfrak{O}^2\mathfrak{P}.\mathfrak{S}$;

$x^3y \ldots 6.\mathfrak{O}O.\mathfrak{P}.\mathfrak{S} + 3\mathfrak{O}^2P.\mathfrak{S} + 3\mathfrak{O}^2\mathfrak{P}.S$

$x^2y^2 \ldots 2O^2\mathfrak{P}.\mathfrak{S} + 4\mathfrak{O}OP\mathfrak{S} + 4\mathfrak{O}O\mathfrak{P}.S$.
$+ 2\mathfrak{O}^2PS$.

$xy^3 \ldots 6\mathfrak{O}OPS + 3.O^2\mathfrak{P}.S + 3O^2P.\mathfrak{S}$

$y^4 \ldots 12\,O^2P.S$.

§. 115.

Ordo septimus: $\varkappa$.

x praebet $\mathfrak{U}$; $y \ldots U$; $x^2 \ldots 2\mathfrak{O}\mathfrak{U}$;
$xy \ldots \mathfrak{O}U + O\mathfrak{U}$; $y^2 \ldots 2OU$;
$x^3 \ldots 3\mathfrak{O}^2\mathfrak{U}$; $x^2y \ldots 2\mathfrak{O}O\mathfrak{U} + \mathfrak{O}^2U$;
$xy^2 \ldots 2\mathfrak{O}O.U + O^2\mathfrak{U}$; $y^3 \ldots 3O^2U$.

§. 116.

Ordo octavus: $\varkappa K$.

x praebet $\mathfrak{V}$; $y \ldots V$; $x^2 \ldots 2\mathfrak{O}\mathfrak{V} + 2\mathfrak{P}.\mathfrak{U}$
$xy \ldots O\mathfrak{V} + P\mathfrak{U} + \mathfrak{P}.U + \mathfrak{O}V$
$y^2 \ldots 2OV + 2PU$; $x^3 \ldots 3\mathfrak{O}^2\mathfrak{V} + 6\mathfrak{O}\mathfrak{P}.\mathfrak{U}$;
$x^2y \ldots 2.\mathfrak{O}O\mathfrak{V} + 2O\mathfrak{P}.\mathfrak{U} + 2\mathfrak{O}P\mathfrak{U} + 2\mathfrak{O}\mathfrak{P}.U$
$+ \mathfrak{O}^2V$.
$xy^2 \ldots 2\mathfrak{O}OV + 2\mathfrak{O}PU + 2O\mathfrak{P}.U + 2OP.\mathfrak{U}$
$+ O^2.\mathfrak{V}$.
$y^3 \ldots 3O^2.V + 6.OP.U$
$x^4 \ldots 12.\mathfrak{O}^2\mathfrak{P}\mathfrak{U}$.
$x^3y \ldots 6.\mathfrak{O}O\mathfrak{P}.\mathfrak{U} + 3\mathfrak{O}^2P\mathfrak{U} + 3\mathfrak{O}^2\mathfrak{P}.U$
$x^2y^2 \ldots 2O^2\mathfrak{P}.\mathfrak{U} + 4\mathfrak{O}OP\mathfrak{U} + 4\mathfrak{O}O\mathfrak{P}.U$
$+ 2.\mathfrak{O}^2PU$
$xy^3 \ldots 6.\mathfrak{O}OPU + 3O^2\mathfrak{P}U + 3O^2P.\mathfrak{U}$
$y^4 \ldots 12.O^2P.U$.
$x^3 \ldots 3\mathfrak{O}^2.\mathfrak{V} + 6.\mathfrak{O}\mathfrak{P}.\mathfrak{U}$

§. 117.

§. 117.

Ordo nonus: x K.

x praebet $\mathfrak{W}$; y ... W.

$x^2 \ldots 2\mathfrak{O}\mathfrak{W} + 2\mathfrak{P}.\mathfrak{V} + 2\mathfrak{Q}\mathfrak{U}.$

$xy \ldots \mathfrak{O}W + O\mathfrak{W} + \mathfrak{P}.V + P\mathfrak{V} + \mathfrak{Q}U + Q\mathfrak{U}.$

$y^2 \ldots 2OW + 2PV + 2QU.$

$x^3 \ldots 3\mathfrak{O}^2\mathfrak{W} + 6.\mathfrak{O}\mathfrak{P}.\mathfrak{V} + 3.\mathfrak{P}^2\mathfrak{U} + 6\mathfrak{O}\mathfrak{Q}\mathfrak{U}.$

$x^2y \ldots 2\mathfrak{O}O\mathfrak{W} + 2O\mathfrak{P}.\mathfrak{V} + 2O\mathfrak{Q}\mathfrak{U}$
$+ 2\mathfrak{O}P\mathfrak{V} + 2\mathfrak{P}.P.\mathfrak{U} + 2\mathfrak{O}Q\mathfrak{U}$
$+ 2\mathfrak{O}\mathfrak{Q}U + \mathfrak{P}^2.U + 2\mathfrak{O}\mathfrak{P}.V + \mathfrak{O}^2W.$

$xy^2 \ldots 2\mathfrak{O}OW + 2\mathfrak{O}PV + 2\mathfrak{O}QU$
$+ 2O\mathfrak{P}.V + 2\mathfrak{P}.P.U + 2O\mathfrak{Q}U$
$+ 2OQ\mathfrak{U} + P^2\mathfrak{U} + 2OP\mathfrak{V} + O^2.\mathfrak{W}.$

$y^3 \ldots 3O^2W + 6.OPV + 3P^2.U + 6.OQU$

$x^4 \ldots 12\mathfrak{O}^2.\mathfrak{P}.\mathfrak{V} + 12.\mathfrak{O}\mathfrak{P}^2.\mathfrak{U} + 12.\mathfrak{O}^2\mathfrak{Q}\mathfrak{U}$

$x^3y \ldots 6.\mathfrak{O}O.\mathfrak{P}.\mathfrak{V} + 3.O\mathfrak{P}^2.\mathfrak{U}$
$+ 6.\mathfrak{O}.O.\mathfrak{Q}\mathfrak{U} + 3.\mathfrak{O}^2P\mathfrak{V} + 6.\mathfrak{O}\mathfrak{P}.P.\mathfrak{U}$
$+ 3.\mathfrak{O}^2Q\mathfrak{U} + 3\mathfrak{O}^2\mathfrak{Q}U + 3\mathfrak{O}.\mathfrak{P}^2.U$
$+ 3\mathfrak{O}^2\mathfrak{P}.V$

$x^2y^2 \ldots 2O^2\mathfrak{P}.\mathfrak{V} + 2O^2\mathfrak{Q}\mathfrak{U} + 2\mathfrak{O}OP\mathfrak{V}$
$+ 4O.\mathfrak{P}.P.\mathfrak{U} + 4\mathfrak{O}OQ\mathfrak{U} + 2\mathfrak{O}P^2.\mathfrak{U}$
$+ 2\mathfrak{O}^2PV + 2\mathfrak{O}^2QU + 4\mathfrak{O}O\mathfrak{P}.V$
$+ 4\mathfrak{O}\mathfrak{P}.P.U + 4.\mathfrak{O}O\mathfrak{Q}U + 2O\mathfrak{P}^2.U.$

$xy^3 \ldots 6\,\mathfrak{O}\,O\,P\,V + 3\,\mathfrak{O}\,P^2\,U + 6.\,\mathfrak{O}\,O\,Q\,U$
$+ 3.\,O^2\,\mathfrak{P}.\,V + 6.\,O\,\mathfrak{P}.\,P.\,U + 3.\,O^2\,\mathfrak{Q}\,U$
$+ 3\,O^2\,Q\,\mathfrak{U} + 3.\,O.\,P^2.\,\mathfrak{U} + 3\,O^2\,P.\,\mathfrak{V}.$
$y^4 \ldots 12.\,O^2\,P.\,V + 12.\,O.\,P^2.\,U + 12.\,O^2.\,Q.\,U.$

Hic insuper adiungi deberent termini quintae potestatis, sed quia omnes per duas dimensiones ipsarum O et $\mathfrak{O}$ essent multiplicati, eos hic eo magis omittere licebit, quod ipsae inaequalitates Lunae ad hunc ordinem pertinentes iam per se sunt valde paruae neque in earum determinatione tanta praecisione opus est, quam in superioribus ordinibus; vnde etiam in sequente ordine istos terminos omittere poterimus. Ceterum in sequentibus facile erit hos ipsos terminos deficientes supplere.

§. 118.

Ordo decimus: $\alpha\,x$.

x praebet $\mathfrak{w}$; $y \ldots w$; $x^2 \ldots 2\,\mathfrak{O}\,\mathfrak{w} + 2\,\mathfrak{S}.\,\mathfrak{U}.$
$xy \ldots O\,\mathfrak{w} + \mathfrak{S}\,U + S\,\mathfrak{U} + \mathfrak{O}\,w$
$y^2 \ldots 2\,O\,w + 2\,S\,U$
$x^3 \ldots 6.\,\mathfrak{O}.\,\mathfrak{S}.\,\mathfrak{U}$
$x^2y \ldots 2\,\mathfrak{O}.\,\mathfrak{S}.\,\mathfrak{U} + 2\,\mathfrak{O}\,\mathfrak{S}.\,U + 2\,\mathfrak{O}\,S.\,\mathfrak{U}$
$xy^2 \ldots 2\,O\,S\,U + 2\,O\,S\,\mathfrak{U} + 2\,O\,\mathfrak{S}\,U$
$y^3 \ldots 6.\,O.\,S.\,U.$

§. 119.

§. 119.

Ordo vndecimus: $i.i.$

x praebet $\mathfrak{X}$; $y \ldots X$;

$x^2 \ldots 2\mathfrak{O}\mathfrak{X}$; $xy \ldots \mathfrak{O}X + O\mathfrak{X}$

$y^2 \ldots 2OX$; $z^2 \ldots \text{♄}$.

$xz^2 \ldots \mathfrak{O}.\text{♄}$; $yz^2 \ldots O.\text{♄}$.

§. 120.

Ordo duodecimus: $i\,i\,K$.

x praebet $\mathfrak{Y}$; $y \ldots Y$; $x^2 \ldots 2\mathfrak{O}\mathfrak{Y} + 2\mathfrak{P}.\mathfrak{X}$.

$xy \ldots \mathfrak{O}Y + O\mathfrak{Y} + \mathfrak{P}X + P\mathfrak{X}$

$y^2 \ldots 2OY + 2PX$.; $z^2 \ldots \text{♃}$

$x^3 \ldots 6.\mathfrak{O}\mathfrak{P}\mathfrak{X}$

$x^2.y \ldots 2O.\mathfrak{P}.\mathfrak{X} + 2\mathfrak{O}P\mathfrak{X} + 2\mathfrak{O}\mathfrak{P}.X$

$xy^2 \ldots 2\mathfrak{O}PX + 2O\mathfrak{P}.X + 2O.P.\mathfrak{X}$

$y^3 \ldots 6.OPX$.

$xz^2 \ldots \mathfrak{O}.\text{♃} + \mathfrak{P}.\text{♄}$

$yz^2 \ldots O\text{♃} + P.\text{♄}$

$x^2z^2 \ldots 2\mathfrak{O}\mathfrak{P}.\text{♄}$.

$xyz^2 \ldots \mathfrak{O}P.\text{♄} + O\mathfrak{P}.\text{♄}$

$y^2z^2 \ldots 2OP.\text{♄}$.

§. 125.

Ordo decimus tertius i † K K.

x praebet $\mathfrak{Z}$; $y \ldots Z$; $x^2 \ldots 2\mathfrak{O}\mathfrak{Z} + 2\mathfrak{P}.\mathfrak{Y} + 2\mathfrak{Q}\mathfrak{X}$

$xy \ldots \mathfrak{O}Z + O\mathfrak{Z} + \mathfrak{P}.Y + P.\mathfrak{Y} + \mathfrak{Q}X + Q\mathfrak{X}$;

$y^2 \ldots 2OZ + 2PY + 2QX.$

$z^2 \ldots \text{♂}.$

$x^3 \ldots 6.\mathfrak{O}\mathfrak{P}.\mathfrak{Y} + 6\mathfrak{O}\mathfrak{Q}\mathfrak{X} + 3.\mathfrak{P}^2.\mathfrak{X}$

$x^2y \ldots 2O\mathfrak{P}\mathfrak{Y} + 2O\mathfrak{Q}\mathfrak{X} + 2\mathfrak{O}P.\mathfrak{Y} + 2\mathfrak{P}.P.\mathfrak{X}.$

$+ 2\mathfrak{O}Q.\mathfrak{X} + 2\mathfrak{O}\mathfrak{P}.Y + 2\mathfrak{O}\mathfrak{Q}X + \mathfrak{P}^2X.$

$xy^2 \ldots 2\mathfrak{O}PY + 2\mathfrak{O}QX + 2O\mathfrak{P}.Y$

$+ 2\mathfrak{P}.P.X + 2O\mathfrak{Q}X + 2OP\mathfrak{Y}$

$+ 2OQ\mathfrak{X} + P^2.\mathfrak{X}.$

$y^3 \ldots 6.O.P.Y + 6.OQX + 3P^2X$

$xz^2 \ldots \mathfrak{O}\text{♂} + \mathfrak{P}.\text{♃} + \mathfrak{Q}\text{♄}$

$yz^2 \ldots O\text{♂} + P\text{♃} + Q\text{♄}$

$x^4 \ldots 12.\mathfrak{O}.\mathfrak{P}^2.\mathfrak{X}$

$x^3y \ldots 3\mathfrak{O}.\mathfrak{P}^2.X + 6.\mathfrak{O}\mathfrak{P}.P.\mathfrak{X} + 3O.\mathfrak{P}^2.\mathfrak{X}.$

$x^2y^2 \ldots 4O\mathfrak{P}.P.\mathfrak{X} + 2\mathfrak{O}P^2\mathfrak{X}$

$+ 4\mathfrak{O}\mathfrak{P}.P.X + 2O.\mathfrak{P}^2.X$

$xy^3 \ldots 3OP^2\mathfrak{X} + 6.O\mathfrak{P}.P.X + 3.\mathfrak{O}P^2.X.$

$y^4 \ldots 12.O.P^2.X.$

x^2z^2

$x^2 z^2 \ldots 2\mathfrak{O}\mathfrak{P}.\text{♃} + 2\mathfrak{O}\mathfrak{Q}\text{♄} + \mathfrak{P}^2\text{♄}$

$xyz^2 \ldots \mathfrak{O}P\text{♃} + O\mathfrak{P}.\text{♃} + O\mathfrak{Q}\text{♄} + \mathfrak{P}.P.\text{♄}$
$+ \mathfrak{O}Q.\text{♄}.$

$y^2 z^2 \ldots 2OP.\text{♃} + 2OQ\text{♄} + P^2.\text{♄}$

$x^3 z^2 \ldots 3\mathfrak{O}\mathfrak{P}^2.\text{♄}$

$x^2 y z^2 \ldots O\mathfrak{P}^2.\text{♄} + 2\mathfrak{O}\mathfrak{P}P.\text{♄}$

$xy^2 z^2 \ldots \mathfrak{O}P^2.\text{♄} + 2O.\mathfrak{P}.P.\text{♄}.$

$y^3 z^2 \ldots 3O.P^2.\text{♄}.$

§. 122.

Ecce igitur tredecim noſtros ordines per omnes terminos, qui quidem in binis aequationibus prioribus reperiuntur, euolutos, quibus deinceps vtemur ad iſtas aequationes reſoluendas; tertia autem aequatio peculiari indiget reſolutione, pro qua tantum quinque ordines hic conſiderantur; quorum ergo euolutionem hic ſimili modo ſubiungamus

Ordo primus: i

z praebet $\mathfrak{p}$; $xz \ldots \mathfrak{O}\mathfrak{p}$; $yz \ldots O\mathfrak{p}$

$x^2 z \ldots \mathfrak{O}^2.\mathfrak{p}$; $xyz \ldots \mathfrak{O}O\mathfrak{p}$; $y^2 z \ldots O^2.\mathfrak{p}$.

§. 123.

Ordo ſecundus: iK

$z \ldots \mathfrak{q}$; $xz \ldots \mathfrak{O}\mathfrak{q} + \mathfrak{P}.\mathfrak{p}$; $yz \ldots O\mathfrak{q} + P.\mathfrak{p}$

$x^2 z \ldots \mathfrak{O}^2\mathfrak{q} + 2\mathfrak{O}\mathfrak{P}.\mathfrak{p}$

O xyz

$xyz \ldots \mathfrak{O}O.\mathfrak{q} + \mathfrak{O}P\mathfrak{p} + O.\mathfrak{P}.\mathfrak{p}$

$y^2 z \ldots O^2\mathfrak{q} + 2OP.\mathfrak{p}$

$x^3 z \ldots 3.\mathfrak{O}^2\mathfrak{P}.\mathfrak{p}$

$x^2 yz \ldots 2\mathfrak{O}O.\mathfrak{P}.\mathfrak{p} + \mathfrak{O}^2 P.\mathfrak{p}$

$xy^2.z \ldots 2\mathfrak{O}OP\mathfrak{p} + O^2\mathfrak{P}.\mathfrak{p}$

$y^3.z \ldots 3O^2 P.\mathfrak{p}.$

§. 124.

Ordo tertius: iK^2.

vbi duas dimensiones ipsarum $\mathfrak{O}$ et O negligemus.

$z \ldots \mathfrak{r};\ xz \ldots \mathfrak{O}\mathfrak{r} + \mathfrak{P}.\mathfrak{q} + \mathfrak{Q}.\mathfrak{p}$

$yz \ldots O\mathfrak{r} + P.\mathfrak{q} + Q\mathfrak{p}$

$x^2.z \ldots 2\mathfrak{O}\mathfrak{P}.\mathfrak{q} + 2\mathfrak{O}\mathfrak{Q}\mathfrak{p} + \mathfrak{P}^2.\mathfrak{p}$

$xyz \ldots \mathfrak{O}P.\mathfrak{q} + O\mathfrak{P}.\mathfrak{q} + O\mathfrak{Q}\mathfrak{p} + \mathfrak{P}.P.\mathfrak{p} + \mathfrak{O}Q\mathfrak{p}$

$y^2.z \ldots 2OP.\mathfrak{q} + 2OQ\mathfrak{p} + P^2.\mathfrak{p}$

$x^3 z \ldots 3\mathfrak{O}.\mathfrak{P}^2.\mathfrak{p}$

$xy^2 z \ldots \mathfrak{O}.P^2.\mathfrak{p} + 2.O.\mathfrak{P}.P.\mathfrak{p}$

§. 125.

Ordo quartus: $i\kappa$.

z praebet $\mathfrak{s}$; $xz \ldots \mathfrak{O}\mathfrak{s} + \mathfrak{U}.\mathfrak{p}$

$yz \ldots O\mathfrak{s} + U.\mathfrak{p}$

$x^2 z \ldots 2.\mathfrak{O}\mathfrak{U}.\mathfrak{p}$

xyz

$xyz \ldots \mathfrak{O}\,U\,p + O\,\mathfrak{U}\,p$

$y^2.\,z \ldots 2\,O.\,U.\,p.$

§. 126.

Ordo quintus: i^5.

z praebet t; $xz \ldots \mathfrak{O}\,t + \mathfrak{X}\,p$

$yz \ldots O\,t + X\,p$

$x^2\,z \ldots 2\,\mathfrak{O}\,\mathfrak{X}.\,p$

$xyz \ldots \mathfrak{O}\,X\,p + O\,\mathfrak{X}\,p$

$y^2.\,z \ldots 2\,O\,X.\,p$

$z^3 \ldots p$ ♄.

$x\,z^3 \ldots \mathfrak{O}.\,p.$ ♄.

CAPVT XV.

AEQVATIONES DIFFERENTIALES SPECIALES PRO SINGVLIS ORDINIBVS ANTE CONSTITVTIS.

§. 127.

Constitutis his ordinibus tam pro binis prioribus coordinatis x et y, quam pro tertia z, quas posuimus

$$x = \mathfrak{O} + K\mathfrak{P} + K^2\mathfrak{Q} + K^3\mathfrak{R} + a.\mathfrak{S} + aK\mathfrak{T} + \varkappa\mathfrak{U} + \varkappa K.\mathfrak{V} + \varkappa K^2\mathfrak{W} + a.\varkappa.\mathfrak{w} + ii\mathfrak{X} + iiK\mathfrak{Y} + iiK^2\mathfrak{Z}.$$

$$y = O + KP + K^2Q + K^3R + aS + aKT + \varkappa.U + K\varkappa V + K^2\varkappa.W + a\varkappa.w + iiX + ii.KY + iiK^2Z.$$

$$z = i.\mathfrak{p} + iK.\mathfrak{q} + iK^2.\mathfrak{r} + i\varkappa.\mathfrak{s} + i^3.\mathfrak{t}.$$

vnde fecimus

$$z^2 = ii\,♄ + iiK\,♃ + iiK^2.\,♂ + ii\varkappa.\,♁$$

ita,

ita, vt ſit

$$♄ = p^2;\quad ♃ = 2.p.q;\quad ♂ = 2.pr + q^2;\quad ♁ = 2.ps$$

ſubſtituamus hos valores in noſtris tribus aequationibus differentialibus et ſingula membra ſecundum eosdem ordines diſponamus; quo facto manifeſtum eſt, cuiusque ordinis membra in his aequationibus ſeorſim nihilo aequalia ſtatui debere; vnde plures aequationes differentiales ſpeciales nanciſcemur, ex quibus ſingulas partes hic de nouo introductas determinare licebit.

§. 128.

Primum ergo hos valores in duabus noſtris prioribus aequationibus differentialibus ſubſtituamus et ſingulis membris ſecundum hos ordines diſpoſitis pro ſingulis tredecim ordinibus, quos conſtituimus, binas aequationes differentiales ſpeciales eliciemus, quae ſequenti modo ſe habebunt.

Pro ordine primo abſoluto.

$$\text{I.}\ \frac{dd\Omega}{dt^2} - \frac{2(m+1)dO}{dt} - 3\lambda.\Omega, - \tfrac{3}{2}\,\text{coſ.}\,2p,$$
$$- \tfrac{3}{2}\Omega.\,\text{coſ.}\,2p + \tfrac{3}{2}O.\,\text{ſin.}\,2p, + 3\lambda\Omega^2$$
$$- \tfrac{3}{2}\lambda O^2 \qquad = 0.$$

$$\text{II.}\ \frac{ddO}{dt^2} + \frac{2(m+1)d\Omega}{dt}, + \tfrac{3}{2}\,\text{ſin.}\,2p,$$
$$+ \tfrac{3}{2}\Omega.\,\text{ſin.}\,2p + \tfrac{3}{2}O.\,\text{coſ.}\,2p, - 3\lambda\Omega O$$
$$= 0.$$

 §. 129.

§. 129.

Pro ordine secundo, cuius character K

$$\text{I.}\ \frac{dd\mathfrak{P}}{dt^2} - \frac{2(m+1)dP}{dt} - 3\lambda\mathfrak{P},$$
$$+\mathfrak{P}(-\tfrac{3}{2}\operatorname{cos}.2p + 6\lambda\mathfrak{O} - 12.\lambda\mathfrak{O}^2 + 6.\lambda O^2)\,..\,(\mathfrak{A})$$
$$+P(\tfrac{3}{2}\operatorname{sin}.2p - 3\lambda O + 12\lambda\mathfrak{O}O)\,....\,(\mathfrak{B})$$
$$= 0.$$

$$\text{II.}\ \frac{ddP}{dt^2} + \frac{2(m+1)d\mathfrak{P}}{dt},$$
$$+\mathfrak{P}(\tfrac{3}{2}\operatorname{sin}.2p - 3\lambda O + 12.\lambda\mathfrak{O}O)\,....\,(A)$$
$$+P(\tfrac{3}{2}\operatorname{cos}.2p - 3\lambda\mathfrak{O} + 6\lambda\mathfrak{O}^2 - \tfrac{5}{2}\lambda O^2)\,....\,(B)$$
$$= 0.$$

§. 130.

Pro ordine tertio characteris K^2

$$\text{I.}\ \frac{dd\mathfrak{Q}}{dt^2} - \frac{2(m+1)dQ}{dt} - 3\lambda\mathfrak{Q}$$
$$+\mathfrak{Q}(-\tfrac{3}{2}\operatorname{cos}.2p + 6.\lambda\mathfrak{O} - 12.\lambda\mathfrak{O}^2 + 6\lambda O^2)\,..\,(\mathfrak{A})$$
$$+Q(\tfrac{3}{2}.\operatorname{sin}.2p - 3\lambda O + 12.\lambda\mathfrak{O}O)\,....\,(\mathfrak{B})$$
$$+\mathfrak{P}^2(3\lambda - 12\lambda\mathfrak{O} + 30.\lambda.\mathfrak{O}^2 - 15.\lambda O^2)\,...\,(\mathfrak{C})$$
$$+\mathfrak{P}.P(12.\lambda O - 60.\lambda\mathfrak{O}.O)\,....\,(\mathfrak{D})$$
$$+P^2(-\tfrac{3}{2}\lambda + 6\lambda\mathfrak{O} - 15.\lambda.\mathfrak{O}^2 + \tfrac{45}{4}\lambda.O^2)\,....\,(\mathfrak{E})$$
$$= 0.$$

II.

$$\text{II. } \frac{dd\mathfrak{Q}}{dt^2} + \frac{2(m+1)d\mathfrak{Q}}{dt},$$

$$+ \mathfrak{Q}(\tfrac{3}{2}\sin. 2p - 3\lambda O + 12.\lambda.\mathfrak{O}O) \ldots . (A)$$

$$+ Q(\tfrac{3}{2}\cos. 2p - 3\lambda\mathfrak{O} + 6\lambda\mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2) \ldots . (B)$$

$$+ \mathfrak{P}^2(6\lambda O - 30.\lambda.\mathfrak{O}O) \ldots . (C)$$

$$+ \mathfrak{P}.P(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2) \ldots (D)$$

$$+ P^2(-\tfrac{9}{2}\lambda O + \tfrac{45}{2}\lambda\mathfrak{O}O) \ldots . (E)$$

$$= 0.$$

§. 131.

Pro ordine quarto characteris K^3

$$\text{I. } \frac{dd\mathfrak{R}}{dt^2} - \frac{2(m+1)dR}{dt} - 3\lambda.\mathfrak{R},$$

$$+ \mathfrak{R}(-\tfrac{3}{2}\cos. 2p + 6.\lambda\mathfrak{O} - 12.\lambda.\mathfrak{O}^2 + 6\lambda O^2) \ldots (\mathfrak{A})$$

$$+ R(\tfrac{3}{2}\sin. 2p - 3\lambda O + 12.\lambda.\mathfrak{O}O) \ldots . (\mathfrak{B})$$

$$+ \mathfrak{P}.\mathfrak{Q}(6.\lambda - 24\lambda\mathfrak{O} + 60.\lambda\mathfrak{O}^2 - 30.\lambda O^2) \ldots (2.\mathfrak{C})$$

$$+ (\mathfrak{P}.Q + P.\mathfrak{Q})(12\lambda O - 60\lambda\mathfrak{O}O) \ldots . (\mathfrak{D})$$

$$+ PQ(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2) \ldots (2\,\mathfrak{E})$$

$$+ \mathfrak{P}^3(-4\lambda + 20.\lambda\mathfrak{O} - 60.\lambda.\mathfrak{O}^2 + 30.\lambda O^2) \ldots (\mathfrak{F})$$

$$+ \mathfrak{P}^2 P(-30.\lambda O + 180\lambda\mathfrak{O}O) \ldots . (\mathfrak{G})$$

$$+ \mathfrak{P}.P^2(6\lambda - 30.\lambda\mathfrak{O} + 90.\lambda.\mathfrak{O}^2 - \tfrac{135}{2}\lambda O^2) \ldots (\mathfrak{H})$$

$$+ P^3(\tfrac{15}{2}.\lambda O - 45.\lambda.\mathfrak{O}O) \ldots . (\mathfrak{I})$$

$$= 0.$$

II.

$$\text{II. } \frac{ddR}{dt^2} + \frac{2(m+1)d\mathfrak{R}}{dt}$$
$$+ \mathfrak{R}(\tfrac{3}{2}\,\text{fin.}\,2p - 3\lambda O + 12.\lambda\mathfrak{Q}O) \ldots. (A)$$
$$+ R(\tfrac{3}{2}\,\text{cof.}\,2p - 3\lambda\mathfrak{Q} + 6.\lambda\mathfrak{Q}^2 - \tfrac{9}{2}\lambda O^2) \ldots. (B)$$
$$+ \mathfrak{P}\mathfrak{Q}(12\lambda O - 60.\lambda\mathfrak{Q}O) \ldots. (2C)$$
$$+ (\mathfrak{P}Q + P\mathfrak{Q})(-3\lambda + 12\lambda\mathfrak{Q} - 30.\lambda\mathfrak{Q}^2 + \tfrac{45}{2}\lambda O^2) \ldots (D)$$
$$+ PQ(-9\lambda O + 45.\lambda\mathfrak{Q}O) \ldots. (2E)$$
$$+ \mathfrak{P}^3(-10.\lambda.O + 60.\lambda\mathfrak{Q}O) \ldots. (F)$$
$$+ \mathfrak{P}^2.P(6\lambda - 30.\lambda\mathfrak{Q} + 90.\lambda\mathfrak{Q}^2 - \tfrac{135}{2}\lambda O^2) \ldots (G)$$
$$+ \mathfrak{P}.P^2(\tfrac{45}{2}\lambda O - 135\lambda.\mathfrak{Q}O) \ldots. (H)$$
$$+ P^3(-\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{Q} - \tfrac{45}{2}\lambda\mathfrak{Q}^2 + \tfrac{75}{4}\lambda O^2) \ldots. (I)$$
$$= 0.$$

§. 132.

Pro ordine quinto, characteris *a*

$$\text{I. } \frac{dd\mathfrak{S}}{dt^2} - \frac{2(m+1)dS}{dt} - 3\lambda\mathfrak{S},$$
$$+ \mathfrak{S}(-\tfrac{3}{2}\,\text{cof.}\,2p + 6\lambda\mathfrak{Q} - 12\lambda\mathfrak{Q}^2 + 6.\lambda O^2) \ldots (\mathfrak{A})$$
$$+ S(\tfrac{3}{2}\,\text{fin.}\,2p - 3\lambda O + 12\lambda\mathfrak{Q}O) \ldots. (\mathfrak{B})$$
$$- \tfrac{3}{8}(3.\,\text{cof.}\,p + 5.\,\text{cof.}\,3p)(1 + 2\mathfrak{Q} + \mathfrak{Q}^2)$$
$$+ \tfrac{3}{4}(\text{fin.}\,p + 5.\,\text{fin.}\,3p)(O + \mathfrak{Q}O)$$
$$- \tfrac{3}{8}(\text{cof.}\,p - 5.\,\text{cof.}\,3p).O^2$$
$$= 0.$$

II.

II. $\frac{ddS}{dt^2} + \frac{2(m+1)d\mathfrak{S}}{dt}$,

$+ \mathfrak{S}(\frac{3}{2}\,\text{fin.}\,2p - 3\lambda O + 12\lambda\mathfrak{D}O) \ldots .(A)$

$+ S(\frac{3}{2}\,\text{cof.}\,2p - 3\lambda\mathfrak{D} + 6.\lambda\mathfrak{D}^2 - \frac{9}{2}\lambda O^2) \,.. (B)$

$+ \frac{3}{8}(\text{fin.}\,p + 5.\,\text{fin.}\,3p)(1 + 2.\mathfrak{D} + \mathfrak{D}^2)$

$- \frac{3}{4}(\text{cof.}\,p - 5.\,\text{cof.}\,3p)(O + \mathfrak{D}O)$

$+ \frac{3}{8}(3.\,\text{fin.}\,p - 5.\,\text{fin.}\,3p)\,O^2$

$= 0.$

§. 133.

Pro ordine sexto, characteris a K

I. $\frac{dd\mathfrak{T}}{dt^2} - \frac{2(m+1)dT}{dt} - 3\lambda\mathfrak{T}$,

$+ \mathfrak{T}(-\frac{3}{2}\text{cof.}\,2p + 6.\lambda\mathfrak{D} - 12.\lambda\mathfrak{D}^2 + 6.\lambda O^2) \,.. (\mathfrak{A})$

$+ T(\frac{3}{2}\text{fin.}\,2p - 3\lambda O + 12.\lambda\mathfrak{D}O) \ldots . (\mathfrak{B})$

$+ \mathfrak{P}.\mathfrak{S}(6.\lambda - 24.\lambda.\mathfrak{D} + 60.\lambda\mathfrak{D}^2 - 30.\lambda O^2) \,.. (2\mathfrak{C})$

$+ (\mathfrak{P}S + P\mathfrak{S})(12.\lambda O - 60.\lambda\mathfrak{D}O) \ldots . (\mathfrak{D})$

$+ PS(-3\lambda + 12.\lambda\mathfrak{D} - 30.\lambda.\mathfrak{D}^2 + \frac{45}{2}\lambda O^2) \,.. (2\mathfrak{E})$

$- \frac{3}{4}(3.\,\text{cof.}\,p + 5.\,\text{cof.}\,3p)(\mathfrak{P} + \mathfrak{D}\mathfrak{P})$

$+ \frac{3}{4}(\text{fin.}\,p + 5.\,\text{fin.}\,3p)(P + \mathfrak{D}P + O\mathfrak{P})$

$- \frac{3}{4}(\text{cof.}\,p - 5.\,\text{cof.}\,3p)\,O\,P.$

$= 0.$

II. $\frac{dd\mathrm{T}}{dt^2} + \frac{2(m+1)d\mathfrak{T}}{dt}$,

$+\mathfrak{T}(\frac{3}{2}\sin. 2p - 3\lambda O + 12.\lambda.\mathfrak{O}O) \ldots (A)$

$+T(\frac{3}{2}\cos. 2p - 3\lambda.\mathfrak{O} + 6.\lambda\mathfrak{O}^2 - \frac{9}{2}\lambda O^2) \ldots (B)$

$+\mathfrak{P}\mathfrak{S}(12.\lambda O - 60.\lambda\mathfrak{O}O) \ldots (2C)$

$+(\mathfrak{P}.S + P\mathfrak{S})(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \frac{45}{2}\lambda O^2) \ldots (D)$

$+PS(-9\lambda O + 45.\lambda\mathfrak{O}O) \ldots (2E)$

$+\frac{3}{4}(\sin. p + 5.\sin. 3p)(\mathfrak{P} + \mathfrak{O}\mathfrak{P})$

$-\frac{3}{4}(\cos. p - 5.\cos. 3p)(P + \mathfrak{O}P + O\mathfrak{P})$

$+\frac{3}{4}(3.\sin. p - 5.\sin. 3p)OP.$

$= 0.$

§. 134.

Pro ordine septimo, characteris $\varkappa$.

I. $\frac{dd\mathfrak{U}}{dt^2} - \frac{2(m+1)dU}{dt} - 3\lambda.\mathfrak{U}$,

$+\mathfrak{U}(-\frac{3}{2}\cos. 2p + 6.\lambda\mathfrak{O} - 12.\lambda\mathfrak{O}^2 + 6.\lambda O^2) \ldots (\mathfrak{A})$

$+U(\frac{3}{2}\sin. 2p - 3\lambda O + 12\lambda\mathfrak{O}O) \ldots (\mathfrak{B})$

$+\frac{3}{4}(2.\cos. t + 7.\cos.(2p-t) - \cos.(2p+t))(1+\mathfrak{O})$

$-\frac{3}{4}(7.\sin.(2p-t) - \sin.(2p+t))O$

$= 0.$

II. $\frac{ddU}{dt^2} + \frac{2(m+1)d\mathfrak{U}}{dt}$,

$+\mathfrak{U}(\frac{3}{2}\sin. 2p - 3\lambda O + 12.\lambda\mathfrak{O}O) \ldots (A)$

$+U(\frac{3}{2}\cos. 2p - 3.\lambda.\mathfrak{O} + 6\lambda.\mathfrak{O}^2 - \frac{9}{2}\lambda O^2) \ldots (B)$

$-\frac{3}{4}(7.\sin.(2p-t) - \sin.(2p+t)(1+\mathfrak{O})$

$+\frac{3}{4}(2.\cos. t - 7.\cos.(2p-t) + \cos.(2p+t))O$

$= 0.$

§. 135.

§. 135.

Pro ordine octauo, characteris $\varkappa$ K.

$$\text{I.}\ \frac{dd\mathfrak{V}}{dt^2} - \frac{2(m+1)\,dV}{dt} - 3\,\lambda.\,\mathfrak{V},$$

$$+\mathfrak{V}(-\tfrac{3}{2}\operatorname{cos}.2p + 6.\lambda\mathfrak{O} - 12.\lambda\,\mathfrak{O}^2 + 6.\lambda.O^2)\,..(\mathfrak{A})$$

$$+V(\tfrac{3}{2}\operatorname{sin}.2p - 3\lambda O + 12.\lambda\mathfrak{O}O)\,....(\mathfrak{B})$$

$$+\mathfrak{P}.\mathfrak{U}(6\lambda - 24.\lambda\mathfrak{O} + 60.\lambda.\mathfrak{O}^2 - 30.\lambda O^2)\,..(2\mathfrak{C})$$

$$+(\mathfrak{P}.U + P\mathfrak{U})(12.\lambda O - 60.\lambda\mathfrak{O}O)\,....(\mathfrak{D})$$

$$+PU(-3\lambda + 12.\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{8}\lambda O^2)\,..(2\mathfrak{E})$$

$$+\tfrac{3}{4}(2.\operatorname{cos}.t + 7.\operatorname{cos}.(2p-t) - \operatorname{cos}.(2p+t))\,\mathfrak{P}$$

$$-\tfrac{3}{4}(7.\operatorname{sin}.(2p-t) - \operatorname{sin}.(2p+t))\,P.$$

$$= 0.$$

$$\text{II.}\ \frac{ddV}{dt^2} + \frac{2(m+1)\,d\mathfrak{V}}{dt},$$

$$+\mathfrak{V}(\tfrac{3}{2}\operatorname{sin}.2p - 3\lambda O + 12.\lambda\mathfrak{O}O)\,....(A)$$

$$+V(\tfrac{3}{2}\operatorname{cos}.2p - 3\lambda\mathfrak{O} + 6.\lambda\mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2)\,.....(B)$$

$$+\mathfrak{P}\mathfrak{U}(12.\lambda O - 60.\lambda\mathfrak{O}O)\,....(2C)$$

$$+(\mathfrak{P}U + P\mathfrak{U})(-3\lambda + 12.\lambda\mathfrak{O} - 30\lambda\,\mathfrak{O}^2 + \tfrac{45}{8}\lambda O^2)\,..(D)$$

$$+PU(-9.\lambda O + 45.\lambda\mathfrak{O}O)\,....(2E)$$

$$-\tfrac{3}{4}(7.\operatorname{sin}.(2p-t) - \operatorname{sin}.(2p+t))\,\mathfrak{P}$$

$$+\tfrac{3}{4}(2.\operatorname{cos}.t - 7.\operatorname{cos}.(2p-t) + \operatorname{cos}.(2p+t))\,P$$

$$= 0.$$

§. 136.

Pro ordine nono, characteris $\varkappa K^2$.

$$\text{I.}\quad \frac{dd\mathfrak{W}}{dt^2} - \frac{2(m+1)dW}{dt} - 3\lambda.\mathfrak{W},$$

$$+\mathfrak{W}(-\tfrac{3}{2}\mathrm{cos}.2p + 6.\lambda\mathfrak{O} - 12.\lambda.\mathfrak{O}^2 + 6.\lambda O^2) \ldots (\mathfrak{A})$$

$$+W(\tfrac{3}{2}\mathrm{sin}.2p - 3\lambda O + 12\lambda\mathfrak{O}O) \ldots\ldots (\mathfrak{B})$$

$$+(\mathfrak{P}.\mathfrak{V}+\mathfrak{Q}\mathfrak{U})(6.\lambda - 24\lambda.\mathfrak{O} + 60.\lambda.\mathfrak{O}^2 - 30.\lambda O^2) \ldots (2\mathfrak{C})$$

$$+(\mathfrak{P}V+\mathfrak{Q}U+P\mathfrak{V}+Q\mathfrak{U})(12.\lambda O - 60.\lambda\mathfrak{O}O) \ldots (\mathfrak{D})$$

$$+(PV+QU)(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2) \ldots (2\mathfrak{E})$$

$$+\mathfrak{P}^2.\mathfrak{U}(-12.\lambda + 60.\lambda\mathfrak{O} - 180.\lambda.\mathfrak{O}^2 + 90\lambda.O^2) \ldots (3\mathfrak{F})$$

$$+(\mathfrak{P}^2U + 2\mathfrak{P}P\mathfrak{U})(-30.\lambda O + 180.\lambda\mathfrak{O}O) \ldots (\mathfrak{G})$$

$$+(P^2\mathfrak{U} + 2\mathfrak{P}.P.U)(6.\lambda - 30.\lambda\mathfrak{O} + 90.\lambda\mathfrak{O}^2 - \tfrac{135}{2}\lambda O^2) \ldots (\mathfrak{H})$$

$$+P^2U(\tfrac{45}{2}\lambda O - 135.\lambda\mathfrak{O}O) \ldots\ldots (3.\mathfrak{J})$$

$$+\tfrac{3}{4}(2.\mathrm{cos}.t + 7.\mathrm{cos}.(2p-t) - \mathrm{cos}.(2p+t))\mathfrak{Q}$$

$$-\tfrac{3}{4}(7.\mathrm{sin}.(2p-t) - \mathrm{sin}.(2p+t))Q$$

$$= 0.$$

II.

$$\text{II.}\quad \frac{dd\mathrm{W}}{dt^2} + \frac{2(m+1)\,d.\mathfrak{W}}{dt},$$

$$+\mathfrak{W}\left(\tfrac{1}{2}\,\text{fin.}\,2p - 3\lambda O + 12\lambda\mathfrak{O}O\right) \ldots\ldots (A)$$

$$+\mathrm{W}\left(\tfrac{3}{2}\,\text{cof.}\,2p - 3\lambda\mathfrak{O} + 6\lambda\mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2\right) \ldots\ldots (B)$$

$$+(\mathfrak{P}\mathfrak{V} + \mathfrak{Q}\mathfrak{U})(12\lambda O - 60.\lambda\mathfrak{O}O) \ldots\ldots (2C)$$

$$+(\mathfrak{P}.V + \mathfrak{Q}U + P\mathfrak{V} + Q\mathfrak{U})(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda.\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2) \ldots (D)$$

$$+(PV + QU)(-9.\lambda O + 45.\lambda\mathfrak{O}O) \ldots\ldots (2E)$$

$$+\mathfrak{P}^2\mathfrak{U}(-30.\lambda O + 180.\lambda\mathfrak{O}O) \ldots\ldots (3F)$$

$$+(\mathfrak{P}^2U + 2\mathfrak{P}.P.\mathfrak{U})(6\lambda - 30\lambda\mathfrak{O} + 90\lambda\mathfrak{O}^2 - \tfrac{135}{2}\lambda O^2) \ldots (G)$$

$$+(P^2\mathfrak{U} + 2\mathfrak{P}.P.U)(\tfrac{45}{2}\lambda O - 135.\lambda.\mathfrak{O}O) \ldots\ldots (H)$$

$$+P^2U(-\tfrac{9}{2}\lambda + \tfrac{45}{2}\lambda\mathfrak{O} - \tfrac{135}{2}\lambda\mathfrak{O}^2 + \tfrac{225}{4}\lambda O^2) \ldots\ldots (3I)$$

$$-\tfrac{3}{4}\left(7.\,\text{fin.}\,(2p - t) - \text{fin.}\,(2p + t)\right)\mathfrak{Q}$$

$$+\tfrac{3}{4}\left(2.\,\text{cof.}\,t - 7.\,\text{cof.}\,(2p - t) + \text{cof.}\,(2p + t)\right)Q$$

$$= 0.$$

§. 137.

Pro ordine decimo, characteris $a\varkappa$.

$$\text{I.}\ \frac{dd\mathfrak{w}}{dt^2} - \frac{2(m+1)d\mathfrak{w}}{dt} - 3\lambda.\mathfrak{w},$$

$$+\mathfrak{w}(-\tfrac{3}{2}\operatorname{cof.}2p+6\lambda\mathfrak{D}-12.\lambda.\mathfrak{D}^2+6.\lambda.\mathfrak{O}^2)\,..\,(\mathfrak{A})$$

$$+w(\tfrac{3}{2}\operatorname{fin.}2p-3\lambda\mathfrak{O}+12\lambda\mathfrak{D}\mathfrak{O})\,.....\,(\mathfrak{B})$$

$$+\mathfrak{S}\mathfrak{U}(6.\lambda-24.\lambda\mathfrak{D}+60.\lambda\mathfrak{D}^2-30.\lambda.\mathfrak{O}^2)\,..\,(2\mathfrak{C})$$

$$+(\mathfrak{S}U+S\mathfrak{U})(12.\lambda\mathfrak{O}-60.\lambda\mathfrak{D}\mathfrak{O})\,....\,(\mathfrak{D})$$

$$+SU(-3\lambda+12\lambda\mathfrak{D}-30.\lambda.\mathfrak{D}^2+\tfrac{45}{2}\lambda\mathfrak{O}^2)\,..\,(2.\mathfrak{E})$$

$$-\tfrac{3}{4}(3.\operatorname{cof.}p+5.\operatorname{cof.}3p)(\mathfrak{U}+\mathfrak{D}\mathfrak{U})$$

$$+\tfrac{3}{4}(\operatorname{fin.}p+5.\operatorname{fin.}3p)(U+\mathfrak{D}U+\mathfrak{O}\mathfrak{U})$$

$$-\tfrac{3}{4}(\operatorname{cof.}p-5.\operatorname{cof.}3p)\,\mathfrak{O}\,U.$$

$$+\tfrac{3}{4}(2.\operatorname{cof.}t+7.\operatorname{cof.}(2p-t)-\operatorname{cof.}(2p+t))\,\mathfrak{S}$$

$$-\tfrac{3}{4}(7.\operatorname{fin.}(2p-t)-\operatorname{fin.}(2p+t))\,S$$

$$+\tfrac{3}{8}(9.\operatorname{cof.}(p-t)+3.\operatorname{cof.}(p+t)+25\operatorname{cof.}(3p-t)-5.\operatorname{cof}(3p+t))$$
$$(1+2\mathfrak{D}+\mathfrak{D}^2)$$

$$-\tfrac{3}{8}(3.\operatorname{fin.}(p-t)+\operatorname{fin.}(p+t)+25.\operatorname{fin.}(3p-t)-5.\operatorname{fin.}(3p+t))$$
$$(\mathfrak{O}+\mathfrak{D}\mathfrak{O})$$

$$+\tfrac{3}{8}(3.\operatorname{cof.}(p-t)+\operatorname{cof.}(p+t)-25.\operatorname{cof}(3p-t)+5.\operatorname{cof.}(3p+t))\mathfrak{O}^2$$

$$=0.$$

II.

$$
\begin{aligned}
\text{II. } & \frac{dd\mathfrak{w}}{dt^2} + \frac{2(m+1)\,d\mathfrak{w}}{dt}, \\
& + \mathfrak{w}\left(\tfrac{3}{2}\sin.2p - 3.\lambda O + 12.\lambda \mathfrak{O} O\right) \;\ldots\; (A) \\
& + w\left(\tfrac{3}{2}\cos.2p - 3.\lambda \mathfrak{O} + 6.\lambda \mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2\right) \;\ldots\; (B) \\
& + \mathfrak{S}\mathfrak{U}\left(12.\lambda O - 60.\lambda \mathfrak{O} O\right) \;\ldots\; (2C) \\
& + (\mathfrak{S}U + S\mathfrak{U})\left(-3\lambda + 12.\lambda \mathfrak{O} - 30.\lambda \mathfrak{O}^2 + \tfrac{45}{2}\lambda . O^2\right) \;\ldots\; (D) \\
& + SU\left(-9.\lambda O + 45.\lambda \mathfrak{O} O\right) \;\ldots\; (2E) \\
& + \tfrac{3}{4}\left(\sin.p + 5.\sin.3p\right)\left(\mathfrak{U} + \mathfrak{O}\mathfrak{U}\right) \\
& - \tfrac{3}{4}\left(\cos.p - 5.\cos.3p\right)\left(U + \mathfrak{O}U + O\mathfrak{U}\right) \\
& + \tfrac{3}{4}\left(3.\sin.p - 5.\sin.3p\right) O U. \\
& - \tfrac{3}{4}\left(7.\sin.(2p - t) - \sin.(2p + t)\right)\mathfrak{S} \\
& + \tfrac{3}{4}\left(2.\cos.t - 7.\cos.(2p - t) + \cos.(2p + t)\right) S \\
& - \tfrac{3}{8}\left(3.\sin.(p-t) + \sin(p+t) + 25.\sin.(3p-t) - 5.\sin.(3p+t)\right) \\
& \qquad\qquad (1 + 2\mathfrak{O} + \mathfrak{O}^2) \\
& + \tfrac{3}{4}\left(3.\cos.(p-t) + \cos.(p+t) - 25.\cos.(3p-t) + 5.\cos.(3p+t)\right) \\
& \qquad\qquad (O + \mathfrak{O} O) \\
& - \tfrac{3}{8}\left(9.\sin.(p-t) + 3.\sin.(p+t) - 25.\sin.(3p-t) + 5\sin.(3p+t)\right) O^2. \\
& = 0.
\end{aligned}
$$

§. 138.

Pro ordine vndecimo, characteris $i\,i$.

$$\text{I.}\quad \frac{dd\mathfrak{X}}{dt^2} - \frac{2(m+1)\,dX}{dt} - 3\lambda\mathfrak{X},$$

$$+\mathfrak{X}\left(-\tfrac{3}{2}\operatorname{cos}.2p + 6.\lambda\mathfrak{O} - 12.\lambda.\mathfrak{O}^2 + 6.\lambda O^2\right)\,..\,(\mathfrak{A})$$

$$+X\left(\tfrac{3}{2}\operatorname{sin}.2p - 3\lambda O + 12.\lambda\mathfrak{O}O\right)\;....\;(\mathfrak{B})$$

$$+\mathfrak{h}\left(-\tfrac{3}{2}\lambda + 6\lambda\mathfrak{O} - 15\lambda\mathfrak{O}^2 + \tfrac{15}{4}\lambda O^2\right)$$

$$=0.$$

$$\text{II.}\quad \frac{ddX}{dt^2} + \frac{2(m+1)\,d\mathfrak{X}}{dt},$$

$$+\mathfrak{X}\left(\tfrac{3}{2}\operatorname{sin}.2p - 3.\lambda O + 12.\lambda.\mathfrak{O}O\right)\,.....\,(A)$$

$$+X\left(\tfrac{3}{2}\operatorname{cos}.2p - 3.\lambda.\mathfrak{O} + 6.\lambda\mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2\right)\,....\,(B)$$

$$+\mathfrak{h}.\left(-\tfrac{3}{2}\lambda O + \tfrac{15}{2}\lambda\mathfrak{O}O\right)$$

$$=0.$$

§. 139.

§. 139.

Pro ordine duodecimo, characteris $i\,i$. K.

I. $\frac{dd\mathfrak{Y}}{dt^2} - \frac{2(m+1)dY}{dt} - 3\lambda.\mathfrak{Y},$

$+\mathfrak{Y}(-\tfrac{3}{2}\cos.2p + 6.\lambda\mathfrak{O} - 12.\lambda.\mathfrak{O}^2 + 6\lambda O^2) \ldots (\mathfrak{A})$

$+Y(\tfrac{3}{2}\sin.2p - 3\lambda O + 12.\lambda.\mathfrak{O}O) \ldots (\mathfrak{B})$

$+\mathfrak{P}.\mathfrak{X}(6.\lambda - 24.\lambda\mathfrak{O} + 60.\lambda\mathfrak{O}^2 - 30.\lambda O^2) \ldots (2.\mathfrak{C})$

$+(\mathfrak{P}.X + P\mathfrak{X})(12\lambda O - 60\lambda\mathfrak{O}O) \ldots (\mathfrak{D})$

$+PX(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2) \ldots (2.\mathfrak{E})$

$+\text{♄}(6.\lambda.\mathfrak{P} - 30.\lambda.\mathfrak{O}.\mathfrak{P} + \tfrac{15}{2}\lambda OP)$

$+\text{♃}(-\tfrac{3}{2}\lambda + 6.\lambda\mathfrak{O})$

$= 0.$

II. $\frac{ddY}{dt^2} + \frac{2(m+1)d\mathfrak{Y}}{dt},$

$+\mathfrak{Y}(\tfrac{3}{2}\sin.2p - 3\lambda O + 12.\lambda.\mathfrak{O}O) \ldots (A)$

$+Y(\tfrac{3}{2}\cos.2p - 3\lambda\mathfrak{O} + 6\lambda\mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2) \ldots (B)$

$+\mathfrak{P}\mathfrak{X}(12.\lambda O - 60.\lambda.\mathfrak{O}O) \ldots (2C)$

$+(\mathfrak{P}X + P\mathfrak{X})(-3\lambda + 12.\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2) \ldots (D)$

$+PX(-9\lambda O + 45\lambda\mathfrak{O}O) \ldots (2E)$

$+\text{♄}(-\tfrac{3}{2}\lambda P + \tfrac{15}{2}\lambda\mathfrak{O}P + \tfrac{15}{2}\lambda O\mathfrak{P})$

$-\tfrac{3}{2}.\text{♃}.\lambda O$

$= 0.$

§. 140.

Pro ordine decimo tertio, characteris $i\,i\,K^*$.

I. $\frac{dd\mathfrak{Z}}{dt^2} - \frac{2(m+1)dZ}{dt} - 3\lambda\mathfrak{Z}$

$+\mathfrak{Z}(-\tfrac{3}{2}\cos. 2p + 6.\lambda\mathfrak{O} - 12.\lambda\mathfrak{O}^2 + 6\lambda O^2)\,..\,(\mathfrak{A})$

$+Z(\tfrac{3}{2}.\sin. 2p - 3\lambda O + 12.\lambda\mathfrak{O}O)\,....\,(\mathfrak{B})$

$+(\mathfrak{P}\mathfrak{Y}+\mathfrak{Q}\mathfrak{X})(6\lambda - 24\lambda\mathfrak{O} + 60.\lambda\mathfrak{O}^2 - 30.\lambda O^2)\,..\,(2\mathfrak{C})$

$+(\mathfrak{P}Y+P\mathfrak{Y}+\mathfrak{Q}X+Q\mathfrak{X})(12.\lambda O - 60.\lambda\mathfrak{O}.O)\,..\,(\mathfrak{D})$

$+(PY+QX)(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{4}\lambda.O^2)\,..\,(2\mathfrak{E})$

$+\mathfrak{P}^2\mathfrak{X}(-12\lambda + 60.\lambda\mathfrak{O} - 180.\lambda.\mathfrak{O}^2 + 90.\lambda O^2)\,...\,(3\mathfrak{F})$

$+(\mathfrak{P}^2X + 2\mathfrak{P}P\mathfrak{X})(-30.\lambda O + 180\lambda\mathfrak{O}O)\,....\,(\mathfrak{G})$

$+(P^2\mathfrak{X} + 2\mathfrak{P}PX)(6\lambda - 30.\lambda\mathfrak{O} + 90.\lambda\mathfrak{O}^2 - \tfrac{135}{8}\lambda O^2)\,..\,(\mathfrak{H})$

$+P^2X(\tfrac{45}{8}.\lambda O - 135.\lambda.\mathfrak{O}O)\,.....\,(3\mathfrak{J})$

$+♄\begin{cases} 6\lambda\mathfrak{Q} - 30.\lambda\mathfrak{O}\mathfrak{Q} + \tfrac{15}{8}\lambda OQ - 15.\lambda.\mathfrak{P}^2 \\ +\tfrac{15}{4}\lambda P^2 + 90.\lambda.\mathfrak{O}.\mathfrak{P}^2 - \tfrac{45}{8}\lambda\mathfrak{O}.P^2 - 45.\lambda O\mathfrak{P}.P \end{cases}$

$+♃(6.\lambda.\mathfrak{P} - 30.\lambda\mathfrak{O}.\mathfrak{P} + \tfrac{15}{8}\lambda OP)$

$+♂(-\tfrac{3}{8}\lambda + 6\lambda\mathfrak{O})$

$= 0.$

II.

$$
\begin{aligned}
\text{II}\quad & \frac{ddZ}{dt^2} + \frac{2(m+1)\,d\mathfrak{Z}}{dt}, \\
& + \mathfrak{Z}\left(\tfrac{3}{2}\sin.2p - 3\lambda O + 12.\lambda\,\mathfrak{O}O\right) \ldots\ldots (A) \\
& + Z\left(\tfrac{3}{2}\cos.2p - 3\lambda\mathfrak{O} + 6.\lambda\mathfrak{O}^2 - \tfrac{9}{2}\lambda O^2\right) \ldots\ldots (B) \\
& + (\mathfrak{P}\mathfrak{Y} + \mathfrak{Q}\mathfrak{X})(12.\lambda O - 60.\lambda\mathfrak{O}O) \ldots\ldots (2C) \\
& + (\mathfrak{P}Y + P\mathfrak{Y} + \mathfrak{Q}X + Q\mathfrak{X})\left(-3\lambda + 12\lambda\mathfrak{O} - 30.\lambda\mathfrak{O}^2 + \tfrac{45}{2}\lambda O^2\right) \ldots (D) \\
& + (PY + QX)(-9\lambda O + 45.\lambda\mathfrak{O}O) \ldots (2E) \\
& + \mathfrak{P}^2\mathfrak{X}(-30.\lambda.O + 180.\lambda\mathfrak{O}O) \ldots\ldots (3F) \\
& + (\mathfrak{P}^2X + 2\mathfrak{P}.P.\mathfrak{X})\left(6\lambda - 30.\lambda\mathfrak{O} + 90.\lambda\mathfrak{O}^2 - \tfrac{135}{2}\lambda O^2\right) \ldots (G) \\
& + (P^2\mathfrak{X} + 2\mathfrak{P}.P.X)\left(\tfrac{45}{2}\lambda O - 135\lambda.\mathfrak{O}O\right) \ldots (H) \\
& + P^2X\left(-\tfrac{9}{2}\lambda + \tfrac{45}{2}\lambda\mathfrak{O} - \tfrac{135}{2}\lambda\mathfrak{O}^2 + \tfrac{225}{4}\lambda O^2\right) \ldots (3I) \\
& + \text{♄}\left\{\begin{aligned} & -\tfrac{3}{2}\lambda Q + \tfrac{15}{2}\lambda(O\mathfrak{Q} + \mathfrak{O}Q) + \tfrac{15}{2}\lambda\mathfrak{P}.P \\ & -\tfrac{45}{2}\lambda(O\mathfrak{P}^2 + 2\mathfrak{O}\mathfrak{P}.P) + \tfrac{45}{4}\lambda OP^2 \end{aligned}\right. \\
& + \text{♃}\left(-\tfrac{3}{2}\lambda P + \tfrac{15}{2}\lambda(\mathfrak{O}P + O\mathfrak{P})\right) \\
& + \text{♂}.\left(-\tfrac{3}{2}\lambda O.\right) \\
& \qquad = 0.
\end{aligned}
$$

§. 141.

Quoniam in his formulis plures factores litteras $\mathfrak{D}$ et O inuoluentes saepius recurrebant, eos in prioribus aequationibus litteris ($\mathfrak{A}$) ($\mathfrak{B}$) ($\mathfrak{C}$) etc. in posterioribus vero litteris (A) (B) (C) signauimus; quarum ergo litterarum significatus sequenti modo se habent

$$\mathfrak{A} = -\tfrac{3}{2}\cos 2p + 6.\lambda\mathfrak{D} - 12.\lambda\mathfrak{D}^2 + 6.\lambda.O^2$$

$$\mathfrak{B} = A = \tfrac{3}{2}\sin 2p - 3\lambda O + 12.\lambda\mathfrak{D}O.$$

$$B = \tfrac{3}{2}\cos 2p - 3\lambda\mathfrak{D} + 6.\lambda.\mathfrak{D}^2 - \tfrac{9}{4}\lambda O^2$$

$$\mathfrak{C} = 3\lambda - 12.\lambda\mathfrak{D} + 30.\lambda.\mathfrak{D}^2 - 15.\lambda.O^2$$

$$\mathfrak{D} = 2C = 12.\lambda O - 60.\lambda\mathfrak{D}O.$$

$$\mathfrak{E} = \tfrac{1}{2}D = -\tfrac{3}{2}\lambda + 6\lambda\mathfrak{D} - 15.\lambda\mathfrak{D}^2 + \tfrac{45}{4}\lambda O^2$$

$$E = -\tfrac{3}{8}\mathfrak{D} = -\tfrac{9}{2}\lambda O + \tfrac{45}{2}\lambda\mathfrak{D}O.$$

$$\mathfrak{F} = -4\lambda + 20.\lambda\mathfrak{D} - 60.\lambda.\mathfrak{D}^2 + 30.\lambda.O^2$$

$$\mathfrak{G} = 3F = -30.\lambda O + 180.\lambda\mathfrak{D}O.$$

$$\mathfrak{H} = G = 6.\lambda - 30.\lambda\mathfrak{D} + 90.\lambda\mathfrak{D}^2 - \tfrac{135}{2}\lambda O^2$$

$$\mathfrak{I} = -\tfrac{1}{4}\mathfrak{G} = \tfrac{1}{3}H = \tfrac{15}{2}\lambda O - 45.\lambda\mathfrak{D}O.$$

$$I = -\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{D} - \tfrac{45}{2}\lambda\mathfrak{D}^2 + \tfrac{75}{4}\lambda O^2.$$

§. 142.

§. 142.

Expeditis pro noſtris ordinibus binis prioribus aequationibus ita, vt pro ſingulis ordinibus binas aequationes ſpeciales ſimus adepti, ſupereſt, vt etiam aequationem tertiam generalem ſimili modo euoluamus, vbi quidem tantum quinque ordines conſtituimus.

Pro ordine primo (I) characteris *i*.

$$\frac{ddp}{dt^2} + (\lambda + 1)p,$$
$$+ p(-3\lambda\mathfrak{D} + 6.\lambda\mathfrak{D}^2 - \tfrac{5}{2}\lambda O^2) \ldots\ldots (a)$$
$$= 0.$$

Pro ordine (II), characteris *i* K.

$$\frac{ddq}{dt^2} + (\lambda + 1)q,$$
$$+ q(-3\lambda\mathfrak{D} + 6\lambda\mathfrak{D}^2 - \tfrac{5}{2}\lambda O^2) \ldots\ldots (a)$$
$$+ \mathfrak{P}.p(-3\lambda + 12.\lambda\mathfrak{D} - 30.\lambda\mathfrak{D}^2 + \tfrac{15}{2}\lambda O^2) \ldots (b)$$
$$+ Pp(-3\lambda O + 15.\lambda\mathfrak{D}O) \ldots\ldots (c)$$
$$= 0.$$

Pro

Pro ordine tertio (III), characteris $i K^{\epsilon}$.

$$\frac{ddr}{dt^2} + (\lambda + 1)r,$$

$$+ r(-3\lambda\mathfrak{D} + 6\lambda\mathfrak{D}^2 - \tfrac{3}{2}\lambda O^2) \ldots\ldots (a)$$

$$+ (\mathfrak{P}q + \mathfrak{Q}p)(-3\lambda + 12.\lambda\mathfrak{D} - 30\lambda O^2 + \tfrac{15}{2}\lambda O^2) \ldots (b)$$

$$+ (Pq + Qp)(-3\lambda O + 15\lambda\mathfrak{D}O) \ldots\ldots (c)$$

$$+ \mathfrak{P}^2 p(6.\lambda - 30.\lambda\mathfrak{D} + 90\lambda\mathfrak{D}^2 - \tfrac{45}{2}\lambda O^2) \ldots (d)$$

$$+ \mathfrak{P}.P.p.(15.\lambda O - 90.\lambda\mathfrak{D}O) \ldots\ldots (e)$$

$$+ P^2 p.(-\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{D} - \tfrac{45}{2}\lambda\mathfrak{D}^2 + \tfrac{45}{8}\lambda O^2) \ldots\ldots (f)$$

$$= 0.$$

Pro ordine quarto (IV), characteris $i\kappa$.

$$\frac{dds}{dt^2} + (\lambda + 1)s,$$

$$+ s(-3\lambda\mathfrak{D} + 6.\lambda\mathfrak{D}^2 - \tfrac{3}{2}\lambda O^2) \ldots\ldots (a)$$

$$+ \mathfrak{U}p(-3\lambda + 12\lambda\mathfrak{D} - 30\lambda\mathfrak{D}^2 + \tfrac{15}{2}\lambda O^2) \ldots (b)$$

$$+ U.p(-3\lambda O + 15\lambda\mathfrak{D}O) \ldots\ldots (c)$$

$$- \mathfrak{J}.p.\cos.t.$$

$$= 0.$$

Pro ordine quinto (V), characteris i^{2}.

$$\frac{ddt}{dt^2} + (\lambda + 1)t,$$
$$+ t(-3\lambda\mathfrak{D} + 6\lambda\mathfrak{D}^2 - \tfrac{3}{2}\lambda O^2) \ldots\ldots (a)$$
$$+ \mathfrak{X}p(-3\lambda + 12\lambda\mathfrak{D} - 30\lambda\mathfrak{D}^2 + \tfrac{15}{2}\lambda O^2) \ldots (b)$$
$$+ Xp(-3\lambda O + 15\lambda\mathfrak{D}O) \ldots\ldots (c)$$
$$+ \mathfrak{P}p(-\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{D})$$
$$= 0.$$

§. 143.

Omnes has aequationes speciales perpendenti mox patebit, istas quinque aequationes postremas ante euolui debere, quam aequationum superiorum tres vltimae suscipi queant; propterea quod ibi in vndecimo ordine primum occurrit character $\mathfrak{P}$, cuius valor non nisi postremis formulis tertiae aequationis generalis iam expeditis innotescit; cum sit $\mathfrak{P} = p^2$. quod multo magis locum habet in ordinibus duodecimo et decimo tertio. Quocirca in sequentibus resolutiones nostras incipi oportebit ab aequationibus specialibus primo exhibitis; in quo negotio procedere poterimus vsque ad ordinem decimum inclusiue; tum vero postremae hae quinque aequationes pertractentur, antequam superiorum tres vltimos ordines adgrediamur.

NOVAE THEORIAE MOTVVM LVNAE

LIBER PRIMVS

CONTINENS IPSAM LVNAE THEORIAM.

PARS SECVNDA.

EVOLVTIO NVMERICA AEQVATIONVM PRO BINIS COORDINATIS x ET y, IN PRAECEDENTE PARTE CONSTITVTARVM.

R

CAPUT I.

EVOLUTIO AEQUATIONUM ORDINIS I, PRO LITTERIS $\mathfrak{D}$ ET O.

§. 144.

Aequationes speciales, vnde hos valores inuestigari oportet, ita se habebant:

I. $\frac{dd\mathfrak{D}}{dt^2} - \frac{2(m+1)\,dO}{dt} - 3\lambda\mathfrak{D}, -\frac{3}{2}\,\mathrm{cos.}\,2p,$

$-\frac{3}{2}\mathfrak{D}.\mathrm{cos.}\,2p + \frac{3}{2}O.\mathrm{sin.}\,2p, + 3\lambda\mathfrak{D}^2 - \frac{3}{2}\lambda O^2$

$= 0.$

II. $\frac{ddO}{dt^2} + \frac{2(m+1)\,d\mathfrak{D}}{dt}, + \frac{3}{2}\,\mathrm{sin.}\,2p,$

$+\frac{3}{2}\mathfrak{D}\,\mathrm{sin.}\,2p + \frac{3}{2}O.\mathrm{cos.}\,2p, - 3\lambda\mathfrak{D}O$

$= 0.$

§. 145.

Quoniam facile praeuidere licet, valores litterarum $\mathfrak{O}$ et O prodituros esse valde exiguos prae vnitate; membra postrema in nostris aequationibus primum reiiciamus, vt tantum vero proxime valores istos eliciamus. Huc igitur translatis, quae supra §. 85. sunt praecepta, pars annexa prioris aequationis $-\frac{3}{8}$ cos. $2p$ statim praebet formulam $\mathfrak{M}$. cos. ω; ita, vt hic sit $\omega = 2p$; ideoque $\mu = 2m$, tum vero $\mathfrak{M} = -\frac{3}{8}$. Simili modo ex altera aequatione terminus absolutus $+\frac{3}{4}$ sin. $2p$ pro M. sin. ω dat $M = +\frac{3}{4}$ atque hinc quaerere debemus litteras $\mathfrak{N}$ et N, ex quibus continuo concludemus proxime $\mathfrak{O} = \mathfrak{N}$. cos. $2p$; et $O = N$. sin. $2p$.

§. 146.

Formulae autem valores istarum litterarum $\mathfrak{N}$ et N exhibentes erant

$$\mathfrak{N} = \frac{\frac{2(m+1)}{\mu} . M - \mathfrak{M}}{\lambda - 2 - \mu^2}$$

$$N = \frac{M}{\mu^2} - \frac{2(m+1)}{\mu} . \mathfrak{N}$$

vbi notandum, $m+1$ eundem esse numerum, ex quo supra λ definiuimus; erit ergo ex §. 79. $m+1 = 13,36892$. existente littera pura $m = 12,36892$. in-

indeque ſtatuimus $\lambda = 179,228928$; indeque

$$\lambda - 2 = 177,228928.$$

Quare cum hic habeamus $\mu = 2m = 24,73784$; elementa numerica, quibus hic indigemus, erunt;

$L. 2(m+1) = 1,4271258$	$L. \mu^2 = 2,7867236$
ſubtr. $l. \mu = 1,3933618$	$\mu^2 = 611,960828$
$L. \frac{2(m+1)}{\mu} = 0,0337640$	$\lambda - 2 = 177,228928$
$L. M = 0,1760913$	den. $= -434,731900$
$L. \frac{2(m+1)M}{\mu} = 0,2098553$	$L.$ num. $= 0,4943313$
$\frac{2(m+1)M}{\mu} = 1,6212700$	$L.$ den. $= 2,6382215\ (-)$
$-\mathfrak{M} = +1,5000000$	$L. \mathfrak{N} = 7,8561098\ (-)$
Numer. $= 3,1212700$	$L. \frac{2(m+1)}{\mu} = 0,0337640$
$L. M = 0,1760913$	$7,8898738\ (-)$
$L. \mu^2 = 2,7867236$	$\frac{M}{\mu^2} = 0,0024511$
$L. \frac{M}{\mu^2} = 7,3893677$	$\frac{-2(m+1)\mathfrak{N}}{\mu} = 0,0077602$
	$N = 0,0102113$

at vero $\mathfrak{N} = -0,0071797$.

§. 146.

Haec igitur prima adpropinquatio nobis iam ſuppeditat iſtos valores

$\mathfrak{D} = -0,0071797 \cdot \text{cof.}\ 2p.$

$\mathfrak{O} = +0,0102113 \cdot \text{fin.}\ 2p.$

Quos ergo iam in vltimis membris aequationum nostrarum reiectis fubftituamus, vt deinceps ad veritatem propius accedamus:

Eft vero

$\mathfrak{D} \cdot \text{cof.}\ 2p = -0,0035898 - 0,0035898 \cdot \text{cof.}\ 4p$

$\mathfrak{D} \cdot \text{fin.}\ 2p = -0,0035898 \cdot \text{fin.}\ 4p.$

$\mathfrak{O} \cdot \text{cof.}\ 2p = +0,0051056 \cdot \text{fin}\ 4p.$

$\mathfrak{O} \cdot \text{fin.}\ 2p = +0,0051056 - 0,0051056 \cdot \text{cof.}\ 4p$

$3\lambda\mathfrak{D}^2 = 0,0138583 + 0,0138583 \cdot \text{cof.}\ 4p.$

$\frac{3}{2}\lambda\mathfrak{O}^2 = +0,0140162 - 0,0140162 \cdot \text{cof.}\ 4p.$

$3\lambda\mathfrak{D}\mathfrak{O} = -0,0197099 \cdot \text{fin.}\ 4p.$

§. 147.

His igitur valoribus fubftituendis pro priore aequatione fient partes annexae

$-1,5000000 \cdot \text{cof.}\ 2p + 0,0128852.$

$+0,0256008 \cdot \text{cof.}\ 4p.$

Pofterioris vero aequationis partes annexae erunt

$+1,5000000 \cdot \text{fin.}\ 2p$

$+0,0219836 \cdot \text{fin.}\ 4p.$

vbi cum partes primae angulum $2p$ inuoluentes eaedem fint, quae ante; inde etiam iidem valores in litteras

litteras $\mathfrak{D}$ et O redundabunt; ſcilicet pro $\mathfrak{D}$ habebimus

$$\mathfrak{D} = -0,0071797 \cdot \operatorname{coſ.} 2p$$

pro O autem

$$O = +0,0102113 \cdot \operatorname{ſin.} 2p$$

Numerus autem abſolutus in priore ſtatim praebet pro $\mathfrak{D}$ ex §. 86. partem $\frac{\alpha}{s\lambda} = 0,0000240$.

§. 148.

Praeterea vero vtrinque termini angulum $4p$ inuoluentes accedent, ſicque pro iis inueniendis erit $\omega = 4p$; hinc $\mu = 4m$; atque

$$\mathfrak{M} = +0,0256008. \text{ et}$$
$$M = +0,0219836.$$

vnde igitur litteras $\mathfrak{N}$ et N quaeri oportet; quod ſequenti calculo praeſtabitur:

L, μ =

$L.\,\mu = 1,6943918$	$L.\,2(m+1) = 1,4271258$
$L.\,\mu^2 = 3,3887836$	$L.\,\mu = 1,6943918$
$\mu^2 = 2447,8434000$	$9,7327340$
$\lambda - 2 = 177,2289280$	$L.\,M = 8,3420989$
$\text{denom.} = -2270,6144720$	$8,0748329$
$L.\,\text{Num.} = 8,1373858\, -$	$\frac{2(m+1)}{\mu} M = 0,011880$
$L.\,\text{denom.} = 3,3561432\, -$	$-\mathfrak{M} = -0,025601$
$L.\,\mathfrak{N} = 4,7812426\, +$	$\text{Numer.} = -0,013721$
$\mathfrak{N} = 0,0000060$	
$L.\,M = 8,3420989$	$L.\,\frac{2(m+1)}{\mu} = 9,7327340$
$L.\,\mu^2 = 3,3887836$	$L.\,\mathfrak{N} = 4,7812426$
$4,9533153$	$4,5139766$
$\frac{M}{\mu^2} = 0,0000090$	$\frac{2(m+1)}{\mu}.\,\mathfrak{N} = 0,0000033$
$-\frac{2(m+1)}{\mu}\mathfrak{N} = -0,0000033$	
$N = +0,0000057$	

Hinc ergo nanciſcimur hos valores correctos

$$\mathfrak{P} = -0,0071797.\,\text{coſ.}\,2p + 0,0000240 + 0,0000060.\,\text{coſ.}\,4p.$$

$$O = +0,0102113.\,\text{ſin.}\,2p + 0,0000057.\,\text{ſin.}\,4p.$$

§. 149.

Videamus nunc, an iſti valores denuo correctione indigeant; hunc in finem ponamus breuitatis gratia

$\mathfrak{P} =$

$$\mathfrak{D} = \beta.\ \text{cos.}\ 2p + \alpha + \gamma.\ \text{cos.}\ 4p$$

$$O = b.\ \text{sin.}\ 2p + c.\ \text{sin.}\ 4p$$

hisque valoribus ſubſtitutis pro aequatione priore erit

$$-\tfrac{3}{2}\mathfrak{D}.\ \text{cos.}\ 2p = -\tfrac{3}{4}\beta - \tfrac{3}{4}\beta.\ \text{cos.}\ 4p - \tfrac{3}{2}\alpha.\ \text{cos.}\ 2p$$
$$-\tfrac{3}{4}\gamma.\ \text{cos.}\ 2p - \tfrac{3}{4}\gamma.\ \text{cos.}\ 6.p.$$

$$+\tfrac{3}{2}O.\ \text{sin.}\ 2p = \tfrac{3}{4}b - \tfrac{3}{4}b.\ \text{cos.}\ 4p$$
$$+\tfrac{3}{4}c.\ \text{cos.}\ 2p - \tfrac{3}{4}c.\ \text{cos.}\ 6p.$$

$$+3\lambda.\mathfrak{D}^2 = \tfrac{3}{2}\lambda\beta^2 + \tfrac{3}{2}\lambda.\beta^2.\ \text{cos.}\ 4p + 6\lambda\alpha\beta.\ \text{cos.}\ 2p$$
$$+3\lambda\beta\gamma.\ \text{cos.}\ 2p + 3\lambda.\beta\gamma.\ \text{cos.}\ 6p$$

$$-\tfrac{3}{2}\lambda.O^2 = -\tfrac{3}{4}\lambda b^2 + \tfrac{3}{4}\lambda b^2.\ \text{cos.}\ 4p$$
$$-\tfrac{3}{2}\lambda.bc.\ \text{cos.}\ 2p + \tfrac{3}{2}\lambda bc.\ \text{cos.}\ 6p.$$

pro poſteriore vero aequatione colligimus

$$+\tfrac{3}{2}\mathfrak{D}.\ \text{sin.}\ 2p = \tfrac{3}{4}\beta.\ \text{sin.}\ 4p + \tfrac{3}{2}\alpha.\ \text{sin.}\ 2p$$
$$+\tfrac{3}{4}\gamma.\ \text{sin.}\ 6p - \tfrac{3}{4}\gamma.\ \text{sin.}\ 2p$$

$$+\tfrac{3}{2}O.\ \text{cos.}\ 2p = \tfrac{3}{4}b.\ \text{sin.}\ 4p + \tfrac{3}{4}c.\ \text{sin.}\ 6p$$
$$+\tfrac{3}{4}c.\ \text{sin.}\ 2p$$

$$-3\lambda\mathfrak{D}O = -\tfrac{3}{2}\lambda\beta.b.\ \text{sin.}\ 4p - 3\lambda\alpha b.\ \text{sin.}\ 2p$$
$$-\tfrac{3}{2}\lambda\gamma b.\ \text{sin.}\ 6p + \tfrac{3}{2}\lambda\gamma b.\ \text{sin.}\ 2p$$
$$-\tfrac{3}{2}\lambda\beta c.\ \text{sin.}\ 6p - \tfrac{3}{2}\lambda\beta c.\ \text{sin.}\ 2p$$

vbi producta ex binis minimis particulis, quales ſunt α, γ et c, omiſimus, quandoquidem ea intra ſeptem figuras decimales nihil producunt.

§. 150.

§. 150.

Hinc igitur pro angulo $2p$ litterae $\mathfrak{M}$ et M quaedam incrementa accipiunt, ſcilicet

$$\mathfrak{M} = -\tfrac{3}{2} - \tfrac{3}{2}\alpha - \tfrac{3}{4}\gamma + \tfrac{3}{4}c + 6\lambda\alpha\beta + 3\lambda\beta\gamma - \tfrac{3}{2}\lambda b c.$$

$$M = +\tfrac{3}{2} + \tfrac{3}{2}\alpha - \tfrac{3}{4}\gamma + \tfrac{3}{4}c - 3\lambda\alpha b + \tfrac{3}{2}\lambda\gamma b - \tfrac{3}{2}\lambda\beta c.$$

Quoniam igitur

$$\beta = -0{,}0071797;\quad \alpha = +0{,}0000240.$$
$$\gamma = +0{,}0000060;\quad b = +0{,}0102113$$
$$c = +0{,}0000057.$$

prodibunt ſequentes valorés

$$\mathfrak{M} = -\tfrac{3}{2} - 0{,}0002602 = -1{,}5002602.$$
$$M = +\tfrac{3}{2} - 0{,}0000684 = +1{,}4999316.$$

ex his igitur nouis valoribus inueſtigemus reſpondentes litteras $\mathfrak{N}$ et N, ſequenti calculo pro angulo $2p$

Log.

$$L.\,\frac{2(m+1)}{\mu} = 0{,}0337640$$
$$L.\,M = 0{,}1760705$$
$$L.\,\frac{2(m+1)}{\mu}M = 0{,}2098345$$
$$1{,}6211926$$
$$-\mathfrak{M} = +\,1{,}5002602$$
$$\text{Numer.} = +\,3{,}1214528$$
$$L.\,\text{Num.} = 0{,}4943566$$
$$L.\,\text{den.} = 2{,}6382215 -$$
$$\text{Log.}\,\mathfrak{N} = 7{,}8561351 -$$
$$\mathfrak{N} = -\,0{,}0071801$$

$$L.\,M = 0{,}1760705$$
$$L.\,\mu^2 = 2{,}7867236$$
$$L.\,\frac{M}{\mu^2} = 7{,}3893469$$
$$L.\,\mathfrak{N} = 7{,}8561351 -$$
$$L.\,\frac{2(m+1)}{\mu} = 0{,}0337640$$
$$L.\,\frac{2(m+1)}{\mu}\mathfrak{N} = 7{,}8898991 -$$

ergo

$$\frac{M}{\mu^2} = +\,0{,}0024510$$
$$-\frac{2(m+1)}{\mu}\mathfrak{N} = +\,0{,}0077607$$
$$N = +\,0{,}0102117$$

§. 151.

Reliquae autem partes litterarum $\mathfrak{N}$ et N hinc nullam plane correctionem accipiunt; pars enim constans litterae $\mathfrak{M}$ est $-\frac{3}{4}\beta + \frac{3}{4}b + \frac{3}{2}\lambda'\beta^2 - \frac{3}{4}\lambda b^2$ prorsus vt ante; simili modo partes angulo $4p$ respondentes a praecedentibus non discrepant; tantum accedent termini angulum $6p$ inuoluentes, qui vero ob paruitatem nihil plane ad vsum nostrum producunt; quocirca nunc affirmare licebit, veros valores litterarum nostrarum $\mathfrak{O}$ et O fore sequentes:

$$\mathfrak{O} = -\,0{,}0071801 \,.\, \cos.\, 2p + 0{,}0000240$$
$$+\,0{,}0000060 \,.\, \cos.\, 4p.$$
$$O = +\,0{,}0102117 \,.\, \sin.\, 2p + 0{,}0000057 \,.\, \sin.\, 4p.$$

§. 152.

Quoniam hic iam euoluimus valores formularum $3\lambda \mho^2$; $\frac{5}{2}\lambda O^2$; $3\lambda \mho O$ in sequentibus autem calculis similes formulae pariterque etiam tales $3\lambda\mho$ et $3\lambda O$ vel earum certa multipla vbique occurrent; huiusmodi formulas hic data opera euoluamus

$$3\lambda\mho = -3,8606454.\ \text{cos}.\ 2p + 0,0129045 + 0,0032261.\ \text{cos}.\ 4p$$

$$3\lambda O = +5,4906964.\ \text{sin}.\ 2p + 0,0030648.\ \text{sin}.\ 4p.$$

$$3\lambda\mho^2 = 0,0138599. + 0,0138599.\ \text{cos}.\ 4p.$$

$$3\lambda O^2 = 0,0280347. - 0,0280347.\ \text{cos}.\ 4p$$

$$3\lambda\mho O = -0,0197114.\ \text{sin}.\ 4p.$$

§. 153.

Hinc igitur iam in subsidium sequentium calculorum valores earum formularum, quas supra litteris siue germanicis $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ etc. siue latinis A, B, C etc. signauimus (§. 141.), adcurate exhibere poterimus; reperiemus scilicet

$$\mathfrak{A} = -9,2212908\ \text{cos}.\ 2p; + 0,0264388 - 0,1050568.\ \text{cos}.\ 4p.$$

$$\mathfrak{B} = * - 3,9906964\ \text{sin}.\ 2p, - 0,0819104.\ \text{sin}.\ 4p.$$

$$\mathfrak{C} = 537,6335924, + 15,4425816.\ \text{cos}.\ 2p, + 0,2658681.\ \text{cos}.\ 4p.$$

$\mathfrak{D} =$

$\mathfrak{D} = * + 21,9627856 \sin. 2p, + 0,4064872. \sin. 4p.$

$\mathfrak{E} = - 268,7817528 - 7,721290. \cos. 2p,$
$\quad - 0,1679774. \cos. 4p.$

$\mathfrak{F} = - 716,826657, - 25,737686. \cos. 2p$
$\quad - 0,536038. \cos. 4p.$

$\mathfrak{G} = * - 54,906964. \sin. 2p, - 1,213332. \sin. 4p.$

$\mathfrak{H} = + 1075,030228, + 38,606454. \cos. 2p,$
$\quad + 1,014310. \cos. 4p.$

$\mathfrak{I} = * + 13,726741. \sin. 2p, + 0,3033327. \sin. 4p.$

$A = * - 3,9906964. \sin. 2p, - 0,0819104. \sin. 4p.$

$B = + 5,3606454. \cos. 2p, - 0,0272367$
$\quad + 0,0665457. \cos. 4p.$

$C = * + 10,9813978. \sin. 2p + 0,2032436. \sin. 4p.$

$D = - 537,5635056 - 15,442580. \cos. 2p$
$\quad - 0,3359548. \cos. 4p.$

$E = * - 8,2360446. \sin. 2p - 0,1524327. \sin. 4p.$

$F = * - 18,302321. \sin. 2p - 0,404444. \sin. 4p.$

$G = + 1075,030228, + 38,606454. \cos. 2p$
$\quad + 1,014310. \cos. 4p.$

$H = * + 41,180223. \sin. 2p + 0,909998. \sin. 4p.$

$I = - 268,739922, - 9,651610. \cos. 2p$
$\quad - 0,271080. \cos. 4p.$

CAPVT II.

EVOLVTIO AEQVATIONVM ORDINIS II. PRO LITTERIS 𝔓 ET P.

§. 154.

Quoniam in his euolutionibus non amplius ad partes principales aequationum ſpecialium reſpicimus; ſuperfluum foret, iſtas partes hic repetere; quamobrem tantum partes annexas exhibebimus.

I. $0 = \ldots\ldots + \mathfrak{P}.\mathfrak{A} + P.\mathfrak{B}.$

II. $0 = \ldots\ldots + \mathfrak{P}.A + P.B.$

ex quibus partibus annexis valores litterarum 𝔐 et M pro ſingulis angulis, qui in determinationem litterarum 𝔓 et P ingrediuntur, inueſtigemus, vt deinceps valores reſpondentes 𝔑 et N inde eruamus, quippe qui valores quaeſitos litterarum 𝔓 et P exhibebunt.

§. 155.

Cum autem ipſae hae litterae 𝔓 et P etiamnum ſint incognitae, eas quoque tales in calculo conſiderari oportet, et quia manifeſtum eſt, angulum prin-

principalem hic fore ipſam anomaliam mediam q, hic ipſe angulus inſuper cum iis, qui in litteras $\mathfrak{O}$ et O ingrediuntur, combinabitur; ex quo perſpicuum eſt, hos valores, quos quaerimus, huiusmodi formas eſſe habituros:

$$\mathfrak{P} = \beta.\ \text{cof.}\ q + \gamma\ \text{cof.}\ (2p - q) + \delta.\ \text{cof.}\ (2p + q)$$
$$+ \varepsilon.\ \text{cof.}\ (4p - q) + \zeta\ \text{cof.}\ (4p + q)$$
$$P = b.\ \text{fin.}\ q + c.\ \text{fin.}\ (2p - q) + d.\ \text{fin.}\ (2p + q)$$
$$+ e.\ \text{fin.}\ (4p - q) + f.\ \text{fin.}\ (4p + q)$$

ita, vt labor noſter ad determinationem horum coefficientium perducatur.

§. 156.

Commodiſſime autem hanc inueſtigationem per adproximationes inſtituemus; vnde primo quidem ex valoribus litterarum $\mathfrak{A}$, $\mathfrak{B}$ et A, B tantum partes primas principales conſiderabimus, neglectis tantisper minoribus, in quibus deinceps ſufficiet, valores prope veros pro litteris $\mathfrak{P}$ et P ſubſtituere. Hinc ex prima aequatione pro ſingulis his angulis valores litterae $\mathfrak{M}$ ſequenti modo colligentur.

	cof. q	cof. $(2p-q)$	cof. $(2p+q)$
$-9{,}2212908.\text{cof.}\,2p.\mathfrak{P}$	$-4{,}6106454.\gamma$	$-4{,}6106454.\beta$	$-4{,}6106454.\beta$
	$-4{,}6106454.\delta$	$-4{,}6106454.\varepsilon$	$-4{,}6106454.\zeta$
$-3{,}9906964.\text{fin.}\,2p.P$	$-1{,}9953482.\,c$	$-1{,}9953482.\,b$	$+1{,}9953482.\,b$
	$-1{,}9953482.\,d$	$-1{,}9953482.\,e$	$-1{,}9953482.\,f$

Simili modo pro angulis $4p - q$ et $4p + q$.

	cof. $(4p-q)$	cof. $(4p+q)$
$-9{,}2212908.\text{cof}\,2p.\mathfrak{P}$	$-4{,}6106454.\gamma$	$-4{,}6106454.\delta$
$-3{,}9906964.\text{fin.}\,2p.P$	$+1{,}9953482.\,c$	$+1{,}9953482.\,d$

§. 257.

§. 157.

Simili modo ex altera aequatione pro his iisdem angulis valores litterae M formabimus:

	$\sin. q$	$\sin. (2p-q)$	$\sin. (2p+q)$
$-3{,}9906964.\sin. 2p.\mathfrak{P}$	$-1{,}9953482.\gamma$	$-1{,}9953482.\beta$	$-1{,}9953482\ \beta$
	$+1{,}9953482.\delta$	$+1{,}9953482.\varepsilon$	$+1{,}9953482.\zeta$
$+5{,}3606454.\cos. 2p.P.$	$-2{,}6803227.c$	$-2{,}6803227.b$	$+2{,}6803227.b$
	$+2{,}6803227.d$	$+2{,}6803227.e$	$+2{,}6803227.f$

Simili modo pro angulis $4p-q$ et $4p+q$.

	$\sin. (4p-q)$	$\sin. (4p+q)$
$-3{,}9906964.\sin. 2p.\mathfrak{P}$	$-1{,}9953482.\gamma$	$-1{,}9953482.\delta$
$+5{,}3606454.\cos. 2p.P.$	$+2{,}6803227.c$	$+2{,}6803227.d$

§. 158.

Elementa numerica pro his quinque angulis sunt sequentia, vbi meminisse oportet, esse $m = 12{,}36892$ et $\frac{dq}{dt} = n = 13{,}25604$. ex § 97.

$\omega = q$	$2p-q$	$2p+q$	$4p-q$	$4p+q$
$\mu = n$	$2m-n$	$2m+n$	$4m-n$	$4m+n$
$\mu = 13{,}25604$	$11{,}48180$	$37{,}99388$	$36{,}21964$	$62{,}73172$
$L. 2(m+1) = 1{,}4271258$	$1{,}4271258$	$1{,}4271258$	$1{,}4271258$	$1{,}4271258$
$L. \mu = 1{,}1224138$	$1{,}0600100$	$1{,}5797136$	$1{,}5589439$	$1{,}7974872$
$L. \frac{2(m+1)}{\mu} = 0{,}3047120$	$0{,}3671158$	$9{,}8474122$	$9{,}8681819$	$9{,}6296386$
$L. \mu^2 = 2{,}2448276$	$2{,}1200200$	$3{,}1594272$	$3{,}1178878$	$3{,}5949744$
$\lambda - 2 = 177{,}228928$	$177{,}228930$	$177{,}22893$	$177{,}22893$	$177{,}22893$
$\mu^2 = 175{,}722608$	$131{,}831730$	$1443{,}53466$	$1311{,}86121$	$3935{,}26809$
Denom. $= +1{,}506320$	$+45{,}397200$	$-1266{,}30573$	$-1134{,}63228$	$-3758{,}03916$
L. den. $= +0{,}1779403$	$+1{,}6570290$	$-3{,}1025375$	$-3{,}0548559$	$-3{,}5749613$

§. 159.

§. 159.

Valores autem litterarum $\mathfrak{M}$ et M pro singulis angulis clarius ob oculos ponamus:

I. Pro angulo q

$$\mathfrak{M} = -4,6106454(\gamma+\delta) - 1,9953482(c+d)$$
$$M = -1,9953482(\gamma-\delta) - 2,6803227(c-d)$$

II. Pro angulo $2p-q$

$$\mathfrak{M} = -4,6106454(\beta+\varepsilon) - 1,9953482(b+e)$$
$$M = -1,9953482(\beta-\varepsilon) - 2,6803227(b-e)$$

III. Pro angulo $2p+q$

$$\mathfrak{M} = -4,6106454(\beta+\zeta) + 1,9953482(b-f)$$
$$M = -1,9953482(\beta-\zeta) + 2,6803227(b+f)$$

IV. Pro angulo $4p-q$

$$\mathfrak{M} = -4,6106454.\gamma + 1,9953482.c$$
$$M = -1,9953482.\gamma + 2,6803227.c$$

V. Pro angulo $4p+q$

$$\mathfrak{M} = -4,6106454\,\delta + 1,9953482.d$$
$$M = -1,9953482\,\delta + 2,6803227.d.$$

§. 160.

Euolutionem primae columnae seu anguli q, quoniam is singularem diiudicationem requirit, in postremum locum reseruari conueniet; calculum ergo secundum nostram regulam pro quatuor reliquis angulis instituamus, vt inde valores coefficientium $\gamma, c, \delta, d; \varepsilon, e; \zeta, f$ per binos priores β et b expressos eliciamus, atque hic quidem notasse iuuabit, si necesse fuit quantitates $\mathfrak{O}$ et O ad septimam figuram in fractionibus decimalibus producere, siquidem tum vnitas in vltima figura tantum $\frac{1}{50}$ minuti secundi in loco Lunae valet, hic sufficere valores litterarum β, γ, δ etc. tantum ad sextam figuram continuari, quoniam quantitates $\mathfrak{P}$ et P per fractionem K multiplicantur, cuius valor circiter est $\frac{1}{10}$.

§. 161.

Incipiamus ergo nostrum calculum ab angulo $2p-q$, pro quo si ex litteris $\mathfrak{M}$ et M elicuerimus litteras $\mathfrak{N}$ et N, statim habebimus has aequationes:

$$\mathfrak{N} = \gamma \text{ et } N = c.$$

Pro

Pro angulo $2p-q$.

	β.	ε.	b.	e.
L. M.	0,3000186 −	. . . . . . +	0,4281870 −	. . . . . . +
$L.\frac{2(m+1)}{\mu}$	0,3671158 +		0,3671158 +	
	0,6671344 −		0,7953028 −	
$\frac{2(m+1)M}{\mu}$	−4,646590	+4,646590	−6,241700	+6,241700
$-\mathfrak{M}$	+4,610645	+4,610645	+1,995348	+1,995348
Numer.	−0,035945	+9,257235	−4,246352	+8,237048
Log.	−8,5556385	+0,9664812	−0,6280160	+0,9157716
L. Den.	+1,6570290	+1,6570290	+1,6570290	+1,6570290
L. $\mathfrak{N}$	−6,8986095	+9,3094522	−8,9709870	+9,2587426
$L.\frac{2(m+1)}{\mu}$	+0,3671158	+0,3671158	+0,3671158	+0,3671158
L. P. II.	−7,2657253	+9,6765680	−9,3381028	+9,6258584
L. M.	0,3000186 −	. . . . . . +	0,4281870 −	. . . . . . +
L. μ^2	2,1200200		2,1200200	
L. P. I.	8,1799986 −		8,3081670 −	
P. I.	−0,015136	+0,015136	−0,020331	+0,020331
− P. II.	+0,001844	−0,474863	+0,217822	−0,422531
N =	−0,013292	−0,459727	+0,197491	−0,402200
at $\mathfrak{N}$ =	−0,000792	+0,203916	−0,093538	+0,181444

En ergo ambas aequationes, quas hinc consequimur

$$\gamma = -0,000792.\,\beta + 0,203916.\,\varepsilon - 0,093538.\,b + 0,181444.\,e.$$

$$c = -0,013292.\,\beta - 0,459727.\,\varepsilon + 0,197491.\,b - 0,402200.\,e.$$

§. 162.

Simili modo faciamus calculum

Pro angulo $2p+q$; vnde $\mathfrak{N}=\delta$ et $N=d$.

	β.	ζ.	b.	f.
L. M	0,3000186 −	. . . . . +	0,4281870 +	. . . . +
L. $\frac{2(m+1)}{\mu}$	9,8474122		9,8474122	
L. $\frac{2(m+1)M}{\mu}$	0,1474308 −		0,2755992 +	
− $\frac{2(m+1)M}{\mu}$	− 1,404206	+ 1,404206	+ 1,886250	+ 1,886250
− $\mathfrak{M}$	+ 4,610645	+ 4,610645	− 1,995348	+ 1,995348
Num.	+ 3,206439	+ 6,014851	− 0,109098	+ 3,881598
Log.	+ 0,5060230	+ 0,7792248	− 9,0378170	+ 0,5890106
L. Den.	− 3,1025375	− 3,1025375	− 3,1025375	− 3,1025375
L. $\mathfrak{N}$	− 7,4034855	− 7,6766873	+ 5,9352795	− 7,4864731
L. $\frac{2(m+1)}{\mu}$	9,8474122	9,8474122	9,8474122	9,8474122
L. P. II.	− 7,2508977	− 7,5240995	+ 5,7826917	− 7,3338853
L. M	− 0,3000186	. . . . . +	+ 0,4281870	. . . . . +
L. μ^2	3,1594272		3,1594272	
L. P. I.	− 7,1405914		+ 7,2687598	
P. I.	− 0,001382	+ 0,001382	+ 0,001857	+ 0,001857
− P. II.	+ 0,001782	+ 0,003343	− 0,000061	+ 0,002157
N =	+ 0,000400	+ 0,004725	+ 0,001796	+ 0,004014
at $\mathfrak{N}$ =	− 0,002532	− 0,004750	+ 0,000086	− 0,003065

En ergo ambas aequationes hinc oriundas:

$$\delta = -0{,}002532\,\beta - 0{,}004750\,\zeta$$
$$+\,0{,}000086\,b - 0{,}003065\,f$$
$$d = +\,0{,}000400\,\beta + 0{,}004725\,\zeta$$
$$+\,0{,}001796\,b + 0{,}004014\,f.$$

§. 163.

§. 163.

Binos tandem reliquos angulos $4p-q$ et $4p+q$ quia vterque binis tantum partibus constat, coniunctim expedire poterimus:

Ex priore colligemus $\mathfrak{N} = \varepsilon$; $N = e$.

Ex posteriore autem $\mathfrak{N} = \zeta$; $N = f$.

	pro $4p-q$ γ.	c.	pro $4p+q$ δ.	d.
L. M =	0,3000186 −	0,4281870 +	0,3000186 −	0,4281870 +
L. $\frac{2(m+1)}{\mu}$	9,8681819	9,8681819	9,6296383	9,6296383
	0,1682005 −	0,2963689 +	9,9296569 −	0,0578253 +
$\frac{2(m+1)M}{\mu}$	− 1,472992	+ 1,978649	− 0,850466	+ 1,142419
− $\mathfrak{M}$	+ 4,610645	− 1,995348	+ 4,610645	− 1,995348
Num.	+ 3,137653	− 0,016699	+ 3,760179	− 0,852929
Log.	0,4966049 +	8,2226905 −	0,5752085 +	9,9309130 −
L. Den.	3,0548550 −	8 0548550 −	3,5749613 −	3,5749613 −
L. $\mathfrak{N}$	7,4417499 −	5,1678355 +	7,0002472 −	6,3559517 +
	9,8681819	9,8681819	9,6296383	9,6296383
L. P. II.	7,3099318 −	5,0360174 +	6,6298855 −	5,9855900 +
L. M	0,3000186 −	0,4281870 +	0,3000186 −	0,4281870 +
L. μ^2	3,1178878	3,1178878	3,5949744	3,5949744
L. P. I.	7,1821308 −	7,3102992 +	6,7050442 −	6,8332126 +
P. I.	− 0,001521	+ 0,002043	− 0,000507	+ 0,000681
− P. II	+ 0,002041	− 0,000011	+ 0,000426	− 0,000097
N	+ 0,000520	+ 0,002032	− 0,000081	+ 0,000584
at $\mathfrak{N}$ =	− 0,002765	+ 0,000015	− 0,001000	+ 0,000217

hinc ergo quatuor ſequentes aequationes reſultant

$$\varepsilon = -0,002765.\gamma + 0,000015.c$$
$$e = +0,000520\,\gamma + 0,002032.c.$$
$$\zeta = -0,001000.\delta + 0,000217.d.$$
$$f = -0,000081.\delta + 0,000584.d.$$

§. 164.

Subſtituamus iam hos poſtremos valores in aequationibus praecedentibus, ac primo quidem pro aequationibus §. 161, cum ſit

$$+0,203916.\varepsilon = -0,000564.\gamma + 0,000003.c.$$
$$+0,181444.e = +0,000094.\gamma + 0,000368.c.$$
$$\overline{\ -0,000470.\gamma + 0,000371.c.}$$

$$-0,459727.\varepsilon = +0,001271.\gamma - 0,000007.c.$$
$$-0,402400.e = -0,000209.\gamma - 0,000818.c.$$
$$\overline{\ +0,001062\,\gamma - 0,000825.c.}$$

obtinebimus ſequentes valores

$$\gamma = -0,000792.\beta - 0,093538.b. \\ -0,000470.\gamma + 0,000371.c.$$

et

$$c = -0,013292.\beta + 0,197491.b. \\ +0,001062.\gamma - 0,000825.c.$$

Ex poſteriore ſtatim colligimus

$$c = -0,013281.\beta + 0,197329.b + 0,001062.\gamma$$

qui

qui valor in priore substitutus praebet

$$\gamma = -0,000797.\beta - 0,093504.b - 0,000470.\gamma$$

adeoque

$$\gamma = -0,000797.\beta - 0,093461.b.$$

hincque vicissim

$$c = -0,013281.\beta + 0,197230.b.$$

§. 165.

Eodem modo litteras δ et d per β et b definire poterimus; cum enim sit

$$\begin{array}{r} -0,00475.\zeta = +0,000004.\delta - 0,000001.d \\ -0,00306.f = +0,000000.\delta - 0,000001.d \\ \hline +0,000004.\delta - 0,000002.d \end{array}$$

$$\begin{array}{r} +0,004725\ \zeta = -0,000005.\delta + 0,000001.d \\ +0,004014.f = -0,000000.\delta + 0,000002.d \\ \hline -0,000005.\delta + 0,000003.d \end{array}$$

erunt nostri valores

$$\begin{aligned} \delta &= -0,002532.\beta + 0,000086.b \\ &\quad -0,000002.d + 0,000004.\delta \\ d &= +0,000400.\beta + 0,001796.b \\ &\quad -0,000005.\delta + 0,000003.d \end{aligned}$$

vnde manifesto sequitur

$$\begin{aligned} \delta &= -0,002532.\beta + 0,000092.b \\ d &= +0,000400.\beta + 0,001796.b. \end{aligned}$$

§. 166.

§. 166.

Adgrediamur nunc primum angulum q, pro quo ſupra vidimus, eſſe

$$\mathfrak{M} = -4,6106454(\gamma+\delta) - 1,9953482(c+d)$$

$$M = -1,9953482(\gamma-\delta) - 2,6803227(c-d)$$

quare cum ſit

$$\gamma+\delta = -0,003329.\beta - 0,093369.b$$

$$\gamma-\delta = +0,001735.\beta - 0,093553.b.$$

$$c+d = -0,012881.\beta + 0,199026.b$$

$$c-d = -0,013681.\beta + 0,195434.b.$$

his valoribus ſubſtitutis inueniemus

$$\mathfrak{M} = +0,041049\,\beta + 0,033365.b.$$

$$M = +0,033207\,\beta - 0,337151.b..$$

§. 167.

Quodſi iam hinc litteras $\mathfrak{N}$ et N deriuemus, ob $\mathfrak{N} = \beta$; et $N = b$, aſſequemur has duas aequationes

$$1^\circ.\ \beta = \frac{\frac{2(m+1)}{n}M - \mathfrak{M}}{\lambda - 2 - nn}$$

$$2^\circ.\ b = \frac{M}{nn} - \frac{2(m+1)\beta}{n}$$

quarum poſterior euoluta praebet

$$b = -2,016832.\beta - 0,001919.b.\ \text{ſeu}$$

$$1,001919.b = -2,016832.\beta.$$

vnde

vnde concluditur

$$b = -2,012968.\,\beta.$$

§. 167.

Quodsi porro hunc valorem loco b in litteris $\mathfrak{M}$ et M substituamus, euidens est, aequationem priorem diuisibilem fore per β, ita, vt littera β prorsus ex calculo excidat; ideoque aequatio identica resultare deberet, siquidem littera n rite fuit assumta, vti iam supra animaduertimus. Mirari autem non debemus, si hoc secus accidat; primo enim hic tantum valores prope veros pro coefficientibus assumtis elicuimus; deinde vero inprimis perpendendum est, in nonnullis sequentibus ordinibus hunc eundem angulum q quoque occurrere, qui simul cum isto coniungi deberent, vt verus valor litterae n inde sit proditurus; scilicet si pro hoc angulo q in sequentibus ordinibus reperiantur litterae $\mathfrak{M}'$ et M'; $\mathfrak{M}''$ et M''; tum demum esse debebit aequatio identica

$$\beta = \frac{\frac{2(m+1)}{n}(M + M' + M'') - \mathfrak{M} - \mathfrak{M}' - \mathfrak{M}''}{\lambda - 2 - n^2}$$

§. 168.

Verum tamen videamus, quantum haec aequatio nunc iam ab identica recedat. Hunc in finem habebimus

$$\mathfrak{M} = -0,026114.\,\beta;\quad M = +0,711879.\,\beta.$$

et quia aequatio identica esse deberet

$$(\lambda - 2 - n^2)\beta = \frac{2(m+1)}{n} M - \mathfrak{M}$$

facta euolutione numerica prodit

$$1,50640.\ \beta = 1,46199.\ \beta$$

vbi discrimen tantum non est, vt adhibitis correctionibus memoratis perfectus consensus sperari nequeat. Ceterum euidens est, si numero n leuissima mutatio induceretur, totum negotium compositum iri.

§. 169.

Cum quantitas β maneat indeterminata atque adeo talis per ipsam rei naturam esse debeat, commodissime statuemus $\beta = 1$; quia enim terminus cos. q iam per K multiplicatus in valorem ipsius x ingreditur, hoc modo ipsa littera K more astronomorum excentricitatem exprimet; quae alioquin foret $= K\beta$. Sumto ergo $\beta = 1$; valores coefficientium ita se habebunt, vt sequens tabula exhibet

$\beta = +1,000000.$	$b = -2,012968.$
$\gamma = +0,187336.$	$c = -0,410299.$
$\delta = -0,002717.$	$d = -0,003215.$
$\epsilon = -0,000524.$	$e = -0,000739.$
$\zeta = -0,000001.$	$f = -0,000003.$

§. 170.

§. 170.

Hactenus ergo valores eruimus prope veros pro litteris $\mathfrak{P}$ et P; qui igitur tam parum a veritate aberrabunt, vt fere sine sensibili errore adhiberi possent; interim tamen in correctiones, quae ex partibus minoribus litterarum $\mathfrak{A}$, A; $\mathfrak{B}$, B supra neglectis originem trahunt, inquirere operae erit pretium; atque dum harum partium exiguarum rationem habebimus, tutissime his valoribus prope veris vti poterimus, qui ita se habent

$$\mathfrak{P} = \operatorname{cos}. q + 0,187336.\operatorname{cos}.(2p-q) - 0,000524.\operatorname{cos}.(4p-q)$$
$$- 0,002717 \operatorname{cos}.(2p+q) - 0,000000.\operatorname{cos}.(4p+q)$$

$$P = -2,012960.\operatorname{fin}. q - 0,410299.\operatorname{fin}.(2p-q)$$
$$- 0,003215.\operatorname{fin}.(2p+q)$$
$$- 0,000739.\operatorname{fin}.(4p-q)$$
$$- 0,000003.\operatorname{fin}.(4p+q).$$

§. 171.

Pro priore ergo aequatione hos valores $\mathfrak{P}$ et P insuper in particulas $+0,0264388 - 0,1050568.\operatorname{cos}.4p$ et $-0,0819104.\operatorname{fin}.4p$ duci oportet, vnde supplementa orientur litteris $\mathfrak{M}$ adiicienda, quae signo $\mathfrak{M}'$ designemus.

Calculus ergo ita se habebit

	cos q.	cos. $(2p-q)$	cos. $(2p+q)$	cos. $(4p-q)$	cos $(4p+q)$
$+0,02644.\mathfrak{P}$	+0,026438	+0,004953	−0,000072	−0,000014	−0,000000
$-0,105.\cos.4p.\mathfrak{P}$	+0,000027	+0,000142	−0,009840	−0,052528	−0,052528
	+0,026465	+0,005095	−0,009912	−0,052542	−0,052528
$-0,082.\sin.4p.P$	+0,000030	+0,000131	+0,016765	+0,082441	−0,082441
$\mathfrak{M}' =$	+0,026495	+0,005226	+0,006853	+0,029899	−0,134969

§. 172.

Simili modo pro altera aequatione istos valores $\mathfrak{P}$ et P insuper in particulas litterarum A et B duci oportet, quae sunt, $-0,0819104.\sin.4p$, et $-0,0272367 + 0,0665457.\cos.4p$.

	sin. q.	sin. $(2p-q)$	sin. $(2p+q)$	sin. $(4p-q)$	sin. $(4p+q)$
$-0,081.\sin.4p.\mathfrak{P}$	+0,000021	+0,000111	−0,007672	−0,040955	−0,040955
$-0,027.P$	+0,054824	+0,011149	+0,000088	+0,000020	
	+0,054845	+0,011260	−0,007584	−0,040935	−0,040955
$+0,066.\cos.4p.P$	+0,000024	+0,000107	+0,013620	+0,066977	−0,066977
$M' =$	+0,054869	+0,011367	+0,006036	+0,026042	−0,107932

§. 173.

Haec ergo supplementa $\mathfrak{M}'$ et M' certa generabunt supplementa pro numeris $\mathfrak{N}$ et N, quae itidem litteris $\mathfrak{N}'$ et N' indicabimus. Ea autem ope nostrae regulae generalis ex illis deriuabimus; vbi quidem ob rationes allegatas primam columnam anguli q singulari iudicio reseruabimus.

En

En ergo calculum pro quatuor reliquis columnis.

	$2p-q$	$2p+q$	$4p-q$	$4p+q$
ad Log. $M' =$	+ 8,0556459	+ 7,7807492	+ 8,4156743	− 9,0331502
adde L. $\frac{2(m+1)}{\mu} =$	0,3671158	9,8474122	9,8681819	9,6296386
Log. $\frac{2(m+1)M'}{\mu} =$	+ 8,4227617	+ 7,6281614	+ 8,2838562	− 8,6627888
$\frac{2(m+1)M'}{\mu}$	+ 0,026470	+ 0,004248	+ 0,019224	− 0,046003
− $\mathfrak{M}'$	− 0,005226	− 0,006853	− 0,029899	+ 0,134969
Numer.	+ 0,021244	− 0,002605	− 0,010675	+ 0,088966
Log Numer.	+ 8,3272363	− 7,4158077	− 8,0283679	+ 8,9492241
subtr. Log. Den.	+ 1,6570290	− 3,1025375	− 3,0548550	− 3,5749613
Log. $\mathfrak{N}' =$	+ 6,6702073	+ 4,3132702	+ 4,9735129	− 5,3742628
adde L. $\frac{2(m+1)}{\mu} =$	0,3671158	9,8474122	9,8681819	9,6296386
L. P. II.	+ 7,0373231	+ 4,1606824	+ 4,8416948	− 5,0039014
Log. $M' =$	+ 8,0556459	+ 7,7807492	+ 8,4156743	− 9,0331502
subtr. L. $\mu^2 =$	2,1200200	3,1594272	3,1178878	3,5949744
L. P. I. $=$	+ 5,9356259	+ 4,6213220	+ 5,2977865	− 5,4381758
P. I. $=$	+ 0,000086	+ 0,000004	+ 0,000020	− 0,000027
− P. II.	− 0,001090	− 0,000001	− 0,000007	+ 0,000010
$N' =$	− 0,001004	+ 0,000003	+ 0,000013	− 0,000017
at $\mathfrak{N}' =$	+ 0,000467	+ 0,000002	+ 0,000009	− 0,000024

§. 174.

His supplementis inuentis perspicuum est, litteras nostras graecas supra definitas γ, δ, ε, ζ reuera per aggregatum $\mathfrak{N}+\mathfrak{N}'$; latinas vero litteras c, d, e, f per aggregatum $N+N'$ determinari, vnde octo illae praecedentes determinationes sequenti modo se habebunt:

1°. $\gamma = - 0,000792.\beta + 0,203916.\varepsilon - 0,093538.b + 0,181444.e + 0,000467.$

2°. $c = - 0,013292.\beta - 0,459727.\varepsilon + 0,197491.b - 0,402200.e - 0,001004.$

3°. $\delta = - 0,002532.\beta - 0,004750.\zeta + 0,000086.b - 0,003065.f + 0,000002.$

4°. $d = + 0,000400.\beta + 0,004725.\zeta + 0,001796.b + 0,004014.f + 0,000003.$

5°. $\varepsilon = - 0,002765.\gamma + 0,000015.c + 0,000009.$

6°. $e = + 0,000520.\gamma + 0,002032.c + 0,000013.$

7°. $\zeta = - 0,001000\,\delta + 0,000217.d - 0,000024.$

8°. $f = - 0,000081.\delta + 0,000584.d - 0,000017.$

§. 175.

§. 175.

Nunc igitur rursus easdem reductiones instituamus, quas supra fecimus, vbi quidem statim euidens est, omnes coefficientes litterarum eosdem manere, quos ante inuenimus; tantum insuper accedent particulae quaedam absolutae, quas igitur solas calculo definiuisse sufficiet.

Primum ergo substituamus quaternos vltimos valores in praecedentibus ac nunc reperiemus:

$$\gamma = -0,000792\,\beta - 0,093538.\,b$$
$$-0,000470.\,\gamma + 0,000372.\,c$$
$$+0,000467.$$
$$c = -0,013292.\,\beta + 0,197491.\,b$$
$$+0,001062.\,\gamma - 0,000824.\,c$$
$$-0,001013.$$
$$\delta = -0,002532.\,\beta + 0,000086.\,b$$
$$+0,000005.\,\delta - 0,000003.\,d$$
$$+0,000002.$$
$$d = +0,000400.\,\beta + 0,001796.\,b$$
$$-0,000005.\,\delta + 0,000004.\,d$$
$$+0,000003.$$

atque hinc facile concluditur, fore

$$\gamma = -0,000797.\,\beta - 0,093422.\,b + 0,000467.$$
$$c = -0,013281.\,\beta + 0,197230.\,b - 0,001013.$$
$$\delta = -0,002532.\,\beta + 0,000086.\,b + 0,000002.$$
$$d = +0,000400.\,\beta + 0,001796.\,b + 0,000003.$$

§. 176.

§. 176.

Substituantur iam hi valores in litteris $\mathfrak{M}$ et M angulo q conuenientibus, quo facto ex §. 159. adipiscimur

$$\mathfrak{M} = + 0,043046.\beta + 0,033213.b - 0,000170.$$

$$M = + 0,035888.\beta - 0,337245.b + 0,001790.$$

quibus addantur supplementa supra inuenta, vt obtineamus valores completos

$$\mathfrak{M} + \mathfrak{M}' = + 0,043046.\beta + 0,033213.b + 0,026325.$$

$$M + M' = + 0,035888.\beta - 0,337245.b + 0,056659.$$

§. 177.

Ex his valoribus pro angulo q quaeri deberent valores itidem completi $\mathfrak{N} + \mathfrak{N}'$ et $N + N''$. Quia autem per hypothesin esse debet

$$\mathfrak{N} + \mathfrak{N}' = \beta \text{ et } N + N' = b$$

posterior formula nobis dat

$$b = \frac{M + M'}{2n} - \frac{2(m+1)}{n}\beta$$

vnde ex euolutione colligitur

$$b = - 2,016824.\beta - 0,001919.b + 0,000322.$$

siue

siue ob $\beta = 1$.

$$1,001919 . b = -2,016502.$$

hincque $b = -2,012639$.

§. 178.

Videamus nunc etiam, quemadmodum se sit habitura prior aequatio

$$(\lambda - 1 - nn)\beta = \frac{2(m+1)}{n}(M + M') - \mathfrak{M} - \mathfrak{M}'$$

quae deberet esse identica; at ob $\beta = 1$, erit

$$M + M' = 0,771199 \text{ et}$$

$$\mathfrak{M} + \mathfrak{M}' = 0,002525$$

hincque ista aequatio euadet

$$1,50640 = 1,55301$$

vbi iam aberratio in alteram partem cadit; quae cum sit perexigua, per terminos formae cos. q sequentium ordinum eo facilius tolli posse intelligitur.

§. 179.

Inuento ergo vero valore litterae b sequentes coefficientes ita determinati reperiuntur, vt sit

$\beta = +1,000000$	$b = -2,012639$
$\gamma = +0,187695$	$c = -0,411247$
$\delta = -0,002703$	$d = -0,003212$
$\varepsilon = -0,000514$	$e = -0,000724$
$\zeta = -0,000021.$	$f = -0,000019.$

§. 180.

En ergo valores, hoc et superiori capite erutos

$$\mathfrak{Q} = +\,0,0000240 - 0,0071801.\ \text{cof.}\ 2p + 0,0000060.\ \text{cof.}\ 4p.$$

$$O = +\,0,0102117.\ \text{fin.}\ 2p + 0,0000057.\ \text{fin.}\ 4p.$$

$$\begin{aligned}\mathfrak{P} = +\,\text{cof.}\ q &+ 0,187695.\ \text{cof.}\ (2p-q)\\ &- 0,002703.\ \text{cof.}\ (2p+q)\\ &- 0,000514.\ \text{cof.}\ (4p-q)\\ &- 0,000021.\ \text{cof.}\ (4p+q)\end{aligned}$$

$$\begin{aligned}P = -\,2,012639.\ \text{fin.}\ q &- 0,411247.\ \text{fin.}\ (2p-q)\\ &- 0,003212.\ \text{fin.}\ (2p+q)\\ &- 0,000724.\ \text{fin.}\ (4p-q)\\ &- 0,000019.\ \text{fin.}\ (4p+q)\end{aligned}$$

CAPUT III.

EVOLUTIO AEQUATIONUM ORDINIS III. PRO LITTERIS $\mathfrak{Q}$ ET Q.

§. 181.

Aequationum differentialium, ex quibus has litteras $\mathfrak{Q}$ et Q erui oportet, partes annexae, quas hic solas considerare sufficit, ita se habebant

$$\text{I. } 0 = \ldots\ldots + \mathfrak{Q}\mathfrak{A} + Q\mathfrak{B} + \mathfrak{P}^2 . \mathfrak{C} + \mathfrak{P} . P . \mathfrak{D} + P^2 . \mathfrak{E}.$$

$$\text{II. } 0 = \ldots\ldots + \mathfrak{Q}A + QB + \mathfrak{P}^2 . C + \mathfrak{P} . P . D + P^2 . E.$$

vbi duplicis generis membra occurrunt, cognita scilicet et incognita; cognita enim sunt membra posteriora formulis $\mathfrak{P}^2$; $\mathfrak{P}P$ et P^2 contenta; at membra priora litteras $\mathfrak{Q}$ et Q innoluentia prorsus sunt incognita. Ceterum litterae vtriusque alphabeti initiales peti debent ex §. 153.

§. 182.

Ante omnia igitur ex valoribus litterarum $\mathfrak{P}$ et P modo inuentis colligere debemus valores formularum $\mathfrak{P}^2$; $\mathfrak{P}$. P et P^2; qui sequenti modo expressi deprehenduntur:

$$\begin{aligned}\mathfrak{P}^2 = &+0,51762 + 0,18490.\cos.2p - 0,00104.\cos.4p\\ &+0,49949.\cos.2q + 0,18770.\cos.(2p-2q)\\ &-0,00271.\cos.(2p+2q) + 0,01710.\cos.(4p-2q)\\ &-0,00002.\cos.(4p+2q)\end{aligned}$$

$$\begin{aligned}\mathfrak{P}.P = &+0,39900.\sin.2p + 0,00038.\sin.4p\\ &-1,00713.\sin.2q - 0,01674.\sin.(2p-2q)\\ &+0,00112.\sin.(2p+2q) - 0,03948.\sin.(4p-2q)\\ &+0,00002.\sin.(4p+2q)\end{aligned}$$

$$\begin{aligned}P^2 = &+2,10992 - 0,82093.\operatorname{cosin}.2p\\ &-0,00274.\operatorname{cosin}.4p - 2,02404.\cos.2q\\ &+0,82769.\cos.(2p-2q) - 0,00646.\cos.(2p+2q)\\ &-0,08310.\cos.(4p-2q) - 0,00004.\cos.(4p+2q)\end{aligned}$$

quas fractiones tantum ad quinque figuras produximus, quia litterae quaesitae $\mathfrak{Q}$ et Q per quadratum KK multiplicantur.

§ 183.

§. 183.

Hae expressiones sponte in duas partes dirimuntur, quarum priores tantum angulum p; posteriores vero insuper anomaliam duplicatam inuoluunt. Euidens autem est, ipsas quantitates quaesitas $\mathfrak{Q}$ et Q simili modo per duas species expressum iri; quae ita essentialiter a se inuicem discrepant, vt vtramque seorsim expedire liceat; ex quo inuestigationem nostram bipartitam exhibere conueniet.

I. Euolutio partium ab anomalia q immunium.

§. 184.

Hic igitur formularum $\mathfrak{P}^2$; $\mathfrak{P}$ P; P^2 partes tantum priores considerabimus, et quia litterae $\mathfrak{Q}$ et Q similes partes continere debent, statuamus

$$\mathfrak{Q} = \alpha + \beta . \operatorname{cos}. 2p + \gamma . \operatorname{cos}. 4p.$$

$$Q = b . \operatorname{sin}. 2p + c . \operatorname{sin}. 4p.$$

quo posito ex nostris aequationibus formulas $\mathfrak{M}$ et M euoluamus; quarum quidem partes ex membris cognitis oriundas ipsis litteris $\mathfrak{M}$ et M; quae vero ex incognitis nascuntur, signis $\mathfrak{M}'$ et M′ indicemus, vt completi valores censendi sint $\mathfrak{M} + \mathfrak{M}'$ et M + M′.

§. 185.

Ordiamur ergo a membris cognitis ac pro priore aequatione singuli termini litterarum $\mathfrak{C}$, $\mathfrak{D}$, etc. seorsim in computum ducantur:

	constans	cos. $2p$	cos. $4p$.
$537,6335924.\mathfrak{P}^2$	$+278,28983$	$+\ 99,40305$	$-0,55914$
$+15,4425816.\cos.2p\mathfrak{P}^2$	$+\ \ 1,42767$	$+\ \ 7,99338$	$+1,42767$
	$-\ \ 0,00014$	$-\ \ 0,00802$	$+0,13740$
$+0,2658681.\cos.4p\mathfrak{P}^2$		$+\ \ 0,02458$	
$\mathfrak{P}^2.\mathfrak{C} =$	$+279,71736$	$+107,41299$	$+1,00593$
$+21,96278.\sin.2p\mathfrak{P}P$	$-\ \ 4,38158$	$+\ \ 0,00417$	$+4,38158$
$+0,40648.\sin.4p.\mathfrak{P}P$	$+\ \ 0,00008$	$-\ \ 0,08109$	
$\mathfrak{P}.P.\mathfrak{D} =$	$-\ \ 4,38150$	$-\ \ 0,07692$	$+4,38158$
$-268,7817.P^2$	$-567,10913$	$+220,65091$	$+0,73646$
$-7,72129.\cos.2p.P^2$	$+\ \ 3,16929$	$-\ 16,29130$	$+3,16929$
$-0,16797.\cos.4p.P^2$	$+\ \ 0,00023$	$+\ \ 0,01058$	$-0,35442$
		$+\ \ 0,06895$	
$P^2.\mathfrak{E} =$	$-563,93961$	$+204,43914$	$+3,55133$
$\mathfrak{M} =$	$-288,60375$	$+311,77521$	$+8,93884$

§. 186.

§. 186.

Simili modo ex altera nostra aequatione litteram M colligamus sequenti calculo:

	sin. $2p$.	sin. $4p$
$+10,9813978.\sin.2p.\mathfrak{P}^2$	$+\ 5,68989$	$+1,01523$
$+0,2032436.\sin.4p.\mathfrak{P}^2$	$+\ 0,01879$	$+0,10520$
$\mathfrak{P}^2.C =$	$+\ 5,69868$	$+1,12043$
$-537,5635056.\mathfrak{P}.P$	$+214,48780$	$-0,20427$
$-15,442580.\cos.2p.\mathfrak{P}P$	$-\ 0,00293$	$+3,08076$
$-0,3359548.\cos.4p\ \mathfrak{P}P$	$-\ 0,06702$	
$\mathfrak{P}.P.D =$	$+214,41785$	$+2,87649$
$-8,2360446.\sin.2p.P^2$	$-\ 17,37740$	$+3,38061$
	$-\ 0,01128$	
$-0,1524327.\sin.4p.P^2$	$+\ 0,06257$	$-0,32162$
$P^2.E =$	$-\ 17,32611$	$+3,05899$
$M =$	$+202,80042$	$+7,05591$

§. 187.

His iam valoribus $\mathfrak{M}$ et M respondeant vi regulae nostrae litterae $\mathfrak{N}$ et N, quibus autem deinceps adiungi debebunt litterae $\mathfrak{N}'$ et N′, quae scilicet ex $\mathfrak{M}'$ et M′ oriuntur; vt istae litterae completae obtineantur; ac primo quidem quoniam $\mathfrak{M}$ etiam partem absolutam inuoluit; inde statim adipiscimur

$$\mathfrak{N} = \frac{-288,60368}{2\lambda} = -0,53675.$$

pro binis autem angulis $2p$ et $4p$ calculus hoc modo abſoluetur:

	$2p.$	$4p.$
ad log. M =	+2,3070684	+0,8485530
adde L. $\frac{2(m+1)}{\mu}$	+0,0337640	+9,7327340
	+2,3408324	+0,5812870
$\frac{2(m+1)M}{\mu}$	+219,19599	+3,81318
$-\mathfrak{M}$	−311,77521	−8,93884
Numerat.	−92,57922	−5,12566
Log. Numerat.	−1,9665134	−0,7097498
Log. Denom.	−2,6382215	−3,3561432
L. $\mathfrak{N}$ =	+9,3282919	+7,3536066
adde L. $\frac{2(m+1)}{\mu}$	+0,0337640	+9,7327340
L. Pars II.	+9,3620559	+7,0863406
L. M =	+2,3070473	+0,8485530
L. μ^2 =	+2,7867236	+3,3887836
L. P. I. =	+9,5203237	+7,4597694
P. I. =	+0,33138	+0,00288
− P. II. =	−0,23017	−0,00122
N =	+0,10121	+0,00166
at $\mathfrak{N}$ =	+0,21299	+0,00226

§. 188.

Nunc igitur, postquam posuimus

$\Omega = \alpha + \beta.\cos.\,2p + \gamma.\cos.\,4p.$

$Q = b.\sin.\,2p + c.\sin.\,4p.$

primum ex aequatione priore colligamus valorem litterae $\mathfrak{M}'$.

	const.	cos. 2 p.	cos. 4 p.
$-9{,}22129.\cos.2p.\Omega$	$-4{,}61064.\beta$	$-9{,}22129.\alpha$	$-4{,}61064.\beta$
		$-4{,}61064.\gamma$	
$+0{,}02664.\Omega$	$+0{,}02664.\alpha$	$+0{,}02664.\beta$	$+0{,}02664.\gamma$
$-0{,}10506.\cos.4p.\Omega$	$-0{,}05253\ \gamma$	$-0{,}05253.\beta$	$-0{,}10506.\alpha$
$-3{,}99069.\sin.2p.Q$	$-1{,}99534.b$	$-1{,}99534\ c$	$+1{,}99534.b$
$-0{,}08191.\sin.4p.Q$	$-0{,}04095.c$	$-0{,}04095.b$	

Ex *altera* aequatione colligamus valores litterae M'.

	sin. 2 p	sin. 4 p.
$-3{,}99069.\sin.2p\Omega$	$-3{,}99069.\alpha$	$-1{,}99534.\beta$
	$+1{,}99534.\gamma$	
$-0{,}08191.\sin.4p\Omega$	$-0{,}04095.\beta$	$-0{,}08191.\alpha$
$+5{,}36064.\cos.2p.Q$	$+2{,}68032.c$	$+2{,}68032.b$
$-0{,}02724.Q$	$-0{,}02724.b$	$-0{,}02724.c$
$+0{,}06654.\cos.4p.Q$	$-0{,}03327.b$	

§. 189.

Partem absolutam pro $\mathfrak{M}'$ tantisper seponamus, donec ipsos angulos $2p$ et $4p$ expediuerimus; adeoque calculum nostrum ab angulo $2p$ inchoemus, pro quo cum sit

$\mathfrak{M}' = -9{,}22129.\alpha - 0{,}02589.\beta - 4{,}61064.\gamma - 0{,}04095.b - 1{,}99534.c.$

$M' = -3{,}99069.\alpha - 0{,}04095.\beta + 1{,}99534.\gamma - 0{,}06051.b + 2{,}68032.c.$

hinc litteras $\mathfrak{N}'$ et N' eruamus, per quinas colum-

	$\alpha.$	$\beta.$
ad Log. $M' =$	$-0,6010479$	$-8,6122539$
adde L. $\frac{2(m+1)}{\mu} =$	$+0,0337640$	$+0,0337640$
Log. $\frac{2(m+1)M'}{\mu} =$	$-0,6348119$	$-8,6460179$
$\frac{2(m+1)M'}{\mu}$	$-4,31332$	$-0,04426$
$-\mathfrak{M}'$	$+9,22129$	$+0,02589$
Numer.	$+4,90797$	$-0,01837$
Log. Numer.	$+0,6909019$	$-8,2641092$
subtr. Log. Den.	$-2,6382215$	$-2,6382215$
Log. $\mathfrak{N}' =$	$-8,0526804$	$+5,6258877$
adde L. $\frac{2(m+1)}{\mu} =$	$+0,0337640$	$+0,0337640$
L. P. II.	$-8,0864444$	$+5,6596517$
Log. $M' =$	$-0,6010479$	$-8,6122539$
subtr. L. $\mu^2 =$	$+2,7867236$	$+2,7867236$
L. P. I. $=$	$-7,8143243$	$-5,8255303$
P. I. $=$	$-0,00652$	$-0,00007$
$-$ P. II.	$+0,01220$	$-0,00004$
$N' =$	$+0,00568$	$-0,00011$
at $\mathfrak{N}' =$	$-0,01129$	$+0,00004$

quoniam igitur fieri oportet $\mathfrak{N} + \mathfrak{N}' = \beta$; et

$$\beta = +0,21299 - 0,01129.\alpha + 0,00004.\beta$$

$$b = +0,10117 + 0,00568.\alpha - 0,00011.\beta$$

nas litteris α, β, γ, b, c. insigniendas:

γ.	b.	c.
+0,3000169	−8,7818271	+0,4281866
+0,0337640	+0,0337640	+0,0337640
+0,3337809	−8,8155911	+0,4619506
+2,15665	−0,06540	+2,89701
+4,61064	+0,04095	+1,99534
+6,76729	−0,02445	+4,89235
+0,8304148	−8,3882789	+0,6895175
−2,6382215	−2,6382215	−2,6382215
−8,1921933	+5,7500574	−8,0512960
+0,0337640	+0,0337640	+0,0337640
−8,2259573	+5,7838214	−8,0850600
+0,3000169	−8,7818271	+0,4281866
+2,7867236	+2,7867236	+2,7867236
+7,5132933	−5,9951035	+7,6414630
+0,00326	−0,00010	+0,00438
+0,01682	−0,00006	+0,01216
+0,02008	−0,00016	+0,01654
−0,01556	+0,00006	−0,01125

$N + N' = b$; habebimus hinc has duas aequationes

$$-0,01556.\gamma + 0,00006.b - 0,01125.c.$$

$$+0,02008.\gamma - 0,00016.b + 0,01654.c.$$

Eodem modo pro angulo $4p$ calculum in-

$$\mathfrak{M}' = -0,10506 . \alpha - 4,61064 . \beta$$
$$M' = -0,08191 . \alpha - 1,99534 . \beta$$

habebimus pro angulo $4p$

	α.	β.
ad log. $M' =$	$-8,9133369$	$-0,3000169$
adde L. $\frac{2(m+1)}{\mu}$	$+9,7327340$	$+9,7327340$
	$-8,6460709$	$-0,0327509$
$\frac{2(m+1)M}{\mu}$	$-0,04426$	$-1,07833$
$-\mathfrak{M}'$	$+0,10506$	$+4,61064$
Numerat.	$+0,06080$	$+3,53231$
Log. Numerat.	$+8,7839036$	$+0,5480588$
Log. Denom.	$-3,3561432$	$-3,3561432$
L. $\mathfrak{N}' =$	$-5,4277604$	$-7,1919156$
adde L. $\frac{2(m+1)}{\mu}$	$+9,7327340$	$+9,7327340$
L. Pars II.	$-5,1604944$	$-6,9246496$
L. $M' =$	$-8,9133369$	$-0,3000169$
subtr. L. $\mu^2 =$	$+3,3887836$	$+3,3887836$
L. P. I. $=$	$-5,5245533$	$-6,9112333$
P. I. $=$	$-0,00003$	$-0,00082$
$-$ P. II. $=$	$+0,00001$	$+0,00084$
$N' =$	$-0,00002$	$+0,00002$
at $\mathfrak{N}' =$	$-0,00003$	$-0,00155$

190.
ſtituamus; et cum ſit:
$+0,02664.\gamma + 1,99534.b.$
$+2,68032.b - 0,02724.c.$
calculum ſequentem:

γ.	b.	c.
	+0,4281866	−8,4352071
	+9,7327340	+9,7327340
	+0,1609206	−8,1679411
	+1,44850	−0,01472
−0,02664	−1,99534	
−0,02664	−0,54684	−0,01472
−8,4255342	−9,7378603	−8,1679078
−3,3561432	−3,3561432	−3,3561432
+5,0693910	+6,3817171	+4,8117646
+9,7327340	+9,7327340	+9,7327340
+4,8021250	+6,1144511	+4,5444986
	+0,4281866	−8,4352071
	+3,3887836	+3,3887836
	+7,0394030	−5,0464235
	+0,00109	−0,00001
−0,00000	−0,00013	−0,00000
−0,00000	+0,00096	−0,00001
+0,00001	+0,00024	+0,00000

vnde hae aequationes naſcuntur:

$$\gamma = +0,00226 - 0,00003.\alpha - 0,00155.\beta$$
$$+0,00001.\gamma + 0,00024.b$$
$$c = +0,00166 - 0,00002.\alpha + 0,00002.\beta$$
$$+0,00096.b - 0,00001.c.$$

§. 191.

Ex his poſtremis aequationibus manifeſto fluunt ſequentes valores:

$$\gamma = +0,00226 - 0,00003.\alpha - 0,00155.\beta$$
$$+0,00024.b.$$
$$c = +0,00166 - 0,00002.\alpha + 0,00002.\beta$$
$$+0,00096.b.$$

qui valores in ſuperioribus aequationibus ſubſtituti producunt:

$$\beta = +0,21294 - 0,01129.\alpha + 0,00006.\beta$$
$$+0,00005.b.$$
$$b = +0,10110 + 0,00568.\alpha - 0,00014.\beta$$
$$-0,00015.b.$$

hincque

$$\beta = +0,21295 - 0,01129.\alpha$$
$$b = +0,10106 + 0,00568.\alpha.$$

atque

atque hi valores retro substituti praebent

$$\gamma = + 0,00195 - 0,00001 . \alpha .$$

$$c = + 0,00175 - 0,00002 . \alpha .$$

§. 192.

Nunc istos valores substituamus in parte absoluta litterae $\mathfrak{M}'$, quae erat

$$\mathfrak{M}' = + 0,02664 \alpha - 4,61064 . \beta$$
$$- 0,05253 \gamma - 1,99534 . b$$
$$- 0,04095 . c .$$

atque obtinebimus

$$\mathfrak{M}' = + 0,06736 . \alpha - 1,18365 . \text{ at}$$
$$\mathfrak{M} = - 288,60368 .$$

vnde fit

$$\mathfrak{M} + \mathfrak{M}' = - 289,78733 + 0,06736 . \alpha$$

quod quia per 3λ diuisum producere debet α, habebitur nunc

$$3 \lambda \alpha = 537,68674 . \alpha = - 289,78733$$
$$+ 0,06736 . \alpha$$

adeoque

$$537,67938 . \alpha = - 289,78733$$

conse-

consequenter

$$\alpha = -0,53896.$$

atque hinc

$$\beta = +0,21903.$$
$$\gamma = +0,00195.$$
$$b = +0,09800.$$
$$c = +0,00175.$$

§. 193.

En ergo valores principales, qui partes priores litterarum $\mathfrak{Q}$ et Q constituunt:

$$\mathfrak{Q} = -0,53896 + 0,21903 . \text{cos}. 2p$$
$$+ 0,00195 . \text{cos}. 4p.$$

$$Q = +0,09800 . \text{sin}. 2p + 0,00175 . \text{sin}. 4p.$$

II. Euolutio partium anomaliam duplicatam 2 *q* continentium.

§. 194.

Remotis igitur partibus iam inuentis pro reliquis vti conueniet istis formulis:

$$\mathfrak{P}^2 = \ldots . + 0,49949 . \text{cos}. 2q$$
$$+ 0,18770 . \text{cos}. (2p - 2q)$$
$$- 0,00271 . \text{cos}. (2p + 2q)$$
$$+ 0,01710 . \text{cos}. (4p - 2q)$$
$$- 0,00002 . \text{cos}. (4p + 2q).$$

$\mathfrak{P} . P =$

$$\mathfrak{P}.\ P = \ldots\ldots - 1{,}00713.\ \sin.\ 2q$$
$$- 0{,}01674.\ \sin.\ (2p - 2q)$$
$$+ 0{,}00112.\ \sin.\ (2p + 2q)$$
$$- 0{,}03948.\ \sin.\ (4p - 2q)$$
$$+ 0{,}00002.\ \sin.\ (4p + 2q).$$

$$P^2 = \ldots\ldots - 2{,}02404.\ \cos.\ 2q$$
$$+ 0{,}82769.\ \cos.\ (2p - 2q)$$
$$- 0{,}00646.\ \cos.\ (2p + 2q)$$
$$- 0{,}08310.\ \cos.\ (4p - 2q)$$
$$- 0{,}00004.\ \cos.\ (4p + 2q).$$

Hos igitur valores in partibus cognitis nostrarum aequationum substituamus, et formulas inde oriundas litteris $\mathfrak{M}$ et M designemus.

§.

Pro priore ergo aequatione haec

	$\cos. 2q$	$\cos.(2p-2q)$
$537,63359.\mathfrak{P}^2 \ldots$	$+268,54257$	$+100,91384$
$+15,44258.\cos. 2p.\mathfrak{P}^2$	$+1,44928$	$+3,85670$
	$-0,02092$	$+0,13203$
	$+269,97093$	$+104,90252$
$+0,26586.\cos. 4p.\mathfrak{P}^2$	$+0,00227$	$-0,00036$
	$+269,97320$	$+104,90216$
$+21,96278.\sin. 2p\,\mathfrak{P}.P$	$-0,18382$	$-11,05969$
	$+0,01230$	$-0,43355$
	$+269,80168$	$+93,40892$
$+0,40649.\sin. 4p.\mathfrak{P}P$	$-0,00802$	$+0,00023$
	$+269,79366$	$+93,40915$
$-268,78175.P^2 \ldots$	$+544,02475$	$-222,46787$
	$+813,81841$	$-129,05872$
$-7,72129.\cos 2p.P^2$	$-3,19542$	$+7,81409$
	$+0,02494$	$+0,32082$
	$+810,64793$	$-120,92381$
$-0,16798.\cos. 4p.P^2$	$+0,00698$	$+0,00054$
ergo $\mathfrak{M} =$	$+810,65491$	$-120,92327$

195.

euolutio ita se habebit:

$\cos(2p+2q)$	$\cos(4p-2q)$	$\cos(4p+2q)$
−1,45699	+9,19353	−0,01075
+3,85670	+1,44928	−0,02092
−0,00015		
+2,39953	+10,64281	−0,03167
+0,02495	+0,06640	+0,06640
+2,42448	+10,70921	+0,03473
+11,05969	+0,18382	−0,01230
+0,00022		
+13,48439	+10,89303	+0,02243
−0,00340	−0,20469	+0,20469
+13,48099	+10,68834	+0,22712
+1,73633	+22,33575	+0,01075
+15,21732	+33,02409	+0,23787
+7,81409	−3,19542	+0,02494
+0,00015		
+23,03156	+29,82867	+0,26281
−0,06952	+0,17000	+0,17000
+22,96204	+29,99867	+0,43281

§.

Simili modo ex altera aequatione

	sin. $2q$.	sin. $(2p-2q)$
$+10,98139 . \sin. 2p . \mathfrak{P}^2$	$+\ 1,03060$	$+\ 2,74255$
	$+\ 0,01488$	$-\ 0,09389$
	$+\ 1,04548$	$+\ 2,64866$
$+0,20324 . \sin. 4p . \mathfrak{P}^2$	$+\ 0,00174$	$-\ 0,00028$
	$+\ 1,04722$	$+\ 2,64838$
$-537,56350 . \mathfrak{P} P$	$+541,39625$	$+\ 8,99881$
	$+542,44347$	$+11,64719$
$-15,44258 . \cos. 2p \mathfrak{P} P$	$-\ 0,12926$	$-\ 7,77634$
	$-\ 0,00865$	$+\ 0,30483$
	$+542,30556$	$+\ 4,17568$
$-0,33595 . \cos. 4p \mathfrak{P} P$	$-\ 0,00663$	$+\ 0,00019$
	$+542,29893$	$+\ 4,17587$
$-8,23604 . \sin. 2p P^2$	$-\ 3,40844$	$+\ 8,33504$
	$-\ 0,02660$	$-\ 0,34221$
	$+538,86389$	$+12,16870$
$-0,15243 . \sin. 4p P^2$	$+\ 0,00633$	$+\ 0,00049$
Ergo $M =$	$+538,87022$	$+12,16919$

196.

valores litterae M eliciamus:

sin. $(2p+2q)$	sin. $(4p-2q)$	sin. $(4p+2q)$
+ 2,74255	+ 1,03060	− 0,01488
+ 0,00011		
+ 2,74266	+ 1,03060	− 0,01488
+ 0,01907	+ 0,05076	+ 0,05076
+ 2,76173	+ 1,08136	+ 0,03588
− 0,60207	+ 21,22300	− 0,01075
+ 2,15966	+ 22,30436	+ 0,02513
+ 7,77634	+ 0,12926	− 0,00865
− 0,00015		
+ 9,93585	+ 22,43362	+ 0,01648
− 0,00281	− 0,16916	+ 0,16916
+ 9,93304	+ 22,26446	+ 0,18564
+ 8,33504	− 3,40844	+ 0,02660
− 0,00016		
+ 18,26792	+ 18,85602	+ 0,21224
− 0,06308	+ 0,15425	+ 0,15425
+ 18,20484	+ 19,01027	+ 0,36649

§.

Antequam hinc litteras $\mathfrak{N}$ et N de-

nostris quinque angu-

$\omega =$	$2q$	$2p-2q$
$\mu =$	$2n$	$2m-2n$
$\mu =$	$+26,51208$	$-1,77424$
Log. $2(m+1) =$	$+1,4271258$	$+1,4271258$
Log. $\mu =$	$+1,4234438$	$-0,2490124$
Log. $\frac{2(m+1)}{\mu} =$	$+0,0036820$	$-1,1781134$
Log. $\mu^2 =$	$+2,8468876$	$+0,4980248$
$\lambda - 2 =$	$+177,22893$	$+177,22893$
$-\mu^2 =$	$-702,89033$	$-3,14793$
Denom. $=$	$-525,66140$	$+174,08100$
Log. den. $=$	$-2,7207050$	$+2,2407514$

197.

finire liceat; elementa numerica pro lis euolui oportet:

$2p+2q$	$4p-2q$	$4p+2q$
$2m+2n$	$4m-2n$	$4m+2n$
+51,24992	+ 22,96360	+75,98776
+ 1,4271258	+ 1,4271258	+ 1,4271258
+ 1,7096932	+1,3610399	+ 1,8807436
+ 9,7174326	+0,0660859	+ 9,5463822
+ 3,4193864	+ 2,7220798	+ 3,7614872
+ 177,22893	+177,22893	+ 177,22893
− 2626,55485	−527,32672	−5774,13940
−2449,32592	−350,09779	−5596,91047
− 3,3880465	−2,5441893	− 3,7479483

§. 198.

Nunc igitur calculum nostrum pro lit-

	$2q$	$2p-2q$
L. M	$+2,7314840$	$+1,0852617$
L. $\frac{2(m+1)}{\mu}$	$+0,0036820$	$-1,1781134$
	$+2,7351660$	$-2,2633751$
$\frac{2(m+1)M}{\mu}$	$+543,45800$	$-183,38976$
$-\mathfrak{M}$	$-810,65491$	$+120,92327$
Num.	$-267,19691$	$-62,46649$
L. Num.	$-2,4268312$	$-1,7956471$
L. Den.	$-2,7207050$	$+2,2407514$
L. $\mathfrak{N}$	$+9,7061262$	$-9,5548957$
L. $\frac{2(m+1)}{\mu}$	$+0,0036820$	$-1,1781134$
L. P. II.	$+9,7098082$	$+0,7330091$
L. M	$+2,7314840$	$+1,0852617$
L. μ^2	$+2,8468876$	$+0,4980248$
L. P. I.	$+9,8845964$	$+0,5872369$
P. I.	$+0,76665$	$+3,86578$
$-$ P. II.	$-0,51253$	$-5,40766$
N =	$+0,25404$	$-1,54188$
at $\mathfrak{N}$ =	$+0,50830$	$-0,35883$

198.

teris $\mathfrak{N}$ et N pro more nostro expediamus:

$2p+2q$	$4p-2q$	$4p+2q$
+ 1,2601868	+ 1,2789883	+ 9,5640621
+ 9,7174326	+ 0,0660859	+ 9,5463822
+ 0,9776194	+ 1,3450742	+ 9,1104443
+ 9,49772	+ 22,13472	+ 0,12896
− 22,96204	− 29,99867	− 0,43281
− 13,46432	− 7,86395	− 0,30385
− 1,1291844	− 0,8956408	− 9,4826592
− 3,3880465	− 2,5441893	− 3,7479483
+ 7,7411379	+ 8,3514515	+ 5,7347109
+ 9,7174326	+ 0,0660859	+ 9,5463822
+ 7,4585705	+ 8,4175374	+ 5,2810931
+ 1,2601868	+ 1,2789883	+ 9,5640621
+ 3,4193864	+ 2,7220798	+ 3,7614872
+ 7,8408004	+ 8,5569085	+ 5,8025749
+ 0,00693	+ 0,03605	+ 0,00006
− 0,00287	− 0,02615	− 0,00002
+ 0,00406	+ 0,00990	+ 0,00004
+ 0,00551	+ 0,02246	+ 0,00005

§. 199.

His expeditis, quae ad partes cognitas nostrarum aequationum referuntur: pro partibus incognitis statuamus

$$\mathfrak{Q} = \ldots + \beta.\cos.2q + \gamma.\cos(2p-2q) + \delta.\cos.(2p+2q) + \varepsilon.\cos.(4p-2q) + \zeta.\cos.(4p+2q)$$

$$Q = \ldots + b.\sin.2q + c.\sin.(2p-2q) + d.\sin.(2p+2q) + e.\sin.(4p-2q) + f.\sin.(4p+2q).$$

Has autem formas primum tantum in partes priores litterarum $\mathfrak{A}$; $\mathfrak{B}$; A; B; ducamus, vt saltem valores prope veros coefficientium assumtorum obtineamus; quibus inuentis multiplicatio demum per partes posteriores litterarum $\mathfrak{A}$, $\mathfrak{B}$, A, B, instituatur; hocque modo veros valores coefficientium poterimus definire.

§. 200.

Formulas autem, quae ex multiplicatione litterarum $\mathfrak{Q}$ et Q per partes principales litterarum $\mathfrak{A}$, $\mathfrak{B}$, A, B oriuntur, litteris $\mathfrak{M}'$ et M' indigitemus.

Pro

Pro littera $\mathfrak{M}'$.

	cos. $2q$	cos. $(2p-2q)$	cos. $(2p+q)$
$-9{,}22129 . \cos . 2p . \mathfrak{Q}$	$-4{,}61064 . \gamma$	$-4{,}61064 . \beta$	$-4{,}61064 . \beta$
	$-4{,}61064 . \delta$	$-4{,}61064 . \varepsilon$	$-4{,}61064 . \zeta$
$-3{,}99069 . \sin . 2p . Q$	$-1{,}99534 . c$	$-1{,}99534 . b$	$+1{,}99534 . b$
	$-1{,}99534 . d$	$-1{,}99534 . e$	$-1{,}99534 . f$

Simili modo pro angulis cos. $(4p-2q)$ et cos. $(4p+2q)$

	cos. $(4p-2q)$	cos. $(4p+2q)$
$-9{,}22129 . \cos . 2p . \mathfrak{Q}$	$-4{,}61064 . \gamma$	$-4{,}61064 . \delta$
$-3{,}99069 . \sin . 2p . Q$	$+1{,}99534 . c$	$+1{,}99534 . d$

Pro littera M'.

	sin. $2q$	sin. $(2p-2q)$	sin. $(2p+2q)$
$-3{,}99069 . \sin . 2p . \mathfrak{Q}$	$-1{,}99534 . \gamma$	$-1{,}99534 . \beta$	$-1{,}99534 . \beta$
	$+1{,}99534 . \delta$	$+1{,}99534 . \varepsilon$	$+1{,}99534 . \zeta$
$+5{,}36064 . \cos . 2p . Q$	$-2{,}68032 . c$	$-2{,}68032 . b$	$+2{,}68032 . b$
	$+2{,}68032 . d$	$+2{,}68032 . e$	$+2{,}68032 . f$

Simili modo pro angulis sin. $(4p-2q)$ et sin. $(4p+2q)$

	sin. $(4p-2q)$	sin. $(4p+2q)$
$-3{,}99069 . \sin . 2p . \mathfrak{Q}$	$-1{,}99534 . \gamma$	$-1{,}99534 . \delta$
$+5{,}36064 . \cos . 2p . Q$	$+2{,}68032 . c$	$+2{,}68032 . d$

§. 201.

Faciamus igitur calculum pro singulis columnis seorsim, uti sequitur:

I. Pro angulo $2q$.

	γ.	δ.	c.	d.
L. M′ =	− 0,3000169	+	− 0,4281866	+ . . . . . .
L. $\frac{2(m+1)}{\mu}$	+ 0,0036820		+ 0,0036820	
	− 0,3036989	+	− 0,4318686	+ . . . . . .
$\frac{2(m+1)M'}{\mu}$	− 2,01233	+ 2,01233	− 2,70314	+ 2,70314
− 𝔐′	+ 4,61064	+ 4,61064	+ 1,99534	+ 1,99534
Num.	+ 2,58831	+ 6,62297	− 0,70780	+ 4,69848
L. Num.	+ 0,4146910	+ 0,8210528	− 9,8499106	+ 0,6719574
L. Den.	− 2,7207050	− 2,7207050	− 2,7207050	− 2,7207050
L. 𝔑′ =	− 7,6939860	− 8,1003478	+ 7,1292056	− 7,9512524
L. $\frac{2(m+1)}{\mu}$	+ 0,0036820	+ 0,0036820	+ 0,0036820	+ 0,0036820
L. P. II.	− 7,6976680	− 8,1040298	+ 7,1328876	− 7,9549344
L. M′	− 0,3000169	+	− 0,4281866	+ . . . . . .
L. μ^2	+ 2,8468876		+ 2,8468876	
L. P. I.	− 7,4531293	+	− 7,5812990	+ . . . . . .
P. I.	− 0,00284	+ 0,00284	− 0,00381	+ 0,00381
− P. II.	+ 0,00498	+ 0,01271	− 0,00136	+ 0,00901
N′	+ 0,00214	+ 0,01555	− 0,00517	+ 0,01282
at 𝔑′ =	− 0,00494	− 0,01260	+ 0,00135	− 0,00894

Cum

Cum igitur per hypothesin fieri debeat

$$\beta = \mathfrak{N} + \mathfrak{N}' \text{ et } b = N + N'$$

nanciscimur has duas aequalitates

$$\beta = + 0,50830 - 0,00494 . \gamma - 0,01260 . \delta + 0,00135 . c - 0,00894 . d.$$

$$b = + 0,25404 + 0,00214 . \gamma + 0,01555 . \delta - 0,00517 . c + 0,01282 . d.$$

§. 202.

Eodem modo secunda columna tractata praebebit valores prope veros

$$\gamma = \mathfrak{N} + \mathfrak{N}' \text{ et } c = N + N'.$$

II. Pro angulo $2p - 2q$.

	β.	ε.	b.	e.
L. M'.	$-0,3000169$	$+ \ldots\ldots$	$-0,4281866$	$+ \ldots\ldots$
L. $\frac{2(m+1)}{\mu}$	$-1,1781134$	$- \ldots\ldots$	$-1,1781134$	$- \ldots\ldots$
	$+1,4781303$	$- \ldots\ldots$	$+1,6063000$	$- \ldots\ldots$
$\frac{2(m+1)M'}{\mu}$	$+30,06978$	$-30,06978$	$+40,39244$	$-40,39244$
$-\mathfrak{M}'$	$+4,61064$	$+4,61064$	$+1,99534$	$+1,99534$
Numer.	$+34,68042$	$-25,45914$	$+42,38778$	$-38,39710$
L. Num	$+1,5400843$	$-1,4058436$	$+1,6272407$	$-1,5842984$
L. Den.	$+2,2407514$	$+2,2407514$	$+2,2407514$	$+2,2407514$
L. $\mathfrak{N}'$	$+9,2993329$	$-9,1650922$	$+9,3864893$	$-9,3435470$
L. $\frac{2(m+1)}{\mu}$	$-1,1781134$	$-1,1781134$	$-1,1781134$	$-1,1781134$
L. P. II	$-0,4774463$	$+0,3432056$	$-0,5646027$	$+0,5216604$
L. M'.	$-0,3000169$	$+ \ldots\ldots$	$-0,4281866$	$+ \ldots\ldots$
L. μ^2	$+0,4980248$	$+ \ldots\ldots$	$+0,4980248$	$+ \ldots\ldots$
L. P. I.	$-9,8019921$	$+ \ldots\ldots$	$-9,9301618$	$+ \ldots\ldots$
P. I.	$-0,63386$	$+0,63386$	$-0,85145$	$+0,85145$
$-$ P. II.	$+3,00224$	$-2,20397$	$+3,66946$	$-3,32399$
N' $=$	$+2,36838$	$-1,57011$	$+2,81801$	$-2,47254$
at $\mathfrak{N}' =$	$+0,19922$	$-0,14625$	$+0,24349$	$-0,22057$

quare valores hinc oriundi ita se habebunt:

$$\gamma = -0,35883 + 0,19922\,\beta - 0,14625.\,\varepsilon + 0,24349.\,b - 0,22057.\,e.$$

$$c = -1,54188 + 2,36838\,\beta - 1,57011.\,\varepsilon + 2,81801.\,b - 2,47254.\,e.$$

§. 203.

§. 203.

Simili modo tertia columna dabit valores prope veros $\delta = \mathfrak{N} + \mathfrak{N}'$; $d = N + N'$.

Pro angulo $2p + 2q$.

	β.	ζ.	b.	f.
L. M'	−0,3000169	+	+0,4281866	+ . . . . .
L. $\frac{2(m+1)}{\mu}$	+9,7174326	+	+9,7174326	+
	−0,0174495	+	+0,1456192	+
$\frac{2(m+1)M'}{\mu}$	−1,04099	+1,04099	+1,39836	+1,39836
−$\mathfrak{M}'$	+4,61064	+4,61064	−1,99534	+1,99534
Numer.	+3,56965	+5,65163	−0,59698	+3,39370
L. Num.	+0,5526255	+0,7521737	−9,7759598	+0,5306734
L. Den.	−3,3880465	−3,3880465	−3,3880465	−3,3880465
Log. $\mathfrak{N}'$	−7,1645790	−7,3641272	+6,3879133	−7,1426269
L. $\frac{2(m+1)}{\mu}$	+9,7174326	+9,7174326	+9,7174326	+9,7174326
L. P. II	−6,8820116	−7,0815598	+6,1053459	−6,8600595
L. M'	−0,3000169	+	+0,4281866	+ . . . . . .
L. μ^2	+3,4193864	+	+3,4193864	+
L. P. I.	−6,8806305	+	+7,0088002	+
P. I.	−0,00076	+0,00076	+0,00102	+0,00102
−P. II.	+0,00076	+0,00121	−0,00013	+0,00072
N'	+0,00000	+0,00197	+0,00089	+0,00174
at $\mathfrak{N}'$	−0,00146	−0,00231	+0,00024	−0,00139

Vnde valores ita se habebunt:

$$\delta = +0,00551 - 0,00146.\beta - 0,00231.\zeta$$
$$+ 0,00024.b - 0,00139.f.$$

$$d = +0,00406 + 0,00197.\zeta$$
$$+ 0,00089.b + 0,00174.f.$$

§ 204.

§. 204.

Quartam et quintam columnam coniunctim poterimus expedire in hunc modum:

Pro angulo $4p - 2q$.

	γ.	c.
L. M′	− 0,3000169	+ 0,4281866
L. $\frac{2(m+1)}{\mu}$	+ 0,0660859	+ 0,0660859
	− 0,8661028	+ 0,4942725
$\frac{2(m+1)M'}{\mu}$	− 2,32330	+ 3,12085
− 𝔐′	+ 4,61064	− 1,99534
Numer.	+ 2,28734	+ 1,12551
L.Num.	+ 0,3593307	+ 0,0513494
L. Den.	− 2,5441893	− 2,5441893
L. 𝔑′	− 7,8151414	− 7,5071601
L. $\frac{2(m+1)}{\mu}$	+ 0,0660859	+ 0,0660859
L. P. II.	− 7,8812273	− 7,5732460
L. M′	− 0 3000169	+ 0,4281866
L. μ^2	+ 2,7220798	+ 2,7220798
L. P. I.	− 7,5779371	+ 7,7061068
P. I.	− 0,00378	+ 0,00508
− P. II.	+ 0,00761	+ 0,00374
N′	+ 0,00383	+ 0,00882
at 𝔑′	− 0,00653	− 0,00321

Pro

Pro angulo $4p + 2q$.

	δ.	d.
L. M′	− 0,3000169	+ 0,4281866
$L\frac{2(m+1)}{\mu}$	+ 9,5463822	+ 9,5463822
	− 9,8463991	+ 9,9745688
$\frac{2(m+1)M'}{\mu}$	− 0,70210	+ 0,94312
$-\mathfrak{M}'$	+ 4,61064	− 1,99534
Numer.	+ 3,90854	− 1,05322
L. Num.	+ 0,5920146	− 0,0225191
L. Den.	− 3,7479483	− 3,7479483
L. $\mathfrak{N}'$	− 6,8440663	+ 6,2745708
$L.\frac{2(m+1)}{\mu}$	+ 9,5463822	+ 9,5463822
L. P. II.	− 6,3904485	+ 5,8209530
L. M′	− 0,3000169	+ 0,4281866
L. μ^2	+ 3,7614872	+ 3,7614872
L. P. I.	− 6,5385297	+ 6,6666994
P. I.	− 0,00035	+ 0,00046
− P. II.	+ 0,00024	− 0,00007
N′	− 0,00011	+ 0,00039
at $\mathfrak{N}'$	− 0,00070	+ 0,00019

vnde ſequentes quatuor valores vero proximi deducuntur:

$$\varepsilon = +0,02246 - 0,00653.\gamma + 0,00321.c$$
$$e = +0,00990 + 0,00383.\gamma + 0,00882.c.$$
$$\zeta = +0,00005 - 0,00070.\delta + 0,00019.d.$$
$$f = +0,00004 - 0,00011.\delta + 0,00039.d.$$

§. 205.

Subſtituamus iam valores ε et e in valoribus paragraphi 202, atque perueniemus ad ſequentes formulas:

$$\gamma = -0,36429 + 0,19922.\beta + 0,24349.b$$
$$+ 0,00012.\gamma - 0,00148.c.$$
$$c = -1,60162 + 2,36838.\beta + 2,81801.b$$
$$+ 0,00078.\gamma - 0,01677.c.$$

ex priore colligitur

$$\gamma = -0,36433 + 0,19924\beta + 0,24351.b - 0,00148.c.$$

qui valor in altera ſubſtitutus praebet

$$c = -1,60190 + 2,36853.\beta + 2,81820.b - 0,01677.c.$$

ſiue

$$1,01677.c = -1,60190 + 2,36853.\beta + 2,81820.b.$$

conſequenter

$$c = -1,57548 + 2,32946.\beta + 2,77172.b$$

tum vero fit

$$\gamma = -0,36200 + 0,19579\beta + 0,23941.b.$$

§. 206.

Deinde etiam valores pro ζ et f inuentos in formulis pro δ et d (§. 203) ſubſtituamus, atque ſtatim obtinebimus

$$\delta = +0,00551 - 0,00146.\beta + 0,00024.b$$
$$d = +0,00406 + 0,00089.b.$$

§. 207.

Tantum igitur ſupereſt, vt iſtos valores in formulis

mulis §. 201. pro β et b inuentis introducamus; eritque

$$\beta = +0,50786 + 0,00215.\beta + 0,00256.b.$$
$$b = +0,26154 - 0,01163.\beta - 0,01382.b.$$

hincque $\beta = +0,50893 + 0,00257.b.$

qui valor in altera substitutus praebet

$b = +0,25562 - 0,01385.b.$ siue

$1,01385.b = +0,25562.$

consequenter

$$b = +0,25213;\quad \beta = +0,50958.$$

His autem valoribus substitutis inueniemus porro

$\gamma = -0,20188.$	$c = +0,31041.$
$\delta = +0,00481.$	$d = +0,00428.$
$\varepsilon = +0,02278.$	$e = +0,01186.$
$\zeta = +0,00004.$	$f = +0,00004.$

§. 208.

En ergo valores prope veros, quos nobis hactenus definire licuit:

$$\mathfrak{Q} = \ldots +0,50958.\text{cos}.2q - 0,20188.\text{cos}.(2p-2q) + 0,00481.\text{cos}.(2p+2q) + 0,02278.\text{cos}.(4p-2q) + 0,00004.\text{cos}.(4p+2q)$$

$$Q = \ldots +0,25213.\text{sin}.2q + 0,31041.\text{sin}.(2p-2q) + 0,00428.\text{sin}.(2p+2q) + 0,01186.\text{sin}.(4p-2q) + 0,00004.\text{sin}.(4p+2q)$$

quorum ergo correctionem vlteriorem inuestigari oportet.

 Cor-

Correctio horum po-

§.

Quoniam nondum rationem habuimus minorum
has ipsas particulas nunc in valores prope veros mo-
$\mathfrak{M}''$ et M'' designemus; quarum valo-

Pro

	cos. $2q$	cos. $2p-2q$
$\mathfrak{Q} + 0,02644$ - -	$+0,01346$	$-0,00533$
$\mathfrak{Q} - 0,10506$. cos. $4p$	$-0,00067$	$+0,00026$
	$+0,01279$	$-0,00507$
$Q - 0,08191$. sin. $4p$	$-0,00048$	$-0,00017$
$\mathfrak{M}''$	$+0,01231$	$-0,00524$

Pro

	sin. $2q$	sin. $2p-2q$
$\mathfrak{Q} - 0,08191$. sin. $4p$.	$-0,00052$	$-0,00021$
		$+0,00825$
$Q - 0,02724$. - -	$-0,00663$	$+0,00804$
		$-0,00796$
	$-0,00715$	$-0,00018$
$Q + 0,06654$. cos. $4p$	$-0,00039$	$-0,00014$
M''	$-0,00754$	$-0,00032$

steriorum valorum.

209.

particularum, quibus litterae 𝔄, 𝔅, A, B constant, do inuentos ducamus et formulas inde ortas litteris res sequenti modo colligemus:

𝔐″.

cos. $2p+2q$	cos. $4p-2q$	cos. $4p+2q$
+0,00012	+0,00034	−0,02674
+0,01059	−0,02674	
+0,01071	−0,02640	−0,02674
−0,01235	−0,00997	+0,00997
−0,00164	−0,03637	−0,01677

M″.

sin. $2p+2q$	sin. $4p-2q$	sin. $4p+2q$
+0,00825	−0,02084	−0,02084
−0,00011	−0,00032	
+0,00814	−0,02116	−0,02084
−0,01004	−0,00810	+0,00810
−0,00190	−0,02926	−0,01274

§.

His iam litteris $\mathfrak{M}''$ et M'' quaeramus ana-

	$2q$	$2p-2q$
ad Log. $M'' =$	$-7,8773713$	$-6,5051500$
adde L. $\frac{2(m+1)}{\mu} =$	$+0,0036820$	$-1,1781134$
	$-7,8810533$	$+7,6832634$
$\frac{2(m+1)M''}{\mu} =$	$-0,00760$	$+0,00482$
$-\mathfrak{M}''$	$-0,01231$	$+0,00524$
Numer.	$-0,01991$	$+0,01006$
Log. Numer.	$-8,2990713$	$+8,0025980$
subtr. Log. Den.	$-2,7207050$	$+2,2407514$
Log. $\mathfrak{N}'' =$	$+5,5783663$	$+5,7618466$
adde L. $\frac{2(m+1)}{\mu} =$	$+0,0036820$	$-1,1781134$
L. P. II.	$+5,5820483$	$-6,9399600$
Log. $M'' =$	$-7,8773713$	$-6,5051500$
L. $\mu^2 =$	$+2,8468876$	$+0,4980248$
Log $\frac{M''}{\mu^2} =$	$-5,0304837$	$+6,0071252$
P. I. $=$	$-0,00001$	$+0,00010$
$-$ P. II.	$-0,00004$	$+0,00087$
$N'' =$	$-0,00005$	$+0,00097$
at $\mathfrak{N}'' =$	$+0,00004$	$+0,00006$

210.

logas $\mathfrak{N}''$ et N'' sequenti modo:

$2p+2q$	$4p-2q$	$4p+2q$
−7,2787536	−8,4662743	−8,1051694
+9,7174326	+0,0660859	+9,5463822
−6,9961862	−8,5323602	−7,6515516
−0,00099	−0,03407	−0,00448
+0,00164	+0,03637	+0,01677
+0,00065	+0,00230	+0,01229
+6,8129134	+7,3617278	+8,0895519
−3,3880465	−2,5441893	−3,7479483
−3,424	−4,8175385	−4,3416036
+9,717	+0,0660859	+9,5463822
−3,141	−4,8836244	−3,8879858
−7,3364597	−8,4662743	−8,1051694
+3,4193864	+2,7220798	+3,7614872
−3,9170733	−5,7441945	−4,3436822
−0,00000	−0,00005	−0,00000
	+0,00000	
−0,00000	−0,00005	−0,00000
−0,00000	−0,00000	−0,00000

§. 211.

§. 211.

Has igitur exiguas correctiones inſuper ad formulas ſuperiores $\mathfrak{N} + \mathfrak{N}'$ et $N + N'$ adiici oportet, vt obtineantur ſequentes aequationes verae:

$$\beta = +0,50834 - 0,00494.\gamma - 0,01260.\delta$$
$$+0,00135.c - 0,00894.d.$$

$$b = +0,25399 + 0,00212.\gamma + 0,01555.\delta$$
$$-0,00517.c + 0,01282.d.$$

$$\gamma = -0,35877 + 0,19922.\beta - 0,14625.\varepsilon$$
$$+0,24349.b - 0,22057.e.$$

$$c = -1,54091 + 2,36838.\beta - 1,57011.\varepsilon$$
$$+2,81801.b - 2,47254.e.$$

$$\delta = +0,00551 - 0,00146.\beta - 0,00231.\zeta$$
$$+0,00024.b - 0,00139.f.$$

$$d = +0,00406 + 0,00001.\beta + 0,00197.\zeta$$
$$+0,00089.b + 0,00174.f.$$

$$\varepsilon = +0,02246 - 0,00653.\gamma - 0,00321.c.$$

$$e = +0,00985 + 0,00383.\gamma + 0,00882.c.$$

$$\zeta = +0,00005 - 0,00070\,\delta + 0,00019.d.$$

$$f = +0,00004 - 0,00011.\delta + 0,00039.d.$$

§. 212.

§. 212.

Repetamus ergo easdem operationes, quibus supra vsi sumus §. 205; ac primo pro ε et e valores in γ et c substituentes inueniemus:

$$\gamma = -0,36422 + 0,19922.\beta + 0,24349.b$$
$$+ 0,00012.\gamma - 0,00148.c.$$
$$c = -1,60052 + 2,36838.\beta + 2,81801.b$$
$$+ 0,00078.\gamma - 0,01677.c.$$

vnde fit

$$\gamma = -0,36426 + 0,19924.\beta + 0,24352.b$$
$$-0,00148.c.$$

qui valor in posteriore substitutus praebet

$$c = -1,60081 + 2,36854.\beta + 2,81820.b$$
$$-0,01677.c.$$

adeoque

$$c = -1,57450 + 2,32959\beta + 2,77191.b.$$

tum vero

$$\gamma = -0,36193 + 0,19580.\beta + 0,23942.b.$$

§. 213.

Simili modo valores ζ et f substituantur in δ et d ac reperitur

$$\delta = +0,00551 - 0,00146.\beta + 0,00024.b.$$
$$d = +0,00406 + 0,00001.\beta + 0,00089.b.$$

Hi iam valores in duabus prioribus aequationibus substituantur, vnde prodit

$$\beta = +0,50790 + 0,00220.\beta + 0,00256.b.$$
$$b = +0,26150 - 0,01164.\beta - 0,01381.b.$$

§. 214.

Ex priore harum postremarum aequationum colligimus

$$\beta = +0,50902 + 0,00257.b.$$

qui valor in altera substitutus praebet

$$b = +0,25558 - 0,01384.b.$$

sicque tandem concluditur

$$b = +0,25209.$$
$$\beta = +0,50967.$$

hincque porro

$\gamma = -0,20179.$	$c = +0,31159.$
$\delta = +0,00482.$	$d = +0,00428.$
$\varepsilon = +0,02279.$	$e = +0,01183.$
$\zeta = +0,00005.$	$f = +0,00004.$

§. 215.

Ecce ergo veros et completos valores litterarum Ω et Q, quos haec euolutio nobis suppeditauit:

$\Omega =$

$$\Omega = -0,53896 + 0,21903.\,\text{cos}.\,2p$$
$$+ 0,00195.\,\text{cos}.\,4p$$
$$+ 0,50967.\,\text{cos}.\,2q - 0,20179.\,\text{cos}.\,(2p-2q)$$
$$+ 0,00482.\,\text{cos}.\,(2p+2q)$$
$$+ 0,02279.\,\text{cos}.\,(4p-2q)$$
$$+ 0,00005.\,\text{cos}.\,(4p+2q)$$

$$Q = + 0,09800.\,\text{fin}.\,2p + 0,00175.\,\text{fin}.\,4p$$
$$+ 0,25209.\,\text{fin}.\,2q + 0,31159.\,\text{fin}.\,(2p-2q)$$
$$+ 0,00428.\,\text{fin}.\,(2p+2q)$$
$$+ 0,01183.\,\text{fin}.\,(4p-2q)$$
$$+ 0,00004.\,\text{fin}.\,(4p+2q)$$

CAPVT IV.

EVOLVTIO AEQVATIONVM ORDINIS IV. PRO LITTERIS $\mathfrak{R}$ ET R.

§. 217.

Harum aequationum partes annexae, vnde haec euolutio est petenda, ita se habent:

$$\text{I. } 0 = \ldots\ldots + \mathfrak{R}\mathfrak{A} + R.\mathfrak{B}$$
$$+ \mathfrak{P}.\mathfrak{Q}\ 2\mathfrak{C} + (\mathfrak{P}Q + P\mathfrak{Q})\mathfrak{D} + PQ.\ 2\mathfrak{E}$$
$$+ \mathfrak{P}^3\mathfrak{F} + \mathfrak{P}^2 P.\mathfrak{G} + \mathfrak{P}.P^2.\mathfrak{H} + P^3.\mathfrak{I}.$$

$$\text{II. } 0 = \ldots\ldots + \mathfrak{R}A + RB$$
$$+ \mathfrak{P}\mathfrak{Q}\ 2C + (\mathfrak{P}Q + P\mathfrak{Q})D + PQ.\ 2E.$$
$$+ \mathfrak{P}^3F + \mathfrak{P}^2PG + \mathfrak{P}.P^2.H + P^3.I.$$

vbi in vtraque aequatione membra priora litteras $\mathfrak{R}$ et R continentia, quippe quae sunt incognita, a sequentibus probe sunt distinguenda, siquidem haec penitus sunt cognita.

§. 218.

Incipiamus igitur a membris postremis et quia iam supra euoluimus formulas $\mathfrak{P}^2$; $\mathfrak{P}\,P$ et P^2, ex iis ante omnia valores formularum $\mathfrak{P}^3$; $\mathfrak{P}^2 P$; $\mathfrak{P}\,P^2$ et P^3 quaeri oportet, qui sequenti modo expressi reperiuntur:

$$\begin{aligned}
\mathfrak{P}^3 = &+0,8021.\cos.q + 0,2843.\cos.(2p-q)\\
&\qquad\qquad + 0,1364.\cos.(2p+q)\\
&\qquad\qquad + 0,0249.\cos.(4p-q)\\
&\qquad\qquad - 0,0012.\cos.(4p+q)\\
&+0,2492.\cos.3q + 0,1407.\cos.(2p-3q)\\
&\qquad\qquad - 0,0020.\cos.(2p+3q)\\
&\qquad\qquad + 0,0260\ \cos\ (4p-3q)\\
&\qquad\qquad + 0,0000.\cos.(4p+3q)
\end{aligned}$$

$$\begin{aligned}
\mathfrak{P}^2 P = &-0,5400.\sin.q - 0,2131.\sin.(2p-q)\\
&\qquad\qquad - 0,2934.\sin.(2p+q)\\
&\qquad\qquad + 0,0570.\sin.(4p-q)\\
&\qquad\qquad + 0,0011.\sin.(4p+q)\\
&-0,5035.\sin.3q + 0,0862.\sin.(2p-3q)\\
&\qquad\qquad + 0,0019.\sin.(2p+3q)\\
&\qquad\qquad - 0,0215.\sin.(4p-3q)\\
&\qquad\qquad + 0,0000.\sin.(4p+3q)
\end{aligned}$$

$$\mathfrak{P}P' = +1,0996.\cos q + 0,3945.\cos(2p-q)$$
$$-0,6098.\cos(2p+q)$$
$$+0,1221.\cos(4p-q)$$
$$-0,0004.\cos(4p+q)$$
$$-1,0138.\cos 3q + 0,2240.\cos(2p-3q)$$
$$-0,0005.\cos(2p+3q)$$
$$+0,0366.\cos(4p-3q)$$
$$+0,0000.\cos(4p+3q)$$

$$P' = -6,6210.\sin q - 2,5403.\sin(2p-q)$$
$$+1,2282.\sin(2p+q)$$
$$+0,2468.\sin(4p-q)$$
$$+0,0061.\sin(4p+q)$$
$$+2,0342.\sin 3q + 1,2490.\sin(2p-3q)$$
$$+0,0097.\sin(2p+3q)$$
$$-0,2531.\sin(4p-3q)$$
$$+0,0001.\sin(4p+3q)$$

§. 220.

Eodem modo ex inuentis valoribus litterarum $\mathfrak{Q}$ et Q definiamus formulas sequentes:

$\mathfrak{P}\mathfrak{Q} =$

$$
\begin{aligned}
\mathfrak{P}\mathfrak{Q} = & -0{,}2829.\cos.q - 0{,}0911.\cos.(2p-q) \\
& \qquad +0{,}1614.\cos.(2p+q) \\
& \qquad +0{,}0335.\cos.(4p-q) \\
& \qquad +0{,}0011.\cos.(4p+q) \\
& +0{,}2556.\cos.3q - 0{,}0531.\cos.(2p-3q) \\
& \qquad +0{,}0017.\cos.(2p+3q) \\
& \qquad -0{,}0076.\cos.(4p-3q) \\
& \qquad +0{,}0000.\cos.(4p+3q)
\end{aligned}
$$

$$
\begin{aligned}
P\mathfrak{Q} = & +1{,}6838.\sin.q + 0{,}6490.\sin.(2p-q) \\
& \qquad -0{,}3182.\sin.(2p+q) \\
& \qquad -0{,}0653.\sin.(4p-q) \\
& \qquad -0{,}0034.\sin.(4p+q) \\
& -0{,}5116.\sin.3q - 0{,}3079.\sin.(2p-3q) \\
& \qquad -0{,}0057.\sin.(2p+3q) \\
& \qquad +0{,}0642.\sin.(4p-3q) \\
& \qquad +0{,}0000.\sin.(4p+3q)
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{P}Q = & +0{,}1061.\sin.q + 0{,}2062.\sin.(2p-q) \\
& \qquad +0{,}0750.\sin.(2p+q) \\
& \qquad +0{,}0156.\sin.(4p-q) \\
& \qquad +0{,}0012.\sin.(4p+q) \\
& +0{,}1268.\sin.3q + 0{,}1321.\sin.(2p-3q) \\
& \qquad +0{,}0018.\sin.(2p+3q) \\
& \qquad +0{,}0351.\sin.(4p-3q) \\
& \qquad +0{,}0000.\sin.(4p+3q)
\end{aligned}
$$

Atque

Atque hinc colligitur

$$
\begin{aligned}
P\mathfrak{Q}+\mathfrak{P}Q = &+1,7899.\sin.q + 0,8552.\sin.(2p-q)\\
&-0,2432.\sin.(2p+q)\\
&-0,0497.\sin.(4p-q)\\
&-0,0022.\sin.(4p+q)\\
&-0,3848.\sin.3q - 0,1758.\sin.(2p-3q)\\
&-0,0039.\sin.(2p+3q)\\
&+0,0993.\sin.(4p-3q)\\
&+0,0000.\sin.(4p+3q)\\
&= \pi.
\end{aligned}
$$

$$
\begin{aligned}
PQ = &-0,3381.\cos.q + 0,2121.\cos.(2p-q)\\
&+0,1457.\cos.(2p+q)\\
&+0,0309.\cos.(4p-q)\\
&+0,0028.\cos.(4p+q)\\
&+0,2522.\cos.3q - 0,3653.\cos.(2p-3q)\\
&+0,0047.\cos.(2p+3q)\\
&+0,0522.\cos.(4p-3q)\\
&+0,0000.\cos.(4p+3q).
\end{aligned}
$$

§. 221.

§. 221.

Isti valores iterum sponte in duas classes distribuuntur, quarum prior angulum tantum simplicem q, posterior vero eius triplum seu angulum $3\,q$ complectitur; quare, vti in capite praecedente fecimus, has duas classes seorsim pertractabimus.

I. Euolutio partium anomaliam simplicem q continentium.

§. 222.

Hic igitur litterae $\mathfrak{M}$ et M eas formulas nobis designabunt, quae ex cognitis partibus aequationum nostrarum orientur, dum eae, quae ex partibus incognitis nascuntur, litteris $\mathfrak{M}'$ et M'; $\mathfrak{M}''$ et M'' indicabuntur, quem calculum perinde expedire licebit, ac si partes posteriores angulum $3\,q$ inuoluentes plane abessent. Ceterum notasse iuuabit, in sequentibus calculis numeros vncinulis inclusos designare partes quantitatum, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$ etc. vel A, B, C etc.; ita, vt exempli gratia $\mathfrak{P}\,P^2.\,\mathfrak{H}^{(III)}$ exprimat valorem ipsius $\mathfrak{P}\,P^2$ ductum in partem tertiam quantitatis $\mathfrak{H}$.

§.

Prima ergo aequatio pro littera

	cos. q	cos. $2p-q$
$\mathfrak{P}^3 . \mathfrak{F}^{(I)}$	$-574,9665$	$-203,7938$
$\mathfrak{P}^2 . \mathfrak{F}^{(II)}$	$-3,6586$	$-10,3220$
	$-1,7553$	$-0,3204$
	$-580,3804$	$-214,4362$
	$-0,0064$	$-0,0365$
	$-580,3868$	$-214,4727$
$\mathfrak{P}^2 . P . \mathfrak{G}^{(I)}$ ...	$+5,8503$	$+14,8247$
	$+8,0548$	$+1,5648$
	$-566,4817$	$-198,0832$
$\mathfrak{P}^2 . P . \mathfrak{G}^{(II)}$ ..	$+0,0340$	$+0,1780$
	$-566,4477$	$-197,9052$
$\mathfrak{P} . P^2 . \mathfrak{H}^{(I)}$ ...	$+1179,3840$	$+424,0994$
	$+612,9363$	$+226,1942$
$\mathfrak{P} . P^2 . \mathfrak{H}^{(II)}$ ..	$+7,6152$	$+21,1770$
	$-11,7711$	$-2,3569$
	$+620,5515$	$+245,0143$
$\mathfrak{P} . P^2 . \mathfrak{H}^{(III)}$ ..	$-0,0620$	$-0,3092$
	$+620,4895$	$+244,7051$
$P^3 . \mathfrak{J}^{(I)}$	$-17,4350$	$-45,4422$
	$+8,4295$	$+1,6938$
	$+611,4840$	$+200,9567$

223.

M sequentem calculum suppeditabit:

$\cos. 2p + q$	$\cos. 4p - q$	$\cos. 4p + q$.
− 97,7752	− 17,8490	+ 0,8602
− 10,3220	− 3,6586	− 1,7553
+ 0,0254		
−108,0718	− 21,5076	− 0,8951
− 0,0762	− 0,2150	− 0,2150
− 108,1480	− 21,7226	− 1,1101
− 14,8247	− 5,8503	− 8,0548
− 0,0302		
− 123,0029	− 27,5729	− 9,1649
+ 0,1293	+ 0,3276	− 0,3276
− 122,8736	− 27,2453	− 9,4925
−655,5535	−131,2612	− 0,4300
− 778,4271	−158,5065	− 9,9225
+ 21,1770	+ 7,6152	− 11,7711
− 0,0077		
− 757,2578	−150,8913	−21,6936
+ 0,2005	+ 0,5565	+ 0,5565
−757,0573	−150,3348	−21,1371
+ 45,4422	+ 17,4350	− 8,4295
− 0,0419		
− 711,6570	− 132,8998	−29,5656

 cos. q

	$\cos. q$	$\cos. 2p - q$
	$+ 611,4840$	$+ 200,9567$
$P^{1}. \mathfrak{J}^{(II)}$	$+ 0,0365$	$+ 0,1862$
	$+ 611,5205$	$+ 201,1429$
$\mathfrak{P}. \mathfrak{Q}. 2. \mathfrak{E}^{(I)}$.	$- 304,1930$	$- 97,9568$
	$+ 307,3275$	$+ 103,1861$
$\mathfrak{P}. \mathfrak{Q}. 2. \mathfrak{E}^{(II)}$.	$- 1,4068$	$- 4,3687$
	$+ 2,4924$	$+ 0,0170$
	$+ 308,4131$	$+ 98,8344$
$\mathfrak{P}. \mathfrak{Q}. 2. \mathfrak{E}^{(III)}$.	$+ 0,0092$	$+ 0,0429$
	$+ 308,4223$	$+ 98,8773$
	$+ 9,3913$	$+ 19,6556$
$\pi. \mathfrak{D}^{(I)}$	$- 2,6706$	$- 0,5458$
	$+ 315,1430$	$+ 117,9871$
$\pi. \mathfrak{D}^{(II)}$	$- 0,0106$	$- 0,0494$
	$+ 315,1324$	$+ 117,9377$
$PQ. 2. \mathfrak{E}^{(I)}$ ..	$+ 181,7500$	$- 114,0172$
	$+ 496,8824$	$+ 3,9205$
$PQ. 2. \mathfrak{E}^{(II)}$ ..	$- 1,6377$	$+ 2,6105$
	$- 1,1250$	$- 0,2386$
	$+ 494,1197$	$+ 6,2924$
$PQ. 2. \mathfrak{E}^{(III)}$ ..	$- 0,0057$	$- 0,0245$
Ergo $\mathfrak{M} =$	$+ 494,1140$	$+ 6,2679$

cos. $2p+q$	cos. $4p-q$	cos. $4p+q$.
−711,6570	−132,8998	−29,5666
− 0,3852	− 1,0042	+ 1,0042
−712,0422	−133,9040	−28,5624
+173,5481	+ 36,0214	+ 1,1828
−538,4941	− 97,8826	− 27,3796
− 4,3687	− 1,4068	+ 2,4924
+ 0,5173		
−542,3455	− 99,2894	−24,8872
− 0,0242	− 0,0752	− 0,0752
−542,3697	− 99,3646	−24,9624
− 19,6556	− 9,3913	
− 0,0242		+ 2,6706
−562,0495	−108,7559	−22,2918
+ 0,1738	+ 0,3637	− 0,3637
−561,8757	−108,3922	−22,6555
− 78,3230	− 16,6107	− 1,5051
−640,1987	−125,0029	−24,1606
+ 2,6105	− 1,6377	− 1,1250
− 0,0216		
−637,6098	−126,6406	−25,2856
− 0,0356	+ 0,0568	+ 0,0568
−637,6454	−126,5838	−25,2288

§.

Eodem modo colligamus

	sin. q	sin. $2p-q$
$\mathfrak{P}^3. -18,3023.$ sin. $2p$	$-$ 2,6017	$-$ 7,3402
	$+$ 1,2483	$+$ 0,2279
	$-$ 1,3534	$-$ 7,1123
$\mathfrak{P}^3. -0,4044.$ sin. $4p$	$-$ 0,0050	$-$ 0,0276
	$-$ 0,0002	
	$-$ 1,3586	$-$ 7,1399
$\mathfrak{P}^2. P. +1075,0302$	$-$ 580,5161	$-$ 229,0889
	$-$ 581,8747	$-$ 236,2288
$\mathfrak{P}^2. P. +38,6064.$ cos. $2p$	$+$ 4,1135	$+$ 10,4237
	$-$ 577,7612	$-$ 225,8051
	$-$ 5,6635	$-$ 1,1003
	$-$ 583,4247	$-$ 226,9054
$\mathfrak{P}^2. P. +1,0143.$ cos. $4p$	$+$ 0,0294	$+$ 0,1488
	$-$ 583,3953	$-$ 226,7566
$\mathfrak{P}. P^2. +41,1802.$ sin. $2p$	$+$ 8,1228	$+$ 22,5888
	$-$ 575,2725	$-$ 204,1678
	$+$ 12,5558	$+$ 2,5140
	$-$ 562,7167	$-$ 201,6538
$\mathfrak{P}. P^2. +0,9100.$ sin. $4p$	$-$ 0,0553	$-$ 0,2775
	$-$ 562,7720	$-$ 201,9313
$P^3. -268,7399$	$+$ 1779,3254	$+$ 682,6800
	$+$ 1216,5534	$+$ 480,7487

224.

valorem litterae M.

sin. $2p + q$	sin. $4p - q$	sin. $4p + q$
− 7,3402	− 2,6017	− 1,2483
− 0,0110		
− 7,3512		
− 0,0575	− 0,1622	− 0,1622
− 7,4087	− 2,7639	− 1,4105
−315,4138	−61,2767	+ 1,1825
−322,8225	−64,0406	− 0,2280
− 10,4237	− 4,1135	
−333,2462	−68,1541	− 0,2280
+ 0,0212		− 5,6635
−333,2250	−68,1541	− 5,8915
+ 0,1080	+ 0,2738	− 0,2738
−333,1170	−67,8803	− 6,1653
+ 22,5888	+ 8,1228	
−310,5282	−59,7575	− 6,1653
+ 0,0082		−12,5558
−310,5200	−59,7575	−18,7211
+ 0,1795	+ 0,4992	+ 0,4992
−310,3405	−59,2583	−18,2219
−330,0665	−66,3250	+ 1,6393
−640,4070	−125,5833	−16,5826

sin. q

	sin. q	sin. $2p - q$
	$+1216,5534$	$+480,7487$
$P^2. - 9,6516. \cos. 2p$	$-$ 12,2589	$-$ 31,9516
	$+$ 1204,2945	$+$ 448,7971
	$-$ 5,9271	$-$ 1,1910
	$+$ 1198,3674	$+$ 447,6061
$P^2. - 0,2711. \cos. 4p$	$+$ 0,0342	$+$ 0,1664
	$+$ 1198,4016	$+$ 447,7725
$P. Q. 2. 10,9814. \sin. 2p$	$-$ 1,0004	$-$ 3,1066
	$-$ 1,7724	$-$ 0,3679
	$+$ 1195,6288	$+$ 444,2980
$P. Q. 2. 0,2032. \sin. 4p$	$+$ 0,0066	$+$ 0,0328
	$+$ 1195,6354	$+$ 444,3308
$\pi. D^{(I)}$ - - - - - - - -	$-$ 962,1849	$-$ 459,7242
	$+$ 233,4505	$-$ 15,3934
$\pi. D^{(II)}$ - - - - - - - -	$+$ 6,6031	$+$ 13,8203
	$+$ 1,8778	$+$ 0,3837
	$+$ 241,9314	$-$ 1,1894
	$-$ 0,0080	$-$ 0,0408
	$+$ 241,9234	$-$ 1,2302
$P. Q. \pi. D^{(III)}$ - - - - -	$-$ 0,5394	$+$ 2,7846
$P. Q. 2 E^{(I)}$ - - - - - -		$+$ 0,2545
	$+$ 241,3840	$+$ 1,8089
$P. Q\ 2 E^{(II)}$ - - - - -	$-$ 0,0043	$-$ 0,0222
Ergo $M =$	$+$ 241,3797	$+$ 1,7867

sin. $2p+q$	sin. $4p-q$	sin. $4p+q$
−640,4070	−125,5833	−16,5826
+ 31,9516	+ 12,2589	− 5,9271
−608,4554	−113,3244	−10,6555
+ 0,0294		
−608,4260	−113,3244	−10,6555
− 0,3442	− 0,8971	+ 0,8971
−608,7702	−114,2215	− 9,7584
− 3,1066	− 1,0004	+ 1,7724
− 0,0121		
−611,8889	−115,2219	− 7,9860
− 0,0185	− 0,0575	− 0,0575
−611,9074	−115,2794	− 8,0435
+130,7354	+ 26,7169	+ 1,1826
−481,1720	− 88,5625	− 6,8609
− 13,8203	− 6,6031	+ 1,8778
+ 0,0170		
−494,9753	− 95,1656	− 4,9831
+ 0,1436	+ 0,3005	− 0,3005
−494,8317	− 94,8651	− 5,2836
+ 2,7846	− 1,7394	− 1,2000
− 0,0231		
−492,0702	− 96,6045	− 6,4836
− 0,0323	+ 0,0515	+ 0,0515
−492,1025	− 96,5530	− 6,4321

§. 225.

Pro calculo litterarum $\mathfrak{N}$ et N elementa numerica iam supra in §. 158. reperiuntur. Ob easdem autem rationes, quae ibi sunt allegatae, primam columnam anguli q, quippe quae singulare iudicium postulat, in vltimum locum reseruemus, pro reliquis igitur columnis calculus sequenti modo instituatur:

	$2p-q$	$2p+q$	$4p-q$	$4p+q$
L. M	+0,2520516	−2,6920556	−1,9847658	−0,8083528
L. $\frac{2(m+1)}{\mu}$	+0,3671158	+9,8474122	+9,8681819	+9,6296386
	+0,6191674	−2,5394678	−1,8529477	−0,4379914
$\frac{2(m+1)M}{\mu}$	+4,1607	−346,3122	− 71,2767	− 2,7416
$-\mathfrak{N}$	−6,2679	+637,6454	+126,5838	+ 25,2288
Num.	− 2,1072	+291,3332	+ 55,3071	+ 22,4872
L. Num.	−0,3237058	+2,4643900	+1,7427809	+1,3519355
L. Den.	+ 1,6570290	−3,1025375	− 3,0548550	−3,5749613
L. $\mathfrak{N}$	−8,6666768	−9,3618525	− 8,6879259	−7,7769742
L. $\frac{2(m+1)}{\mu}$	+0,3671158	+9,8474122	+9,8681819	+9,6296386
L. P. II.	−9,0337926	−9,2092647	− 8,5561078	− 7,4066128
L. M	+0,2520516	− 2,6920556	− 1,9847658	− 0,8083528
L. μ^2	+2,1200200	+3,1594272	+3,1178878	+3,5949744
L. P. I.	+ 8,1320316	− 9,5326284	− 8,8668780	− 7,2133784
P. I.	+0,0136	−0,3409	−0,0736	−0,0016
− P. II.	+0,1081	+0,1619	+0,0360	+0,0026
N =	+ 0,1217	− 0,1790	− 0,0376	+0,0010
at $\mathfrak{N}$ =	−0,0464	−0,2301	−0,0487	−0,0060

§. 226.

§. 226.

Pro partibus autem incognitis ſtatuamus

$\mathfrak{N} = \beta . \cos . q + \gamma . \cos . 2p - q + \delta . \cos . 2p + q + \varepsilon . \cos . 4p - q + \zeta . \cos . 4p + q$

$R = b . \sin . q + c . \sin . 2p - q + d . \sin . 2p + q + e . \sin . 4p - q + f . \sin . 4p + q$

vbi ante omnia obſeruandum eſt, hic neceſſario eſſe debere $\beta = 0$, quandoquidem numerus n ita definitus eſſe aſſumitur, vt ex omnibus ordinibus ſimul ſumtis prodeat coefficiens termini $\cos . q = K$. Has igitur quantitates primo tantum in partes principales litterarum $\mathfrak{A}$, $\mathfrak{B}$, et A, B ducamus, indeque colligamus numeros $\mathfrak{M}'$ et M', quibus deinceps reſpondeant numeri $\mathfrak{N}'$ et N'. Particulas autem minimas eorundem coefficientium ad correctiones finales reſeruemus.

§. 227.

Primum igitur ex priore aequatione numeros $\mathfrak{M}'$ hoc modo colligimus:

Pro numeris $\mathfrak{M}'$.

$\cos . q$	$\cos . 2p-q$	$\cos . 2p+q$	$\cos . 4p-q$	$\cos . 4p+q$
$-4,6106 . \gamma$	$-4,6106 . \varepsilon$	$-4,6106 . \zeta$	$-4,6106 . \gamma$	$-4,6106 \ \delta$
$-4,6106 . \delta$	$-1,9953 . b$	$+1,9953 . b$	$+1,9953 . c$	$+1,9953 . d$
$-1,9953 . c$	$-1,9953 . e$	$-1,9953 . f$		
$-1,9953 . d$				

Simili modo

Pro numeris M′.

$\sin . q$	$\sin . 2p-q$	$\sin . 2p+q$	$\sin . 4p-q$	$\sin . 4p+q$
$-1,9953 \ \gamma$	$+1,9953 . \varepsilon$	$+1,9953 . \zeta$	$-1,9953 . \gamma$	$-1,9953 . \delta$
$+1,9953 . \delta$	$-2,6803 . b$	$+2,6803 . b$	$+2,6803 \ c$	$+2,6803 . d$
$-2,6803 . c$	$+2,6803 . e$	$+2,6803 . f$		
$+2,6803 . d$				

§. 228.

Reseruata prima columna ad peculiare iudicium, pro columna secunda et quarta calculum litterarum $\mathfrak{N}'$ et N' expediamus:

Pro $2p - q$.

	ε	b	e
L. M′	+0,3000082	−0,4281834	+ . . . . . .
L. $\frac{2(m+1)}{\mu}$	+0,3671158	+0,3671158	+ . . . . . .
	+0,6671240	−0,7952992	+0,7952992
$\frac{2(m+1)M'}{\mu}$	+4,6465	−6,2417	+6,2417
$-\mathfrak{M}'$	+4,6106	+1,9953	+1,9953
Numer.	+9,2571	−4,2464	+8,2370
L. Num.	+0,9664750	−0,6280209	+0,9157691
L. den.	+1,6570290	+1,6570290	+1,6570290
Log. $\mathfrak{N}'$	+9,3094460	−8,9709919	+9,2587401
	+0,3671158	+0,3671158	+0,3671158
L. P. II.	+9,6765618	−9,3381077	+9,6258559
Log. M′	+0,3000082	−0,4281834	+ . . . . . .
Log. μ^2	+2,1200200	+2,1200200	+ . . . . . .
L. P. I.	+8,1799882	−8,3081634	+ . . . . . .
P. I.	+0,0151	−0,0203	+0,0203
− P. II.	−0,4749	+0,2178	−0,4225
N′	−0,4598	+0,1975	−0,4022
$\mathfrak{N}'$	+0,2039	−0,0935	+0,1814

Simi-

Simili modo

Pro $4p-q$.

	γ	c
ad Log. M'	$-$ 0,3000082	$+$0,4281834
Log. $\frac{2(m+1)}{\mu}$	$+$ 9,8681819	$+$ 9,8681819
	$-$ 0,1681901	$+$0,2963653
$\frac{2(m+1)M'}{\mu}$	$-$ 1,4730	$+$ 1,9786
$-\mathfrak{M}'$	$+$ 4,6106	$-$ 1,9953
Numerat.	$+$ 3,1376	$-$ 0,0167
Log. Numerat.	$+$0,4965976	$-$ 8,2227165
Log. Denom.	$-$ 3,0548550	$-$ 3,0548550
L. $\mathfrak{N}'$	$-$ 7,4417426	$+$ 5,1678615
	$+$9,8681819	$+$ 9,8681819
L. Pars II.	$-$ 7,3099245	$+$5,0360434
L. M'	$-$ 0,3000082	$+$0,4281834
L. μ^2	$+$3,1178878	$+$ 3,1178878
L. P. I.	$-$ 7,1821204	$+$ 7,3102956
P. I.	$-$ 0,0015	$+$ 0,0020
$-$ P. II.	$+$ 0,0020	$-$ 0,0000
N'	$+$ 0,0005	$+$ 0,0020
at $\mathfrak{N}'$	$-$ 0,0027	$+$ 0,0000

qui totus calculus non discrepat ab eo, qui in secundo capite pro iisdem angulis $2p-q$ et $4p-q$ est institutus (§. 161. et 163), nisi quod hic statuimus $\beta=0$.

 §. 229.

§. 229.

Hinc ergo sequentes deducimus determinationes

$\gamma = -0,0464 + 0,2039.\varepsilon - 0,0935\, b + 0,1814.e,$

$c = +0,1217 - 0,4598.\varepsilon + 0,1975.b - 0,4022.e.$

$\varepsilon = -0,0487 - 0,0027.\gamma + 0,0000.c.$

$e = -0,0376 + 0,0005.\gamma + 0,0020.c.$

Bini hi valores posteriores substituantur in binis prioribus, et habebimus

$\gamma = -0,0631 - 0,0935.b - 0,0005.\gamma + 0,0004.c.$

$c = +0,1592 + 0,1975.b + 0,0011.\gamma - 0,0008.c.$

Ex posteriore statim colligimus

$$c = +0,1580 + 0,1960.b + 0,0011.\gamma.$$

qui valor in priore substitutus dat

$$\gamma = -0,0631 - 0,0935.b - 0,0005.\gamma$$

adeoque

$$\gamma = -0,0631 - 0,0935.b.$$

vnde vicissim

$$c = +0,1579 + 0,1959.b.$$

§. 230.

Simili modo calculus pro columnis tertia et quinta prorsus idem erit, atque supra §. 162. et 163; quo circa consequimur has aequationes:

$\delta =$

$\delta = -0,2301 - 0,0048.\zeta + 0,0001.b - 0,0031.f.$

$d = -0,0376 + 0,0048\,\zeta + 0,0018.b + 0,0040.f.$

$\zeta = -0,0060 - 0,0010.\,\delta + 0,0002.\,d.$

$f = +0,0010 - 0,0001.\,\delta + 0,0006.\,d.$

Bini valores posteriores in prioribus substituti praebent

$\delta = -0,2301 + 0,0001.\,b.$

$d = -0,0376 + 0,0018.\,b.$

§. 231.

Progrediamur iam ad primam columnam, pro qua ob

$\gamma + \delta = -0,2932 - 0,0934.\,b.$ et

$c + d = +0,1203 + 0,1977.\,b.$

obtinebimus

$\mathfrak{M}' = +1,1118 + 0,0361.\,b.$

Eodem modo pro M′ ob

$\gamma - \delta = +0,1670 - 0,0934.\,b.$

$c - d = +0,1955 + 0,1941.\,b.$

inuenimus

$M' = -0,8572 - 0,3338.\,b.$

sicque

ſicque omnino pro hac columna habebimus

$$\mathfrak{M} + \mathfrak{M}' = + 495,2258 + 0,0361 . b.$$
$$M + M' = + 240,5225 - 0,3338 . b.$$

§. 232.

Hinc iam quaerere deberemus reſpondentes valores $\mathfrak{N} + \mathfrak{N}'$ et $N + N'$; verum quia totum valorem $\mathfrak{N} + \mathfrak{N}'$ transferimus in ordinem ſecundum capitis ſecundi; hic eius valor ſumitur $= 0$; ex quo valor ipſius $N + N'$ fiet $= \frac{M + M'}{\mu^2}$; cui propterea aequari debet littera b, vnde adipiſcimur

$$b = + 1,3687 - 0,0019 . b.$$

adeoque tandem colligitur

$$b = + 1,3662.$$

vnde reliquae litterae ita reperiuntur expreſſae

$$\gamma = - 0,1908.$$
$$\delta = - 0,2300.$$
$$\varepsilon = - 0,0482.$$
$$\zeta = - 0,0058.$$
$$c = + 0,4255.$$
$$d = - 0,0353.$$
$$e = - 0,0377.$$
$$f = + 0,0010.$$

§. 233.

§. 233.

Consequenter valor prope verus priorum membrorum litterarum $\mathfrak{R}$ et R erit

$$\mathfrak{R} = 0.\ \text{cos}.\ q - 0,1908.\ \text{cos}.\ 2p - q$$
$$- 0,2300.\ \text{cos}.\ 2p + q$$
$$- 0,0482.\ \text{cos}.\ 4p - q$$
$$- 0,0058.\ \text{cos}.\ 4p + q.$$

$$R = + 1,3662.\ \text{sin}.\ q + 0,4255.\ \text{sin}.\ 2p - q$$
$$- 0,0353.\ \text{sin}.\ 2p + q$$
$$- 0,0377.\ \text{sin}.\ 4p - q$$
$$+ 0,0010.\ \text{sin}.\ 4p + q.$$

Quos ergo valores iam vlterius corrigamus, quod sequenti calculo praestabitur.

 Cor-

Correctio isto-

§.

Hos igitur valores prope veros ducamus in ducta litteris $\mathfrak{M}''$ et M'' designemus, quae ergo

	$\cos. q$	$\cos. 2p - q$
$\mathfrak{R}.\ \mathfrak{A}^{(II)}$		$-0,0050$
$\mathfrak{R}.\ \mathfrak{A}^{(III)}$	$+0,0028$	$+0,0100$
		$+0,0050$
$R.\ \mathfrak{B}^{(II)}$	$+0,0015$	$+0,0014$
$\mathfrak{M}''$	$+0,0043$	$+0,0064$
	$\sin. q$	$\sin. 2p - q$
$\mathfrak{R}.\ A^{(II)}$	$+0,0018$	$+0,0094$
$R.\ B^{(II)}$	$-0,0372$	$-0,0116$
	$-0,0354$	$-0,0022$
$R.\ B^{(III)}$	$+0,0013$	$+0,0012$
M''	$-0,0341$	$-0,0010$

rum valorum.

234.

particulas minimas litterarum 𝔄, 𝔅, A, B et pro-
quaeramus sequenti calculo:

cos. $2p + q$	cos. $4p - q$	cos. $4p + q$
$-0,0061$	$-0,0013$	$-0,0002$
$+0,0121$		
$+0,0060$		
$-0,0174$	$-0,0559$	$+0,0559$
$-0,0114$	$-0,0572$	$+0,0557$

sin. $2p + q$	sin. $4p - q$	sin. $4p + q$
$+0,0078$		
$+0,0010$	$+0,0010$	
$+0,0088$	$-0,0454$	$+0,0454$
$-0,0141$		
$-0,0053$	$-0,0444$	$+0,0454$

§. 235.

Nunc igitur iterum seposita prima columna pro reliquis sequentem calculum instituamus:

	$2p-q$	$2p+q$	$4p-q$	$4p+q$
L. M″	− 7,0000000	− 7,7242759	− 8,6473830	+ 8,6570559
L. $\frac{2(m+1)}{\mu}$	+ 0,3671158	+ 9,8474122	+ 9,8681819	+ 9,6296386
	− 7,3671158	− 7,5716881	− 8,5155649	+ 8,2866945
$\frac{2(m+1)M''}{\mu}$	− 0,0023	− 0,0037	− 0,0328	+ 0,0194
− $\mathfrak{M}''$	+ 0,0010	+ 0,0053	+ 0,0444	− 0,0454
Numer.	− 0,0013	+ 0,0016	+ 0,0116	− 0,0260
L. Num.	− 7,11394	+ 7,20412	+ 8,06446	− 8,41497
L. Den.	+ 1,65703	− 3,10254	− 3,05485	− 3,57496
L. $\mathfrak{N}''$	− 5,45691	− 4,10158	− 5,010	− 4,84
L. $\frac{2(m+1)}{\mu}$	+ 0,36711	+ 9,84741	+ 9,868	+ 9,63
L. P. II.	− 5,82402	− 3,94899	− 4,87	− 4,47
L. M″	− 7,0000	− 7,7242	− 8,6473	+ 8,6570
L. μ^2	+ 2,1200	+ 3,1594	+ 3,1178	+ 3,5949
L. P. I.	− 4,8800	− 4,5648	− 5,5295	+ 5,06

$N'' = 0$ et $\mathfrak{N}'' = 0$.

cum

cum igitur tam $\mathfrak{N}''$, quam N'' nihil pro his columnis produxerit; coefficientes noſtri prorſus vt ſupra per b determinantur; tum vero adiectis pro prima columna litteris $\mathfrak{N}''$ et N'' habebimus omnino

$$\mathfrak{M} + \mathfrak{M}' + \mathfrak{M}'' = +\,495,2301 + 0,0361.\,b. \text{ et}$$

$$M + M' + M'' = +\,240,4884 - 0,3338.\,b.$$

quos numeros ſimpliciter litteris $\mathfrak{M}$ et M indicemus. Cum igitur ſit $b = \frac{M}{\mu^2}$; fiet

$b = +\,1,3686 - 0,0019.\,b$ hinque $b = +\,1,3661.$

Reliqui vero coefficientes manent, vt ante;

$\gamma = -\,0,1908.$	$c = +\,0,4255.$
$\delta = -\,0,2300.$	$d = +\,0,0353.$
$\varepsilon = -\,0,0482.$	$e = -\,0,0377.$
$\zeta = -\,0,0058.$	$f = +\,0,0010.$

§. 236.

Videamus denique, quid hinc in ordinem ſuperiorem ſecundum ſit transferendum; hunc in finem ſubſtituto pro b valore pro prima columna habebimus

$$\mathfrak{M} = +\,495,2794;\quad M = +\,240,0324;$$

hinc ergo colligimus

$$\frac{2(m'+1)}{\mu} M - \mathfrak{M} = +\,11,1273.$$

qui eſt ipſe ille numerator in caput ſecundum transferendus. Cum autem hic multiplicatum ſit per K^3, dum ibi tantum per K multiplicatur, hunc numerum inſuper in KK duci oportet; ita, vt iam pro §. 178. habeamus

$$1,5064 = 1,5530 - 11,1273 . K^2$$

atque adeo haec forma ob $K^2 = \frac{1}{350}$ circiter; iam multo propius ad veritatem perducitur; infra autem denuo ordo occurret, vnde quaepiam particula huc redundat, vnde veritati plene ſatisfiet.

§. 237.

Ecce ergo valores veros priorum partium litterarum $\mathfrak{R}$ et R:

$$\begin{aligned} \mathfrak{R} = 0.\operatorname{cof.} q &- 0,1908.\operatorname{cof.}(2p-q) \\ &- 0,2300.\operatorname{cof.}(2p+q) \\ &- 0,0482.\operatorname{cof.}(4p-q) \\ &- 0,0058.\operatorname{cof.}(4p+q) \end{aligned}$$

$$\begin{aligned} R = +1,3662.\operatorname{fin.} q &+ 0,4255.\operatorname{fin.}(2p-q) \\ &- 0,0353.\operatorname{fin.}(2p+q) \\ &- 0,0377.\operatorname{fin.}(4p-q) \\ &+ 0,0010.\operatorname{fin.}(4p+q) \end{aligned}$$

vnde

vnde simul intelligimus, correctiones, quas adhibuimus, plane fuisse superfluas, ita, vt sine errore partes minimas litterarum 𝔄, 𝔅, A, B negligere potuissemus; quod ergo in sequente euolutione obseruabimus.

II. Euolutio partium anomaliam triplam seu angulum $3q$ inuoluentium.

§. 238.

Hic ergo tantum membra posteriora formularum $\mathfrak{P}^3$; $\mathfrak{P}^2 P$; $\mathfrak{P} P^2$; P^3; $\mathfrak{P} \Omega$; π; $P \Omega$ in computum ducemus, et calculum vt ante expediemus:

Pro

Pro mu-

	$\cos. 3q$	$\cos. 2p - 3q$
$\mathfrak{P}^3. - 716,8266.$	$-$ 178,6332	$-$ 100,8575
$\mathfrak{P}^3. - 25,7376. \cos. 2p.$	$-$ 1,8106	$-$ 3,2069
	$-$ 180,4438	$-$ 0,3346
	$+$ 0,0257	
	$-$ 180,4181	$-$ 104,3990
$\mathfrak{P}^3. - 0,5360. \cos. 4p.$	$-$ 0,0070	$+$ 0,0005
	$-$ 180,4251	$-$ 104,3985
$\mathfrak{P}^2. P. - 54,9070. \sin. 2p.$	$-$ 2,3665	$+$ 13,8228
	$-$ 0,0522	$+$ 0,5903
	$-$ 182,8438	$-$ 89,9854
$\mathfrak{P}^2. P. - 1,2133. \sin. 4p.$	$+$ 0,0130	$-$ 0,0012
	$-$ 182,8308	$-$ 89,9866
$\mathfrak{P}. P^2. + 1075,0302.$	$-$ 1089,8652	$+$ 240,8066
	$-$ 1272,6960	$+$ 150,8200
$\mathfrak{P}. P^2. + 38,6064. \cos. 2p$	$+$ 4,3239	$-$ 19,5696
	$-$ 0,0097	$+$ 0,7065
	$-$ 1268,3818	$+$ 131,9569
$\mathfrak{P}. P^2. + 1,0143. \cos. 4p.$	$+$ 0,0186	$-$ 0,0003
	$-$ 1268,3632	$+$ 131,9566
$P^3. + 13,7267. \sin. 2p.$	$+$ 8,5723	$+$ 13,9613
	$+$ 0,0666	$-$ 1,7371
	$-$ 1259,7243	$+$ 144,1808

meris M.

cos. $2p+3q$	cos. $4p-3q$	cos. $4p+3q$
+ 1,4336	− 18,6375	
− 3,2069	− 1,8106	+ 0,0257
− 1,7733	− 20,4481	+ 0,0257
− 0,0377	− 0,0668	− 0,0668
− 1,8110	− 20,5149	− 0,0411
− 13,8228	+ 2,3665	
		+ 0,0522
− 15,6338	− 18,1484	+ 0,0111
− 0,0523	+ 0,3054	− 0,3054
− 15,6861	− 17,8430	− 0,2943
− 0,5375	+ 39,3461	
− 16,2236	+ 21,5031	− 0,2943
− 19,5696	+ 4,3239	− 0,0097
− 35,7932	+ 25,8270	− 0,3040
+ 0,1136	− 0,5141	− 0,5141
− 35,6796	+ 25,3129	− 0,8181
− 13,9613	− 8,5723	
+ 0,0007		− 0,0666
− 49,6402	+ 16,7406	− 0,8847

	cos. $3q$	cos. $2p - 3q$
	− 1259,7243	+ 144,1808
$P^3. + 0,3033. \sin. 4p.$	− 0,0384	+ 0,0015
	− 1259,7627	+ 144,1823
$\mathfrak{P}\,\mathfrak{Q}.\ 2.\ 537,6336.$	+ 274,8383	− 57,0860
	− 984,9244	+ 87,0963
$\mathfrak{P}\,\mathfrak{Q}.\ 2.\ 15,4426. \cos. 2p.$	− 0,8199	+ 3,9472
	+ 0,0263	− 0,1174
	− 985,7180	+ 90,9261
$\mathfrak{P}\,\mathfrak{Q}.\ 2.\ 0,26. \cos. 4p.$	− 0,0020	+ 0,0005
	− 985,7200	+ 90,9266
$\pi.\ 21,96. \sin. 2p.$	− 1,9733	− 3,1353
	− 987,6933	+ 87,7913
$\pi.\ 0,406. \sin. 4p.$	+ 0,0201	− 0,0008
	− 987,6732	+ 87,7905
$PQ.\ 2. - 268,78.$	− 135,5735	+ 196,3800
	− 1123,2467	+ 284,1705
$PQ.\ 2. - 7,7. \cos. 2p.$	+ 2,7843	− 2,3504
	− 1120,4624	+ 281,8201
$PQ.\ 2. - 0,16. \cos. 4p.$	− 0,0087	− 0,0008
Ergo $\mathfrak{M} =$	− 1120,4711	+ 281,8193

cos. $2p + 3q$	cos. $4p - 3q$	cos. $4p + 3q$
− 49,6402	+ 16,7406	− 0,8847
+ 0,1894	+ 0,3084	− 0,3084
− 49,4508	+ 17,0490	− 1,1931
+ 1,8279	− 8,1720	
− 47,6229	+ 8,8770	− 1,1931
+ 3,9472	− 0,8199	+ 0,0263
− 43,6757	+ 8,0571	− 1,1668
− 0,0141	+ 0,0680	+ 0,0680
− 43,6898	+ 8,1251	− 1,0988
+ 4,2257	+ 1,9305	+ 0,0428
− 39,4641	+ 10,0556	− 1,0560
− 0,0357	− 0,0782	+ 0,0782
− 39,4998	+ 9,9774	− 0,9778
− 2,5265	− 28,0610	
− 42,0263	− 18,0836	− 0,9778
− 1,9473	+ 2,8206	− 0,0363
− 43,9736	− 15,2630	− 1,0141
+ 0,0614	− 0,0424	− 0,0424
− 43,9122	− 15,3054	− 1,0565

Pro nu-

	sin. $3q$	sin. $2p-3q$
$\mathfrak{P}^3. -18,3023.$ sin. $2p$	$-$ 1,2875	$-$ 2,2804
	$-$ 0,0183	$+$ 0,2379
	$-$ 1,3058	$-$ 2,0425
$\mathfrak{P}^3. -0,4044.$ sin. $4p$	$-$ 0,0053	$+$ 0,0004
	$-$ 1,3111	$-$ 2,0421
$\mathfrak{P}^2.$ P. 1075,0302.	$-$ 541,2780	$+$ 92,6676
	$-$ 542,5891	$+$ 90,6255
$\mathfrak{P}^2$ P. 38,6064. cos. $2p$	$-$ 1,6272	$+$ 9,3040
	$-$ 544,2163	$+$ 99,9295
$\mathfrak{P}^2$ P. 1,0143. cos. $4p$.	$+$ 0,0109	$-$ 0,0009
	$-$ 544,2054	$+$ 99,9286
$\mathfrak{P}$. P^2. 41,1802. sin. $2p$	$+$ 4,6205	$-$ 21,6278
	$-$ 539,5849	$+$ 78,3008
$\mathfrak{P}$ P^2. 0,9100. sin. $4p$	$+$ 0,0167	$-$ 0,0002
	$-$ 539,5682	$+$ 78,3006
P^2. $-$ 268,74.	$-$ 546,6710	$-$ 335,6563
	$-$ 1086,2392	$-$ 257,3557
P^2. $-$ 9,65. cos. $2p$.	$+$ 5,9806	$+$ 11,0380
	$-$ 1080,2586	$-$ 246,3177

meris M.

sin. $2p+3q$	sin. $4p-3q$	sin. $4p+3q$
− 2,2804	− 1,2875	+ 0,0183
− 0,0285	− 0,0504	− 0,0504
− 2,3089	− 1,3379	− 0,0321
+ 2,0425	− 23,1132	
− 0,2664	− 24,4511	
− 9,7190	+ 1,6639	+ 0,0367
− 9,9854	− 22,7872	+ 0,0046
− 0,0437	+ 0,2553	− 0,2553
− 10,0291	− 22,5319	− 0,2507
− 20,8742	+ 4,6122	− 0,0103
− 30,9033	− 17,9197	− 0,2610
+ 0,1019	− 0,4613	− 0,4613
− 30,8014	− 18,3810	− 0,7223
− 2,6068	+ 68,0181	− 0,0268
− 33,4082	+ 49,6371	− 0,7491
− 9,8169	− 6,0274	− 0,0468
− 43,2251	+ 43,6097	− 0,7959

	sin. $3q$	sin. $2p - 3q$
	$-1080,2586$	$-246,3177$
$P^3. -0,27. \cos. 4p.$	$-0,0343$	$+0,0013$
	$-1080,2929$	$-246,3164$
$PQ. 2. 10,98. \sin. 2p.$	$-0,6017$	$+2,8903$
	$-1080,8946$	$-243,4261$
$PQ. 2. 0,2032. \sin. 4p.$	$-0,0015$	$+0,0003$
	$-1080,8961$	$-243,4258$
$\pi. -537,56.$	$+206,8545$	$+94,5030$
	$-874,0416$	$-148,9228$
$\pi. -15,4426. \cos. 2p.$	$-1,3575$	$-3,7380$
	$-875,3991$	$-152,6608$
$\pi. -0,336. \cos. 4p.$	$+0,0167$	$-0,0007$
	$-875,3824$	$-152,6615$
$PQ. 2. -8,23. \sin. 2p.$	$+3,0473$	$-1,6472$
	$-872,3351$	$-154,3087$
$PQ. 2. -0,15. \sin. 4p$	$-0,0079$	$-0,0007$
Ergo $M =$	$-872,3430$	$-154,3094$

fin. $2p+3q$	fin. $4p-3q$	fin. $4p+3q$
−43,2251	+43,6097	−0,7959
+ 0,1692	+ 0,2756	−0,2756
−43,0559	+43,8853	−1,0715
+ 2,8068	− 0,5830	+0,0187
−40,2491	+43,3023	−1,0528
− 0,0108	+ 0,0519	+0,0519
−40,2599	+43,3542	−1,0009
+ 2,0965	− 53,3800	
−38,1634	− 10,0258	
+ 2,9713	+ 1,3575	+0,0301
−35,1921	− 8,6683	−0,9708
− 0,0295	− 0,0646	+0,0646
−35,2216	− 8,7329	− 0,9062
− 2,0771	+ 3,0086	−0,0387
−37,2987	− 5,7243	−0,9449
+ 0,0557	− 0,0384	−0,0384
−37,2430	− 5,7627	−0,9833

§. 240.

Iam pro numeris 𝔑 et N ante omnia elemen-

ω	$8q$	$2p - 3q$
μ	$3n$	$2m - 3n$
	$+ 39,7681$	$- 15,0303$
Log. $2(m+1)$	$+ 1,4271258$	$+ 1,4271258$
Log. μ	$+ 1,5995337$	$- 1,1769671$
Log. $\frac{2(m+1)}{\mu}$	$+ 9,8275921$	$- 0,2501587$
Log. μ^2	$+ 3,1990674$	$+ 2,3539342$
$\lambda - 2$	$177,2289$	$177,2289$
μ^2	$1581,5000$	$225,9100$
Denom.	$- 1404,2711$	$- 48,6811$
Log.	$- 3,1474551$	$- 1,6873604$

240.

ta numerica pro his angulis constitui oportet.

$2p+3q$	$4p-3q$	$4p+3q$
$2m+3n$	$4m-3n$	$4m+3n$
$+64,5060$	$+9,7076$	$+89,2438$
$+1,4271258$	$+1,4271258$	$+1,4271258$
$+1,8096061$	$+0,9871119$	$+1,9505782$
$+9,6175257$	$+6,4400139$	$+9,4765476$
$+3,6192002$	$+1,9742238$	$+3,9011564$
$177,2289$	$177,2289$	$177,2289$
$4161,1000$	$94,2370$	$7964,5000$
$-3983,8711$	$+82,9919$	$-7787,2711$
$-3,6003051$	$+1,9190362$	$-3,8913851$

§. 241.

§. 241.

Hic iterum prima columna commode ad vltimum locum remittitur; vnde calculum nostrum tantum pro sequentibus columnis expediamus:

	$2p-3q$	$2p+3q$
L. M	− 2,1883912	− 1,5710447
L. $\frac{2(m+1)}{\mu}$	− 0,2501587	+ 9,6175257
	+ 2,4385499	− 1,1885704
$\frac{2(m+1)M}{\mu}$	+ 274,5050	− 15,4373
− 𝔐	− 281,8193	+ 43,9122
Num.	− 7,3143	+ 28,4749
L. Num.	− 0,8641728	+ 1,4544637
L. Den.	− 1,6873604	− 3,6003051
L. 𝔑	+ 9,1768124	− 7,8541586
L. $\frac{2(m+1)}{\mu}$	− 0,2501587	+ 9,6175257
L. P. II.	− 9,4269711	− 7,4716843
L. M	− 2,1883912	− 1,5710447
L. μ^2	+ 2,3539342	+ 3,6192002
L. P. I.	− 9,8344570	− 7,9518445
P. I.	− 0,6831	− 0,0090
− P. II.	+ 0,2673	+ 0,0030
N =	− 0,4158	− 0,0060
at 𝔑 =	+ 0,1503	− 0,0072

Simili

Simili modo

	$4p-3q$	$4p+3q$
L. M.	$-0,7606260$	$-9,9926860$
L. $\frac{2(m+1)}{\mu}$	$+0,4400139$	$+9,4765476$
	$-1,2006399$	$-9,4692336$
$\frac{2(m+1)M}{\mu}$	$-15,8723$	$-0,2946$
$-\mathfrak{M}$	$+15,3054$	$+1,0565$
Numer.	$-0,5669$	$+0,7619$
L. Num.	$-9,7535065$	$+9,8818980$
L. Den.	$+1,9190362$	$-3,8913851$
L. $\mathfrak{N}$	$-7,8344703$	$-5,9905129$
L. $\frac{2(m+1)}{\mu}$	$+0,4400139$	$+9,4765476$
L. P. II.	$-8,2744842$	$-5,4670605$
L. M.	$-0,7606260$	$-9,9926860$
L. μ^2	$+1,9742238$	$+3,9011564$
L. P. I.	$-8,7864022$	$-6,0915296$
P. I.	$-0,0612$	$-0,0001$
$-$ P. II.	$+0,0188$	$+0,0000$
N $=$	$-0,0424$	$-0,0001$
at $\mathfrak{N} =$	$-0,0068$	$-0,0000$

 §. 242.

§.

Iam igitur pro ipsis partibus incognitis, qui-

$$\mathfrak{N} = \beta.\cos.3q + \gamma.\cos.2p-3q + \delta.\cos.2p+3q$$

$$R = b.\sin.3q + c.\sin.2p-3q + d.\sin.2p+3q$$

atque hinc calculum

Pro

	$\cos.3q$	$\cos.2p-3q$
$\mathfrak{N}.-9,2212.\cos.2p.$	$-4,6106.\gamma$	$-4,6106\ \beta$
	$-4,6106.\delta$	$-4,6106.\varepsilon$
$R.-3,9907.\sin.2p$	$-1,9953.c$	$-1,9953.b$
	$-1,9953.d$	$-1,9953.e$

Pro

	$\sin.3q$	$\sin.2p-3q$
$\mathfrak{N}.-3,9907.\sin.2p$	$-1,9953.\gamma$	$-1,9953.\beta$
	$+1,9953.\delta$	$+1,9953.\varepsilon$
$R.+5,3606.\cos.2p$	$-2,6803.c$	$-2,6803.b$
	$+2,6803.d$	$+2,6803.e$

242.

bus conueniant litterae $\mathfrak{M}'$ et M', ponamus:

$+\varepsilon.\cos.\, 4p-3q+\zeta.\cos.\, 4p+3q$

$+e.\sin.\, 4p-3q+f.\sin.\, 4p+3q$

sequenti modo faciamus:

$\mathfrak{M}'$.

$\cos.\, 2p+3q$	$\cos.\, 4p-3q$	$\cos.\, 4p+3q$
$-4,6106.\beta$	$-4,6106.\gamma$	$-4,6106.\delta$
$-4,6106.\zeta$		
$+1,9953.b$	$+1,9953.c$	$+1,9953.d$
$-1,9953.f$		

M'.

$\sin.\, 2p+3q$	$\sin.\, 4p-3q$	$\sin.\, 4p+3q$
$-1,9953.\beta$	$-1,9953.\gamma$	$-1,9953.\delta$
$+1,9953.\zeta$		
$+2,6803.b$	$+2,6803.c$	$+2,6803.d$
$+2,6803.f$		

§. 243.

Hic igitur iterum pro numeris $\mathfrak{N}'$ et N' columnas secundam et quartam coniunctim expediamus.

Pro $2p - 3q$.

	β	ε	b	e
L. M'	−0,3000082	+	−0,42818	+ . . . . . .
L. $\frac{2(m+1)}{\mu}$	−0,2501587	−	−0,25016	− . . . . . .
	+0,5501669	−	+0,67834	− . . . . . .
$\frac{2(m+1)M'}{\mu}$	+3,5495	−3,5495	+4,7681	−4,7681
− $\mathfrak{M}'$	+4,6106	+4,6106	+1,9953	+1,9953
Numer.	+8,1601	+1,0611	+6,7684	−2,7728
L. Num.	+0,9116955	+0,02576	+0,83017	+0,44292
L. den.	−1,6873604	−1,68736	−1,68736	−1,68736
Log. $\mathfrak{N}'$	−9,2243351	−8,33840	−9,14281	−8,75556
	−0,2501587	−0,25016	−0,25016	−0,25016
L. P. II.	+9,4744938	+8,58856	+9,39297	+9,00572
Log. M'	−0,3000082	+	−0,42818	+ . . . . . .
Log. μ^2	+2,3539342	+	+2,35393	+ . . . . . .
L. P. I.	−7,9460740	+	−8,07425	+ . . . . . .
P. I.	−0,0088	+0,0088	−0,0119	+0,0119
− P. II.	−0,2981	−0,0388	−0,2472	−0,1013
N'	−0,3069	−0,0300	−0,2591	−0,0894
$\mathfrak{N}'$	−0,1676	−0,0218	−0,1389	−0,0570

Simili

Simili modo

Pro $4p - 3q$.

	γ	ϵ
ad Log. M'	$-0,3000082$	$+0,4281834$
Log. $\frac{2(m+1)}{\mu}$	$+0,4400139$	$+0,4400139$
	$-0,7400221$	$+0,8681973$
$\frac{2(m+1)M'}{\mu}$	$-5,4957$	$+7,3823$
$-\mathfrak{M}'$	$+4,6106$	$-1,9953$
Numerat.	$-0,8851$	$+5,3870$
Log. Numerat.	$-9,9469923$	$+0,7313470$
Log. Denom.	$+1,9190362$	$+1,9190362$
L. $\mathfrak{N}'$	$-8,0279561$	$+8,8123108$
	$+0,4400139$	$+0,4400139$
L. Pars II.	$-8,4679700$	$+9,2523247$
L. M'	$-0,3000082$	$+0,4281834$
L. μ^2	$+1,9742238$	$+1,9742238$
L. P. I.	$-8,3257844$	$+8,4539596$
P. I.	$-0,0212$	$+0,0285$
$-$ P. II.	$+0,0294$	$-0,1788$
N'	$+0,0082$	$-0,1503$
at $\mathfrak{N}'$	$-0,0107$	$+0,0649$

§. 244.

§. 244.

Ex his igitur quatuor sequentes determinationes adipiscimur:

$$\gamma = +0,1503 - 0,1676.\beta - 0,0218.\varepsilon - 0,1389.b - 0,0570.e.$$

$$c = -0,4158 - 0,3069.\beta - 0,0300.\varepsilon - 0,2591.b - 0,0894.e.$$

$$\varepsilon = -0,0068 - 0,0107.\gamma + 0,0649.c.$$

$$e = -0,0424 + 0,0082.\gamma - 0,1503.c.$$

Bini hi posteriores valores in prioribus substituantur:

$$\gamma = +0,1528 - 0,1676.\beta - 0,1389.b - 0,0002\gamma + 0,0072\,c$$

$$c = -0,4120 - 0,3069.\beta - 0,2591.b - 0,0004.\gamma + 0,0114.c$$

Inde ergo colligitur

$$\gamma = +0,1528 - 0,1676.\beta - 0,1389.b + 0,0072.c$$

qui valor in altera substitutus producit

$$c = -0,4120 - 0,3069.\beta - 0,2591.b + 0,0114.c.$$

hincque

$$c = -0,4166 - 0,3102.\beta - 0,2620.b.$$

et tandem

$$\gamma = +0,1560 - 0,1700.\beta - 0,1409.b.$$

§. 245.

§. 245.

Simili modo columnam tertiam et quintam euoluamus:

Pro $2p + 3q$

	β	ζ	b	f
L. M'	− 0,3000082	+	+ 0,42818	+ . . . .
L. $\frac{2(m+1)}{\mu}$	+ 9,6175257	+	+ 9,61753	+ . . . .
	− 9,9175339	+	+ 0,04571	+ . . . .
$\frac{2(m+1)M'}{\mu}$	− 0,8270	+ 0,8270	+ 1,1110	+ 1,1110
− 𝔐'	+ 4,6106	+ 4,6106	− 1,9953	+ 1,9953
Numer.	+ 3,7836	+ 5,4376	− 0,8843	+ 3,1063
L. Num.	+ 0,57791	+ 0,73541	− 9,94660	+ 0,49224
L. Den.	− 3,60030	− 3,60030	− 3,60030	− 3,60030
Log. 𝔑'	− 6,97761	− 7,13511	+ 6,34630	− 6,89194
L. $\frac{2(m+1)}{\mu}$	+ 9,61752	+ 9,61752	+ 9,61752	+ 9,61752
L. P. II.	− 6,59513	− 6,75263	+ 5,96382	− 6,50946
L. M'	− 0,30001	+	+ 0,42818	+ . . . .
L. μ^2	+ 3,61920	+	+ 3,61920	+ . . . .
L. P. I.	− 6,68081	+	+ 6,80898	+ . . . .
P. I.	− 0,0005	+ 0,0005	+ 0,0006	+ 0,0006
− P. II.	+ 0,0004	+ 0,0006	+ 0,0000	+ 0,0003
N'	− 0,0001	+ 0,0011	+ 0,0006	+ 0,0009
at 𝔑'	− 0,0010	− 0,0014	+ 0,0002	− 0,0008

Simili modo

Pro $4p+3q$.

	δ	d
L. M′	− 0,3000082	+ 0,42818
L. $\frac{2(m+1)}{\mu}$	+ 9,4765476	+ 9,47655
	− 9,7765558	+ 9,90473
$\frac{2(m+1)M'}{\mu}$	− 0,5978	+ 0,8030
− 𝔐′	+ 4,6106	+ 1,9953
Numer.	+ 4,0128	+ 2,7983
L. Num.	+ 0,60345	+ 0,44685
L. Den.	− 3,89138	− 3,89138
L. 𝔑′	− 6,71207	− 6,55547
L. $\frac{2(m+1)}{\mu}$	+ 9,47654	+ 9,47654
L. P. II.	− 6,18861	− 6,03202
L. M′	− 0,30001	+ 0,42818
L. μ^2	+ 3,90115	+ 3,90115
L. P. I.	+ 6,39886	+ 6,52703
P. I.	− 0,0003	+ 0,0003
− P. II.	+ 0,0002	+ 0,0001
N′	− 0,0001	+ 0,0004
at 𝔑′	− 0,0005	− 0,0004

§. 246.

§. 246.

Quatuor ergo determinationes hinc deductae sunt

$\delta = -0{,}0072 - 0{,}0010.\beta - 0{,}0014.\zeta + 0{,}0002.b - 0{,}0008.f.$

$d = -0{,}0060 - 0{,}0001.\beta + 0{,}0011.\zeta + 0{,}0006.b + 0{,}0009.f.$

$\zeta = -0{,}0005.\delta - 0{,}0004.d.$

$f = -0{,}0001. - 0{,}0001.\delta + 0{,}0004.d.$

ex quibus manifesto sequitur

$\delta = -0{,}0072 - 0{,}0010.\beta + 0{,}0002.b.$

$d = -0{,}0060 - 0{,}0001.\beta + 0{,}0006.b.$

§. 247.

Nunc demum primam columnam adgrediamur, pro qua notari conuenit, fore,

$\gamma = +0{,}1560 - 0{,}1700.\beta - 0{,}1409.b.$

$\delta = -0{,}0072 - 0{,}0010.\beta + 0{,}0002.b.$

$c = -0{,}4166 - 0{,}3102.\beta - 0{,}2620.b.$

$d = -0{,}0060 - 0{,}0001.\beta + 0{,}0006.b.$

$\gamma + \delta = +0{,}1488 - 0{,}1710.\beta - 0{,}1407.b.$

$\gamma - \delta = +0{,}1632 - 0{,}1690\,\beta - 0{,}1411.b.$

$c + d = -0{,}4226 - 0{,}3103.\beta - 0{,}2614\,b.$

$c - d = -0{,}4106 - 0{,}3101.\beta - 0{,}2626\,b.$

atque hinc colligimus pro angulo $3\,q$.

$\mathfrak{M}' = +0{,}1571 + 1{,}4076.\beta + 1{,}1703.b.$

$M' = +0{,}7749 + 1{,}1684.\beta + 0{,}9853.b.$

his addamus ipsos numeros $\mathfrak{M}$ et M, vt obtineamus

$\mathfrak{M} + \mathfrak{M}' = -1120{,}3140 + 1{,}4076.\beta + 1{,}1703.b.$

$M + M' = -871{,}5681 + 1{,}1684.\beta + 0{,}9853.b.$

pro quibus simpliciter scribamus $\mathfrak{M}$ et M.

§. 248.

His numeris iam quaeramus respondentes $\mathfrak{N}$ et N.

Pro angulo $3q$.

Term. absolutus.		β	b
Log. M	− 2,9403013	+ 0,0675915	+ 9,9935685
L. $\frac{2(m+1)}{\mu}$	+ 9,8275921	+ 9,8275921	+ 9,8275921
	− 2,7678934	+ 9,8951836	+ 9,8211606
$\frac{2(m+1)M}{\mu}$	− 585,9943	+ 0,7856	+ 0,6625
− $\mathfrak{M}'$	+ 1120,3140	− 1,4076	− 1,1703
Num.	+ 534,3197	− 0,6220	− 0,5078
L. Num.	+ 2,7278010	+ 9,7937904	− 9,7056927
L. Den.	− 3,1474551	− 3,1474511	− 3,1474511
L. $\mathfrak{N}$	− 9,5803459	+ 6,6463393	+ 6,5582416
	+ 9,8275921	+ 9,8275921	+ 9,8275921
L. P. II.	− 9,4079380	+ 6,4739314	+ 6,3858337
L. M	− 2,9403013	+ 0,0675915	+ 9,9935685
L. μ^2	+ 3,1990674	+ 3,1990674	+ 3,1990674
L. P. I.	− 9,7412339	+ 6,8685241	+ 6,7945011
P. I.	− 0,5511	+ 0,0007	+ 0,0006
− P. II.	+ 0,2558	− 0,0003	− 0,0002
N	− 0,2953	+ 0,0004	+ 0,0004
at $\mathfrak{N}$ =	− 0,3805	+ 0,0004	+ 0,0004

§. 249.

§. 249.

Cum nunc fieri debeat

$$\mathfrak{N} = \beta \text{ et } N = b$$

habebimus

$$\beta = -0,3805 + 0,0004.\beta + 0,0004.b.$$

$$b = -0,2953 + 0,0004.\beta + 0,0004.b.$$

ex posteriore statim fit

$$b = -0,2954 + 0,0004.\beta.$$

qui valor in priore substitutus dat

$$\beta = -0,3806 + 0,0004.\beta$$

ideoque

$$\beta = -0,3807 \text{ et } b = -0,2955.$$

§. 250.

His valoribus inuentis sequentes facile reperiuntur, vt sequitur:

$\gamma = +0,2623.$	$c = -0,2211.$
$\delta = -0,0068.$	$d = -0,0061.$
$\varepsilon = -0,0239.$	$e = -0,0071.$
$\zeta = +0,0000.$	$f = -0,0001.$

Conclusio.

§. 251.

En ergo completos valores nostrarum litterarum $\mathfrak{R}$ et R:

$$\begin{aligned}\mathfrak{R} = 0.\cos.q - 0{,}1908.\cos.(2p-q)\\ -0{,}2300.\cos.(2p+q)\\ -0{,}0482.\cos.(4p-q)\\ -0{,}0058.\cos.(4p+q)\\ -0{,}3807.\cos.3q + 0{,}2623.\cos.(2p-3q)\\ -0{,}0068.\cos.(2p+3q)\\ -0{,}0239.\cos.(4p-3q)\\ +0{,}0000.\cos.(4p+3q).\end{aligned}$$

$$\begin{aligned}R = +1{,}3662.\sin.q + 0{,}4255.\sin.(2p-q)\\ -0{,}0353.\sin.(2p+q)\\ -0{,}0377.\sin.(4p-q)\\ +0{,}0010.\sin.(4p+q)\\ -0{,}2955.\sin.3q - 0{,}2211.\sin.(2p-3q)\\ -0{,}0061.\sin.(2p+3q)\\ -0{,}0071.\sin.(4p-3q)\\ -0{,}0001.\sin.(4p+3q)\end{aligned}$$

CAPVT V.

CAPUT V.

EVOLUTIO AEQUATIONUM ORDINIS V. PRO LITTERIS $\mathfrak{S}$ ET S.

§. 252.

Partes annexae nostrarum aequationum ita in superioribus sunt exhibitae:

$$\text{I. } 0 = \ldots\ldots + \mathfrak{S}.\mathfrak{A} + S.\mathfrak{B}$$
$$- (\tfrac{9}{8} \operatorname{cos}. p + \tfrac{15}{8} \operatorname{cos}. 3p)(1 + 2\mathfrak{O} + \mathfrak{O}^2)$$
$$+ (\tfrac{3}{4} \operatorname{sin}. p + \tfrac{15}{4} \operatorname{sin}. 3p)(O + \mathfrak{O}O)$$
$$- (\tfrac{3}{8} \operatorname{cos}. p - \tfrac{15}{8} \operatorname{cos}. 3p) O^2.$$

$$\text{II. } 0 = \ldots\ldots + \mathfrak{S}.A + S.B$$
$$+ (\tfrac{3}{8} \operatorname{sin}. p + \tfrac{15}{8} \operatorname{sin}. 3p)(1 + 2\mathfrak{O} + \mathfrak{O}^2)$$
$$- (\tfrac{3}{4} \operatorname{cos}. p - \tfrac{15}{4} \operatorname{cos}. 3p)(O + \mathfrak{O}O)$$
$$+ (\tfrac{9}{8} \operatorname{sin}. p - \tfrac{15}{8} \operatorname{sin}. 3p) \mathfrak{O}^2.$$

In praecedentibus autem calculis abunde vidimus, terminos vbi $\mathfrak{O}$ et O ad duas dimensiones assurgunt, atque ob eandem rationem etiam partes minores litterateraterarum $\mathfrak{O}$ et $\mathfrak{O}$ sine sensibili errore negligi posse, vnde partes annexae cognitae pro prima aequatione erunt

$$-\left(\tfrac{9}{8}\,\text{cos.}\,p+\tfrac{15}{8}\,\text{cos.}\,3p\right)\left(1-0,01436.\,\text{cos.}\,2p\right)$$
$$+\left(\tfrac{3}{4}\,\text{sin.}\,p+\tfrac{15}{4}\,\text{sin.}\,3p\right)\left(+0,01021.\,\text{sin.}\,2p\right)$$

Pro altera vero aequatione

$$+\left(\tfrac{3}{8}\,\text{sin.}\,p+\tfrac{15}{8}\,\text{sin.}\,3p\right)\left(1-0,01436.\,\text{cos.}\,2p\right)$$
$$-\left(\tfrac{5}{4}\,\text{cos.}\,p-\tfrac{15}{4}\,\text{cos.}\,3p\right)\left(+0,01021.\,\text{sin.}\,2p\right).$$

§. 253.

Ex his igitur partibus cognitis ambas litteras $\mathfrak{M}$ et M colligamus:

	cos. p.	cos. $3p$.	cos. $5p$.
	− 1,12500	− 1,87500	
	+ 0,00807	+ 0,00807	
	+ 0,01347		+ 0,01347
	− 1,10346	− 1,86693	
	+ 0,00383	− 0,00383	
	+ 0,01913		− 0,01913
Ergo $\mathfrak{M}$	− 1,08050	− 1,87076	− 0,00566

sin. p.

Simili modo:

	$\sin. p.$	$\sin. 3p.$	
$\frac{3}{8}\sin. p + \frac{15}{8}\sin. 3p$	+0,37500	+1,87500	
$(\frac{3}{8}\sin. p + \frac{15}{8}\sin. 3p) - 0,014 . \cos. 2p$	+0,00538	−0,00538	
	−0,01346		−0,01346
	+0,36692	+1,86962	
$-(\frac{3}{8}\cos. p - \frac{15}{8}\cos. 3p) 0,0102 . \sin. 2p$	−0,00383	−0,00383	
	−0,01915		+0,01915
Ergo M =	+0,34394	+1,86579	+0,00569

§. 254.

Constituantur nunc elementa numerica pro his angulis:

ω	m	$3m$	$5m$
μ	12,36892	37,10676	61,84460
$L. 2(m+1) =$	1,4271258	1,4271258	1,4271258
$\text{Log } \mu =$	1,0923314	1,5694527	1,7913014
$L. \frac{2(m+1)}{\mu}$	0,3347944	9,8576731	9,6358244
$\text{Log. } \mu^2 =$	2,1846628	3,1389054	3,5826028
$\lambda - 2 =$	177,22893	177,22893	177,22893
$-\mu^2 =$	−152,98990	−1376,90950	−3824,74700
Denom.	+24,23903	−1199,68057	−3647,51807
Log.	+ 1,3845152	− 3,0780657	− 3,5619966

§. 254.

Nunc ergo faciamus calculum nostrum pro litteris 𝔑 et N.

	p	$3p$	$5p$
L. M	+ 9,5364827	+ 0,2708628	+ 7,7551123
L. $\frac{2(m+1)}{\mu}$	+ 0,3347944	+ 9,8576731	+ 9,6358244
	+ 9,8712771	+ 0,1285359	+ 7,3909367
$\frac{2(m+1)M}{\mu}$	+ 0,74349	+ 1,34442	+ 0,00246
− 𝔐	+ 1,08050	+ 1,87076	+ 0,00566
Numer.	+ 1,82399	+ 3,21518	+ 0,00812
L. Num.	+ 0,2610224	+ 0,5061694	+ 7,9095560
L. Den.	+ 1,3845152	− 3,0780657	− 3,5619966
L. 𝔑	+ 8,8765072	− 7,4281037	− 4,3475594
	+ 0,3347944	+ 9,8576731	+ 9,6358244
L. P. II.	+ 9,2113016	− 7,2857768	− 3,9833838
L. M	+ 9,5364827	+ 0,2708628	+ 7,7551123
L. μ^2	+ 2,1846627	+ 3,1389054	+ 3,5826028
L. P. I.	+ 7,3518200	+ 7,1319574	+ 4,1725095
P. I.	+ 0,00225	+ 0,00136	+ 0,00000
− P. II.	− 0,16267	+ 0,00193	+ 0,00000
N	− 0,16042	+ 0,00329	+ 0,00000
at 𝔑	+ 0,07525	− 0,00268	+ 0,00000

§. 255.

Quoniam tertia columna pro $\mathfrak{N}$ et N nihil praebuit, pro partibus incognitis posuisse sufficiet:

$$\mathfrak{S} = \beta . \cos . p + \gamma . \cos . 3 p .$$

$$S = b . \sin . p + c . \sin . 3 p .$$

Vnde litteras $\mathfrak{M}'$ et M' deriuemus:

Pro $\mathfrak{M}'$.

	$\cos . p .$	$\cos . 3 p .$
$\mathfrak{S} . - 9,22129 . \cos . 2 p$	$- 4,61064 . \beta$ $- 4,61064 . \gamma$	$- 4,61064 . \beta$
$S . - 3,99069 . \sin . 2 p$	$- 1,99534 . b$ $- 1,99534 . c$	$+ 1,99534 . b$

Pro M'.

	$\sin . p .$	$\sin . 3 p .$
$\mathfrak{S} . - 3,99069 . \sin . 2 p$	$- 1,99534 . \beta$ $+ 1,99534 . \gamma$	$- 1,99534 . \beta$
$S . + 5,36064 . \cos . 2 p$	$- 2,68032 . b$ $+ 2,68032 . c$	$+ 2,68032 . b$

§. 256.

Incipiamus a secunda columna seu angulo $3p$ et quaeramus litteras $\mathfrak{N}'$ et N':

Pro angulo $3p$.

	β.	b.
Log. M'	$-0,3000169$	$+0,4281866$
$L.\frac{2(m+1)}{\mu}$	$+9,8576731$	$+9,8576731$
	$-0,1576900$	$+0,2858597$
	$-1,43777$	$+1,93135$
$-\mathfrak{M}'$	$+4,61064$	$-1,99534$
Num.	$+3,17287$	$-0,06399$
Log.	$+0,5014522$	$-8,8061121$
L. den.	$-3,0780657$	$-3,0780657$
Log. $\mathfrak{N}'$	$-7,4233865$	$+5,7280464$
	$+9,8576731$	$+9,8576731$
L. P. II.	$-7,2810596$	$+5,5857195$
Log. M'	$-0,3000169$	$+0,4281866$
Log. μ^2	$+3,1389054$	$+3,1389054$
Log. P.I.	$-7,1611115$	$+7,2892812$
P. I.	$-0,00145$	$+0,00195$
$-$P. II.	$+0,00191$	$-0,00004$
N'	$+0,00046$	$+0,00195$
ad $\mathfrak{N}'$	$-0,00265$	$+0,00005$

vnde statim colliguntur hi valores:

$$\gamma = -0,00268 - 0,00265.\beta + 0,00005.b.$$
$$c = +0,00329 + 0,00046.\beta + 0,00195.b.$$

§. 257.

Pro prima ergo columna seu angulo p propter

$$\beta + \gamma = -0,00268 + 0,99734\beta + 0,00005.b.$$
$$\beta - \gamma = +0,00268 + 1,00265.\beta - 0,00005.b.$$

$b + c$

$$b + c = + 0,00329 + 0,00046.\beta + 1,00195.b.$$
$$b - c = - 0,00329 - 0,00046.\beta + 0,99804.b.$$

reperiemus sequentes valores

$$\mathfrak{M}' = + 0,00580 - 4,59932.\beta - 1,99943.b.$$
$$M' = + 0,00347 - 1,99567.\beta - 2,67505.b.$$

§. 258.

His ergo litteris analogas $\mathfrak{N}'$ et N' quaeramus more consueto: Pro angulo p.

	const.	β.	b.
L. M'	+7,5403295	−0,3000735	−0,4273319
L. $\frac{2(m+1)}{\mu}$	+0,3347944	+0,3347944	+0,3347944
	+7,8751239	−0,6348679	−0,7621263
$\frac{2(m+1)M'}{\mu}$	+0,00750	−4,31388	−5,78264
− $\mathfrak{M}'$	−0,00580	+4,59932	+1,99943
Numer.	+0,00170	+0,28544	−3,78321
L. Num.	+7,2304489	+9,4555148	−0,5778593
L. den.	+1,3845152	+1,3845152	+1,3845152
Log. $\mathfrak{N}'$	+5,8459337	+8,0709996	−9,1933441
	+0,3347944	+0,3347944	+0,3347944
L. P. II.	+6,1807281	+8,4057940	−9,5281385
Log. M'	+7,5403295	−0,3000735	−0,4273319
Log. μ^2	2,1846628	2,1846628	2,1846628
L. P. I.	+5,3556667	−8,1154107	−8,2426691
P. I.	+0,00003	−0,01304	−0,01750
− P. II.	−0,00015	−0,02546	+0,33740
N'	−0,00012	−0,03850	+0,31990
$\mathfrak{N}'$	+0,00007	+0,01778	−0,15608

§. 259.

Hinc igitur ob

$$\beta = \mathfrak{N} + \mathfrak{N}' \text{ et } b = N + N'$$

elicimus

$$\beta = +0,07532 + 0,01778.\,\beta - 0,15608.\,b.$$

$$b = -0,16054 - 0,03850.\,\beta + 0,31990.\,b.$$

atque hinc

$$0,98221.\,\beta = +0,07532 - 0,15608.\,b.$$

adeoque

$$\beta = +0,07668 - 0,15890.\,b. \text{ et}$$

$$b = -0,16349 + 0,32602.\,b. \text{ seu}$$

$$0,67397.\,b = -0,16349.$$

adeoque

$$b = -0,24258. \text{ et } \beta = +0,11523.$$

ac denique

$$\gamma = -0,00300. \text{ et } c = +0,00288.$$

§. 260.

Ecce ergo prope veros valores litterarum $\mathfrak{S}$ et S:

$$\mathfrak{S} = +0,11523.\,\text{cos.}\,p - 0,00300.\,\text{cos.}\,3\,p.$$

$$S = -0,24258.\,\text{sin.}\,p + 0,00288.\,\text{sin.}\,3\,p.$$

§. 261.

§. 261.

Hos vero valores 𝔖 et *S* tantum vt prope veros spectemus; ex iisque in partes minores litterarum 𝔄, 𝔅; A, B ductis litteras 𝔐'' et M'' eliciamus:

	cos. *p*.	cos. 3 *p*.
𝔖. + 0, 02644.	+ 0,03047	− 0,00008
𝔖. − 0, 10506. cos. 4 *p*.	+ 0,00016	− 0,06053
	+ 0,03063	− 0,06061
S. − 0, 08191. sin. 4 *p*.	− 0,00012	+ 0,00993
Ergo 𝔐'' =	+ 0,03051	− 0,05068

	sin. *p*.	sin. 3 *p*.
𝔖. − 0, 08191. sin. 4 *p*.	+ 0,00012	− 0,04719
S. − 0, 02724.	+ 0,00661	− 0,00008
	+ 0,00673	+ 0,04727
S. + 0, 06655. cos. 4 *p*.	− 0,00010	+ 0,00807
Ergo M'' =	+ 0,00663	− 0,03920

§. 262.

§. 262.

His iam numeris respondentes quaeramus $\mathfrak{N}''$ et N''.

	$p.$	$3p.$
L. M.''	+ 7,8215135	− 8,5932861
$L.\frac{2(m+1)}{\mu}$	+ 0,3347944	+ 9,8576731
	+ 8,1563079	− 8,4509592
$\frac{2(m+1)M''}{\mu}$	+ 0,01433	− 0,02825
$-\mathfrak{M}''$	− 0,03051	+ 0,05068
Numer.	− 0,01618	+ 0,02243
L. Num.	− 8,2089785	+ 8,3508293
L. Den.	+ 1,3845152	− 3,0780657
L. $\mathfrak{N}''$	− 6,8244633	− 5,2727636
$L.\frac{2(m+1)}{\mu}$	+ 0,3347944	+ 9,8576731
L. P. II	− 7,1592577	− 5,1304367
L. M''.	+ 7,8215135	− 8,5932861
L. μ^2	+ 2,1846628	+ 3,1389054
L. P. I.	+ 5,6368507	− 5,4543807
P. I.	+ 0,00004	− 0,00003
− P. II.	+ 0,00144	+ 0,00001
N''	+ 0,00148	− 0,00002
at $\mathfrak{N}''$	− 0,00067	− 0,00002

§. 263.

§. 263.

Hos ergo numeros insuper ipsis $\mathfrak{N}$ et N addi oportet; vnde ex §. 256. emergent hae aequationes:

$$\gamma = -0,00270. - 0,00265.\beta + 0,00005.b.$$
$$c = +0,00327. + 0,00046.\beta + 0,00195.b.$$

§. 264.

Pro prima ergo columna seu angulo p propter

$$\beta + \gamma = -0,00270 + 0,99734.\beta + 0,00005.b.$$
$$\beta - \gamma = +0,00270 + 1,00265.\beta - 0,00005.b.$$
$$b + c = +0,00327 + 0,00046.\beta + 1,00195.b.$$
$$b - c = -0,00327 - 0,00046.\beta + 0,99804.b.$$

reperiemus sequentes valores

$$\mathfrak{M}' = +0,00585 - 4,59932.\beta - 1,99943.b.$$
$$M' = +0,00348 - 1,99567.\beta - 2,67505.b.$$

vbi bina posteriora membra cum praecedentibus conveniunt. Primis autem partibus statim iungamus $\mathfrak{M} + \mathfrak{M}''$ et $M + M''$ vt habeamus:

$$\mathfrak{M} + \mathfrak{M}' + \mathfrak{M}'' = -1,04414 - 4,59932.\beta - 1,99943.b.$$
$$M + M' + M'' = +0,35405 - 1,99567.\beta - 2,67505.b.$$

pro quibus breuitatis gratia tantum litteras $\mathfrak{M}$ et M scribamus; iisque respondentes $\mathfrak{N}$ et N quaeramus. Quod quia pro partibus posterioribus §. 258 iam est factum; tantum pro numeris absolutis calculus instituatur:

Pro angulo p.

Log. M	$+9,5490646$
Log. $\frac{2(m+1)}{\mu}$	$+0,3347944$
	$+9,8838590$
$\frac{2(m+1)M}{\mu}$	$+0,76535$
$-\mathfrak{M}$	$+1,04414$
Numerat.	$+1,80949$
Log. Numerat.	$+0,2575563$
Log. Denom.	$+1,3845152$
L. $\mathfrak{N}$	$+8,8730411$
adde Log. $\frac{2(m+1)}{\mu}$	$+0,3347944$
L. Pars II.	$+9,2078355$
L. M	$+9,5490646$
L. μ^2	$+2,1846628$
L. P. I.	$+7,3644018$
P. I.	$+0,00231$
$-$ P. II.	$-0,16137$
N	$-0,15906$
at $\mathfrak{N}$	$+0,07465$

§. 263.

§. 265.

Hinc ergo pro §. 259. habebimus

$$\beta = +0,07465 + 0,01778.\beta - 0,15608.b.$$

$$b = -0,15906 - 0,03850.\beta + 0,31990.b.$$

vnde fit

$$\beta = +0,07600 - 0,15891.b.$$

hic vero facta substitutione prodit

$$b = -0,16199 + 0,32602.b.$$

vnde tandem concluditur

$b = -0,24035$ hincque $\beta = +0,11419.$

hincque

$\gamma = -0,00289$ et $c = +0,00285.$

§. 266.

En ergo veros valores, quos nobis haec correctio suppeditauit:

$$\mathfrak{S} = +0,11419.\operatorname{cos}.p - 0,00289.\operatorname{cos}.3p.$$

$$S = -0,24035.\operatorname{sin}.p + 0,00285.\operatorname{sin}.3p.$$

CAPVT VI.

EVOLVTIO AEQVATIONVM ORDINIS VI, PRO LITTERIS $\mathfrak{T}$ ET T.

§. 267.

Partes annexae nostrarum aequationum ita se habent:

$$
\begin{aligned}
\text{I. } 0 = \ldots\ldots & + \mathfrak{T}.\mathfrak{A} + T.\mathfrak{B} \\
& + \mathfrak{P}.\mathfrak{S}\, 2\mathfrak{C} + (\mathfrak{P}S + P\mathfrak{S})\mathfrak{D} + PS.2\mathfrak{E} \\
& - (\tfrac{5}{4}\operatorname{cos}.p + \tfrac{15}{4}\operatorname{cos}.3p)\mathfrak{P}(1+\mathfrak{O}) \\
& + (\tfrac{3}{4}\operatorname{sin}.p + \tfrac{15}{4}\operatorname{sin}.3p)P(1+\mathfrak{O}) \\
& + (\tfrac{3}{4}\operatorname{sin}.p + \tfrac{15}{4}\operatorname{sin}.3p)\mathfrak{P}O \\
& - (\tfrac{3}{4}\operatorname{cos}.p - \tfrac{15}{4}\operatorname{cos}.3p)PO.
\end{aligned}
$$

$$
\begin{aligned}
\text{II. } 0 = \ldots\ldots & + \mathfrak{T}.A + T.B \\
& + \mathfrak{P}.\mathfrak{S}\, 2C + (\mathfrak{P}S + P\mathfrak{S})D + PS.2E \\
& + (\tfrac{3}{4}\operatorname{sin}.p + \tfrac{15}{4}\operatorname{sin}.3p)\mathfrak{P}(1+\mathfrak{O}) \\
& - (\tfrac{3}{4}\operatorname{cos}.p - \tfrac{15}{4}\operatorname{cos}.3p)P(1+\mathfrak{O}) \\
& - (\tfrac{3}{4}\operatorname{cos}.p - \tfrac{15}{4}\operatorname{cos}.3p)\mathfrak{P}O \\
& + (\tfrac{5}{4}\operatorname{sin}.p - \tfrac{15}{4}\operatorname{sin}.3p)PO.
\end{aligned}
$$

ybi

vbi cum character huius ordinis sit a K, qui valet circiter $\frac{1}{7000}$, sufficit nostros valores numericos ad quatuor figuras decimales extendisse.

§. 268.

Ante omnia ergo hic producta a litteris $\mathfrak{O}$ et O libera euolui oportet; facto autem calculo reperitur.

$$\mathfrak{P}.\mathfrak{S} = +0,0678.\ \text{cos.}\ p-q+0,0093.\ \text{cos.}\ 3p-q. \\ +0,0566.\ \text{cos.}\ p+q-0,0016.\ \text{cos}\ 3p+q.$$

$$\mathfrak{P}S+P\mathfrak{S} = -0,0062.\ \text{sin.}\ p-q-0,0476.\ \text{sin.}\ 3p-q \\ -0,2347.\ \text{sin.}\ p+q+0,0044.\ \text{sin.}\ 3p+q \\ = \pi.$$

$$PS = +0,2913.\ \text{cos.}\ p-q-0,0523.\ \text{cos.}\ 3p-q \\ -0,2421.\ \text{cos.}\ p+q+0,0025.\ \text{cos.}\ 3p+q.$$

quae formulae vtrique aequationum nostrarum sunt communes.

§. 269.

Quoniam autem postrema membra non plane sunt eadem, eorum loco in prima aequatione scribamus $\mathfrak{P}'(1+\mathfrak{O})+P'.\ O.$ ita, vt sit

$$\mathfrak{P}' = +\mathfrak{P}(-2,2500.\ \text{cos.}\ p-3,7500.\ \text{cos.}\ 3p) \\ +P(+0,7500.\ \text{sin.}\ p+3,7500.\ \text{sin.}\ 3p)$$

$$P' = +\mathfrak{P}(+0,7500.\ \text{sin.}\ p+3,7500.\ \text{sin.}\ 3p) \\ +P(-0,7500.\ \text{cos.}\ p+3,7500.\ \text{cos.}\ 3p)$$

 qua-

quarum ergo formularum euolutio ita ſtabit:

	$\cos p-q$	$\cos p+q$	$\cos 3p-q$	$\cos 3p+q$
$\mathfrak{P}'(-2{,}2500.\cos p-3{,}7500.\cos 3p)$	$-1{,}3301$	$-1{,}4739$	$-2{,}0856$	$-1{,}8720$
$P(+0{,}7500.\sin p+3{,}7500.\sin 3p)$	$-0{,}9161$	$-0{,}0176$	$-3{,}6197$	$+3{,}7748$
	$-2{,}2462$	$-1{,}4915$	$-5{,}7053$	$+1{,}9028$

Ergo

$$\mathfrak{P}' = -2{,}2462.\cos p-q - 5{,}7053.\cos 3p-q \\ -1{,}4915.\cos p+q + 1{,}9028.\cos 3p+q$$

	$\sin p-q$	$\sin p+q$	$\sin 3p-q$	$\sin 3p+q$
$\mathfrak{P}'(+0{,}7500.\sin p+3{,}7500.\sin 3p)$	$+0{,}3005$	$+0{,}7279$	$+1{,}9452$	$+1{,}8740$
$P(-0{,}7500\cos p+3{,}7500.\cos 3p)$	$-0{,}6078$	$+1{,}5270$	$+3{,}9281$	$-3{,}7724$
	$-0{,}3073$	$+2{,}2549$	$+5{,}8733$	$-1{,}8984$

Ergo

$$P' = -0{,}3073.\sin p-q + 5{,}8733.\sin 3p-q \\ +2{,}2549.\sin p+q - 1{,}8984.\sin 3p+q$$

§. 270.

Simili modo in altera aequatione membra poſtrema ita repraeſententur:

$$\mathfrak{P}''\mathfrak{O} + P''(1+\mathfrak{O})$$

ita, vt ſit

$$\mathfrak{P}'' = \begin{cases} +\mathfrak{P}(-0{,}7500.\cos p + 3{,}7500.\cos 3p) \\ +P(+2{,}2500.\sin p - 3{,}7500.\sin 3p) \end{cases}$$

$P'' =$

$$P'' = \begin{cases} + \mathfrak{P}\,(+0,7500.\,\text{fin.}\,p + 3,7500.\,\text{fin.}\,3p) \\ + P\,(-0,7500.\,\text{cof.}\,p + 3,7500.\,\text{cof.}\,3p) \end{cases}$$

vbi patet, effe $P'' = P'$.

Prior ergo forma ita euoluatur:

	cof. $p-q$	cof. $p+q$	cof. $3p-q$	cof. $3p+q$
$\mathfrak{P}(-0,7500.\text{cof.}p+3,7500.\text{cof.}3p)$	$-0,4515$	$-0,0221$	$+1,8048$	$+1,8760$
$P(+2,2500.\text{fin.}p-3,7500.\text{fin.}3p)$	$-2,7194$	$+3,0316$	$+4,2353$	$-3,7700$
	$-3,1709$	$+3,0095$	$+6,0401$	$-1,8940$

Ergo

$$\mathfrak{P}'' = \begin{cases} -3,1709.\,\text{cof.}\,p-q + 6,0401.\,\text{cof.}\,3p-q \\ +3,0095.\,\text{cof}\,p+q - 1,8940.\,\text{cof.}\,3p+q \end{cases}$$

et

$$P'' = \begin{cases} -0,3073.\,\text{fin.}\,p-q + 5,8733.\,\text{fin.}\,3p-q \\ +2,2549.\,\text{fin.}\,p+q - 1,8984.\,\text{fin.}\,3p+q \end{cases}$$

§.

Iam ex partibus cognitis primae aequationis nu-
etiam multiplicatorum

	cos. $p-q$.
𝔓. 𝔖. 2. 537,6336.	+ 72,9060
𝔓. 𝔖. 2. 15,4426. cos. $2p$	+ 0,8741
	+ 0,1436
	+ 73,9237
𝔓. 𝔖. 2. 0,2659. cos. $4p$	− 0,0004
	+ 73,9233
π. + 21,9628. sin. $2p$	− 2,5772
	− 0,5227
	+ 70,8234
π. + 0,4065. sin. $4p$	+ 0,0009
	+ 70,8243
P. S. − 2. 268,7817.	− 156,5900
	− 85,7657
P. S. − 2. 7,7213. cos. $2p$	+ 1,8693
	+ 0,4038
	− 83,4926
P. S. − 2. 0,1680. cos. $4p$	− 0,0004
	− 83,4930

271.

meros $\mathfrak{M}$ sequenti modo colligamus, sumtis particulis minimis:

$\cos. p+q$	$\cos. 3p-q$	$\cos. 3p+q$
+ 60,8630	+ 10,0000	− 1,7205
+ 1,0470	+ 1,0470	+ 0,8741
− 0,0247		
+ 61,8853	+ 11,0470	− 0,8464
+ 0,0025	+ 0,0151	+ 0,0180
+ 61,8878	+ 11,0621	− 0,8284
− 0,0681	+ 0,0681	+ 2,5772
+ 0,0483		
+ 61,8680	+ 11,1302	+ 1,7488
− 0,0097	− 0,0477	− 0,0013
+ 61,8583	+ 11,0825	+ 1,7475
+ 130,1400	+ 28,1140	− 1,3439
+ 191,9983	+ 39,1965	+ 0,4036
− 2,2492	− 2,2491	+ 1,8693
− 0,0193		
+ 189,7298	+ 36,9474	+ 2,2729
+ 0,0088	+ 0,0407	− 0,0489
+ 189,7386	+ 36,9881	+ 2,2240

	$\text{cos.}\, p - q$
	$-83{,}4930$
$\mathfrak{P}'.\ 1$ - - - - - -	$-\ 2{,}2462$
	$-85{,}7392$
$\mathfrak{P}'. - 0{,}0072.\ \text{cos.}\ 2p$	$+\ 0{,}0054$
	$+\ 0{,}0205$
	$-85{,}7133$
$P' + 0{,}0102.\ \text{sin.}\ 2p$	$+\ 0.0464$
Ergo $\mathfrak{M} =$	$-85{,}6669$

§.

Pari modo ex partibus cognitis secundae aequa-

	$\text{sin.}\ p - q$
$\mathfrak{P}.\mathfrak{S}.\ 2.\ 10{,}9814.\ \text{sin.}\ 2p$	$+\ 0{,}6215$
	$-\ 0{,}1021$
	$+\ 0{,}5194$
$\mathfrak{P}.\mathfrak{S}.\ 2.\ 0{,}2032.\ \text{sin.}\ 4p$	$-\ 0{,}0003$
	$+\ 0{,}5191$
$\pi. - 537{,}563.$ - - - -	$+\ 3{,}3329$
	$+\ 3{,}8520$
$\pi. - 15{,}4426.\ \text{cos.}\ 2p$	$-\ 1{,}8122$
	$+\ 0{,}3675$
	$+\ 2{,}4073$

cos. $p+q$	cos. $3p-q$	cos. $3p+q$
$+189,7386$	$+36,9881$	$+2,2240$
$-\ \ 1,4915$	$-\ \ 5,7053$	$+1,9028$
$+188,2471$	$+31,2828$	$+4,1268$
$+\ \ 0,0081$	$+\ \ 0,0081$	$+0,0054$
$-\ \ 0,0069$		
$+168,2483$	$+31,2909$	$+4,1322$
$-\ \ 0,0113$	$+\ \ 0,0016$	$-0,0165$
$+188,2370$	$+31,2925$	$+4,1157$

272.

tionis colligamus valores numerorum M.

sin. $p+q$	sin. $3p-q$	sin. $3p+q$
$+\ \ 0,7445$	$+\ \ 0,7445$	$+0,6215$
$+\ \ 0,0176$		
$+\ \ 0,7621$		
$+\ \ 0,0024$	$+\ \ 0,0115$	$+0,0138$
$+\ \ 0,7645$	$+\ \ 0,7560$	$+0,6353$
$+126,1650$	$+25,2880$	$-2,3653$
$+126,9295$	$+26,0440$	$-\ 1,7300$
$-\ \ 0,0478$	$+\ \ 0,0478$	$+1,8122$
$-\ \ 0,0339$		
$+126,8478$	$+26,0918$	$+0,0822$

	ſin. $p - q$
	+ 2,4073
π. − 0,3360. coſ. $4p$	+ 0,0007
	+ 2,4080
P. S. 2. − 8,236. ſin. $2p$	+ 1,9939
	− 0,4308
	+ 3,9711
P. S. 2. − 0,152. ſin. $4p$	− 0,0004
	+ 3,9707
𝔓″. + 0,0102. ſin. $2p$	− 0,0155
	+ 3,9552
P″. 1 - - - - -	− 0,3073
	+ 3,6479
P″. − 0,0072. coſ. $2p$	− 0,0130
Ergo M	+ 3,6349

$\sin p+q$	$\sin. 3p-q$	$\sin 3p+q$
+ 126,8478	+ 26,0918	+ 0,0822
− 0,0080	− 0,0394	− 0,0010
+ 126,8398	+ 26,0524	+ 0,0812
− 2,3992	− 2,3992	+ 1,9939
+ 0,0206		
+ 124,4612	+ 23,6532	+ 2,0751
+ 0,0080	+ 0,0369	− 0,0444
+ 124,4692	+ 23,6901	+ 2,0307
− 0,0065	− 0,0161	+ 0,0153
+ 124,4627	+ 23,6740	+ 2,0460
+ 2,2549	+ 5,8733	− 1,8984
+ 126,7176	+ 29,5473	+ 0,1476
+ 0,0057	+ 0,0011	− 0,0081
+ 126,7233	+ 29,5484	+ 0,1395

§. 273.

Nunc ergo constituamus pro his quatuor angulis elementa numerica.

$\omega =$	$p - q$	$p + q$	$3p - q$	$3p + q$
$\mu =$	$m - n$	$m + n$	$3m - n$	$3m + n$
$\mu =$	$- 0,8871$	$+ 25,6250$	$+ 23,8507$	$+ 50,3628$
$L.2(m+1) =$	$+ 1,4271258$	$+ 1,4271258$	$+ 1,4271258$	$+ 1,4271258$
Log. $\mu =$	$- 9,9479726$	$+ 1,4086639$	$+ 1,3775011$	$+ 1,7021099$
$L.\frac{2(m+1)}{\mu}$	$- 1,4791532$	$+ 0,0184619$	$+ 0,0496247$	$+ 9,7250159$
Log. $\mu^2 =$	$+ 9,8959452$	$+ 2,8173278$	$+ 2,7550022$	$+ 3,4042198$
$\lambda - 2 =$	$+ 177,2289$	$+ 177,2289$	$+ 177,2289$	$+ 177,2289$
$- \mu^2 =$	$- 0,7869$	$- 656,6392$	$- 568,8600$	$- 2536,4000$
Denom.	$+ 176,4420$	$- 479,4103$	$- 391,6311$	$- 2359,1711$
Log.	$+ 2,2465970$	$- 2,6807071$	$- 2,5928759$	$- 3,3727648$

§. 274.

§. 274.

Instituatur ergo calculus pro numeris $\mathfrak{N}$ et N.

	$p-q$	$p+q$	$3p-q$	$3p+q$
L. M	+ 0,5604925	+ 2,1028452	+ 1,4705281	+ 9,1445742
L. $\frac{2(m+1)}{\mu}$	− 1,4791532	+ 0,0184619	+ 0,0496247	+ 9,7250159
	− 2,0396457	+ 2,1213071	+ 1,5201528	+ 8,8695901
$\frac{2(m+1)M}{\mu}$	− 109,5600	+ 132,2200	+ 33,1245	+ 0,0741
− $\mathfrak{M}$	+ 85,6669	− 188,2370	− 31,2925	− 4,1157
Numer.	− 23,8931	− 56,0170	+ 1,8320	− 4,0416
L.Num.	− 1,3782707	− 1,7483198	+ 0,2629255	− 0,6065533
L. Den.	+ 2,2465970	− 2,6807071	− 2,5928759	− 3,3727648
L. $\mathfrak{N}$	− 9,1316737	+ 9,0676127	− 7,6700496	+ 7,2337885
L. $\frac{2(m+1)}{\mu}$	− 1,4791532	+ 0,0184619	+ 0,0496247	+ 9,7250159
L. P. II.	+ 0,6108269	+ 9,0860746	− 7,7196743	+ 6,9588044
L. M	+ 0,5604925	+ 2,1028452	+ 1,4705281	+ 9,1445742
L. μ^2	+ 9,8959452	+ 2,8173278	+ 2,7550022	+ 3,4042198
L. P. I.	+ 0,6645473	+ 9,2855174	+ 8,7155259	+ 5,7403544
P. I.	+ 4,6190	+ 0,1930	+ 0,0519	+ 0,0000
−P. II.	− 4,0815	− 0,1219	+ 0,0052	− 0,0009
N	+ 0,5375	+ 0,0711	+ 0,0571	− 0,0009
at $\mathfrak{N}$	− 0,1354	+ 0,1168	− 0,0047	+ 0,0017

§. 275.

§. 275.

Pro partibus autem incognitis statuamus

$$\mathfrak{T} = +\beta.\ \mathrm{cos}.\ p-q + \delta.\ \mathrm{cos}.\ 3p-q$$
$$+\gamma.\ \mathrm{cos}.\ p+q + \varepsilon\ \mathrm{cos}.\ 3p+q.$$
$$T = +b.\ \mathrm{sin}.\ p-q + d.\ \mathrm{sin}.\ 3p-q$$
$$+c.\ \mathrm{sin}.\ p+q + e.\ \mathrm{sin}.\ 3p+q.$$

vnde facile deriuantur litterae $\mathfrak{M}'$ et M':

Pro litteris $\mathfrak{M}'$.

	$p-q$	$p+q$	$3p-q$	$3p+q$
$\mathfrak{T}. -9,2213.\ \mathrm{cos}.\ 2p$	$-4,6106.\ \gamma$	$-4,6106.\ \beta$	$-4,6106.\ \beta$	$-4,6106.\ \gamma$
	$-4,6106.\ \delta$	$-4,6106.\ \varepsilon$		
$T. -3,9907\ \mathrm{sin}.\ 2p$	$-1,9953.\ c$	$-1,9953.\ b$	$+1,9953.\ b$	$+1,9953.\ c$
	$-1,9953\ d$	$-1,9953.\ e$		

Pro litteris M'.

	$p-q$	$p+q$	$3p-q$	$3p+q$
$\mathfrak{T}. -3,9907\ \mathrm{sin}.\ 2p$	$-1,9953.\ \gamma$	$-1,9953.\ \beta$	$-1,9953.\ \beta$	$-1,9953.\ \gamma$
	$+1,9953.\ \delta$	$+1,9953.\ \varepsilon$		
$T. +5,3606.\ \mathrm{cos}.\ 2p$	$-2,6803.\ c$	$-2,6803.\ b$	$+2,6803.\ b$	$+2,6803.\ c$
	$+2,6803.\ d$	$+2,6803.\ e$		

§. 276.

§. 276.

Hinc calculum litterarum $\mathfrak{N}'$ et N' pro singulis noſtris angulis ſeorſim expediamus ac primo quidem :

Pro angulo $p - q$.

	γ	δ	c	d
L. M′	− 0,3000082	+ . . .	− 0,4281834	+ . . .
L. $\frac{2(m+1)}{\mu}$	− 1,4791532	− . . .	− 1,4791532	− . . .
	+ 1,7791614	− . . .	+ 1,9073366	− . . .
$\frac{2(m+1)M'}{\mu}$	+ 60,1400	− 60,1400	+ 80,7860	− 80,7860
− $\mathfrak{M}'$	+ 4,6106	+ 4,6106	+ 1,9953	+ 1,9953
Numer.	+ 64,7506	− 55,5294	+ 82,7813	− 78,7907
L Num.	+ 1,8112438	− 1,7443198	+ 1,9179307	− 1,8964766
L. den.	+ 2,2465970	+ 2,2465970	+ 2,2465970	+ 2,2465970
Log. $\mathfrak{N}'$	+ 9,5646468	− 9,4977228	+ 9,6713337	− 9,6498796
	− 1,4791532	− 1,4791532	− 1,4791532	− 1,4791532
L. P. II.	− 1,0438000	+ 0,9768760	− 1,1504869	+ 1,1290328
Log. M′	− 0,3000082	+ 0,3000082	− 0,4281834	+ 0,4281834
Log. μ^2	+ 9,8959452	+ 9,8959452	+ 9,8959452	+ 9,8959452
L. P. I.	− 0,4040630	+ 0,4040630	− 0,5322382	+ 0,5322382
P. I.	− 2,5355	+ 2,5355	− 3,4059	+ 3,4059
− P. II.	+ 11,0610	− 9,4815	+ 14,1415	− 13,4595
N′	+ 8,5255	− 6,9460	+ 10,7356	− 10,0536
$\mathfrak{N}'$	+ 0,3670	− 0,3146	+ 0,4692	− 0,4466

§. 277.

Simili modo calculum faciamus

Pro angulo $p + q$.

	β	ε	b	e
L. M′	−0,3000082	+0,3000082	−0,4281834	+ . . .
L. $\frac{2(m+1)}{\mu}$	+0,0184619	+ . . .	+0,0184619	+ . . .
	−0,3184701	+ . . .	−0,4466453	+ . . .
$\frac{2(m+1)M'}{\mu}$	−2,0819	+2,0819	−2,7967	+2,7967
$-\mathfrak{M}'$	+4,6106	+4,6106	+1,9953	+1,9953
Numer.	+2,5287	+6,6925	−0,8014	+4,7920
L. Num.	+0,4028973	+0,8255884	−9,9038493	+0,6805168
L. Den.	−2,6807071	−2,6807071	−2,6807071	−2,6807071
Log. $\mathfrak{N}'$	−7,7221902	−8,1448813	+7,2231422	−7,9998097
L. $\frac{2(m+1)}{\mu}$	+0,0184619	+0,0184619	+0,0184619	+0,0184619
L. P. II	−7,7406521	−8,1633432	+7,2416041	−8,0182716
L. M′	−0,3000082	+ . . .	−0,4281834	+ . . .
L. μ^2	+2,8173278	+ . . .	+2,8173278	+ . . .
L. P. I.	−7,4826804	+ . . .	−7,6108556	+ . . .
P. I.	−0,0030	+0,0030	−0,0041	+0,0041
− P. II.	+0,0055	+0,0146	−0,0017	+0,0104
N′	+0,0025	+0,0176	−0,0058	+0,0145
at $\mathfrak{N}'$	−0,0053	−0,0145	+0,0017	−0,0104

§. 278.

§. 278.

Pro binis reliquis angulis calculum eodem modo instituere licet:

Pro $3p-q$.

	β	b
Log. M'	− 0,3000082	+ 0,4281834
Log. $\frac{2(m+1)}{\mu}$	+ 0,0496247	+ 0,0496247
	− 0,3496329	+ 0,4778081
$\frac{2(m+1)M}{\mu}$	− 2,2368	+ 3,0048
$-\mathfrak{M}'$	+ 4,6106	− 1,9953
Numerat.	+ 2,3738	+ 1,0095
Log. Numerat.	+ 0,3754441	+ 0,0031063
Log. Denom.	− 2,5928759	− 2,5928759
L. $\mathfrak{N}'$	− 7,7825682	− 7,4102304
adde Log. $\frac{2(m+1)}{\mu}$	+ 0,0496247	+ 0,0496247
L. Pars II.	− 7,8321929	− 7,4598551
L. M'	− 0,3000082	+ 0,4281834
L. μ^2	+ 2,7550022	+ 2,7550022
L. P. I.	− 7,5450060	+ 7,6731812
P. I.	− 0,0035	+ 0,0047
− P. II.	+ 0,0068	+ 0,0029
N'	+ 0,0033	+ 0,0076
at $\mathfrak{N}'$	− 0,0061	− 0,0026

 Pro

Pro $3p+q$.

	γ	c
Log. M'	$-$ 0,3000082	$+$ 0,4281834
L. $\frac{2(m+1)}{\mu}$	$+$ 9,7250159	$+$ 9,7250159
	$-$ 0,0250241	$+$ 0,1531993
$\frac{2(m+1)M}{\mu}$	$-$ 1,0593	$+$ 1,4230
$-\mathfrak{M}'$	$+$ 4,6106	$-$ 1,9953
Num.	$+$ 3,5513	$-$ 0,5723
L. Num.	$+$ 0,5503874	$-$ 9,7576237
L. Den.	$-$ 3,3727648	$-$ 3,3727648
L. $\mathfrak{N}'$	$-$ 7,1776226	$+$ 6,3848589
L. $\frac{2(m+1)}{\mu}$	$+$ 9,7250159	$+$ 9,7250159
L. P. II.	$-$ 6,9026385	$+$ 6,1098748
L. M'	$-$ 0,3000082	$+$ 0,4281834
L. μ^2	$+$ 3,4042198	$+$ 3,4042198
L. P. I.	$-$ 6,8957884	$+$ 7,0239636
P. I.	$-$ 0,0008	$+$ 0,0011
$-$ P. II.	$+$ 0,0008	$-$ 0,0001
N'	$+$ 0,0000	$+$ 0,0010
at $\mathfrak{N}'$	$-$ 0,0015	$+$ 0,0002

§. 279.

§. 279.

Ecce ergo valores, ad quos his calculis sumus perducti:

$$\beta = -0,1354 + 0,3670.\gamma - 0,3146.\delta + 0,4692.c - 0,4466.d$$

$$b = +0,5375 + 8,5255.\gamma - 6,9460.\delta + 10,7356.c - 10,0536.d$$

$$\gamma = +0,1168 - 0,0053.\beta - 0,0145.\varepsilon + 0,0017.b - 0,0104.e$$

$$c = +0,0711 + 0,0025.\beta + 0,0176.\varepsilon - 0,0058.b + 0,0145.e$$

$$\delta = -0,0047 - 0,0061.\beta - 0,0026.b$$

$$d = +0,0571 + 0,0033.\beta + 0,0076.b$$

$$\varepsilon = +0,0017 - 0,0015.\gamma + 0,0002.c.$$

$$e = -0,0009 + 0,0000.\gamma + 0,0010.c.$$

§. 280.

Primum igitur quatuor postremos valores in quatuor prioribus substituamus; quod negotium pro prima ita peragatur:

$$-0,3146.\delta = +0,0015 + 0,0019.\beta + 0,0008.b.$$
$$-0,4466.d = -0,0255 - 0,0015.\beta - 0,0033.b.$$

$$\text{iunctim} = -0,0240 + 0,0004.\beta - 0,0025.b.$$

 Ergo

Ergo

$$\beta = -0,1594 + 0,0004.\beta - 0,0025.b$$
$$+ 0,3670.\gamma + 0,4692.c.$$

Simili modo pro secunda:

$$-6,9460.\delta = +0,0326 + 0,0424\,\beta + 0,0181.b.$$
$$-10,0536.d = -0,5740 - 0,0332\,\beta - 0,0764\,b.$$
$$\text{iunctim} = -0,5414 + 0,0092\,\beta - 0,0583.b.$$

Ergo

$$b = -0,0039 + 0,0092.\beta - 0,0583.b$$
$$+ 8,5255.\gamma + 10,7356.c.$$

Eodem modo pro tertia:

$$-0,0145.\varepsilon = 0,0000 + 0,0000\,\gamma + 0,0000.c.$$
$$-0,0104.e = 0,0000 + 0,0000\,\gamma + 0,0000.c.$$
$$\text{iunctim} = 0.$$

Ergo

$$\gamma = +0,1168 - 0,0053.\beta + 0,0017.b.$$

Similique modo

$$c = +0,0711 + 0,0025.\beta - 0,0058.b.$$

§. 281.

Nunc denuo hi bini posteriores valores in duobus prioribus substituti dabunt

Pro Prima:

$$+0,3670.\gamma = +0,0425 - 0,0019.\beta + 0,0006.b.$$
$$+0,4692.c = +0,0333 + 0,0011.\beta - 0,0027.b.$$
$$\text{iunctim} = +0,0758 - 0,0008.\beta - 0,0021.b.$$

Ergo $\beta = -0,0836 - 0,0004.\beta - 0,0046.b$

atque

atque hinc

$$\beta = -0,0836 - 0,0046.\, b.$$

Pari modo

Pro Secunda:

$$+8,5255.\,\gamma = +0,9958 - 0,0451.\,\beta + 0,0145.\,b.$$
$$+10,7356.\,c = +0,7633 + 0,0268.\,\beta - 0,0623.\,b.$$

$$\text{iunctim} = +1,7591 - 0,0183.\,\beta - 0,0478.\,b.$$
$$\text{adde} \quad -0,0039 + 0,0092.\,\beta - 0,0583.\,b.$$

$$\text{Ergo } b = +1,7552 - 0,0091.\,\beta - 0,1061.\,b.$$

Iam loco β substituatur valor modo inuentus scilicet

$$-0,0091.\,\beta = +0,0007.$$

ita, vt fiat

$$b = +1,7559 - 0,1061.\,b. \text{ siue}$$
$$1,1061.\,b = +1,7559.$$

atque hinc $b = +1,5875.$

§. 282.

Hoc valore inuento reliqui ita reperiuntur expressi:

$\beta = -0,0908.$	$b = +1,5875.$
$\gamma = +0,1199.$	$c = +0,0619.$
$\delta = -0,0082.$	$d = +0,0686.$
$\varepsilon = +0,0016.$	$e = -0,0008.$

§ 283.

Valores ergo prope veri hactenus inuenti sunt

$$\mathfrak{T} = -0,0908.\,\text{cos.}\,p-q - 0,0082.\,\text{cos.}\,3p-q$$
$$+0,1199.\,\text{cos.}\,p+q + 0,0016.\,\text{cos.}\,3p+q.$$
$$T = +1,5875.\,\text{sin.}\,p-q + 0,0686.\,\text{sin.}\,3p-q$$
$$+0,0619.\,\text{sin.}\,p+q - 0,0008.\,\text{sin.}\,3p+q.$$

§. 284

§.

Correctio ho-

Hos ergo valores prope veros ducamus in vt obtineamus

Pro

	$\cos. p - q$
$\mathfrak{T}. + 0,0264.$ - -	$- 0,0024$
$\mathfrak{T}. - 0,1051. \cos. 4p$	$- 0,0001$
	$- 0,0025$
$T. - 0,0819. \sin 4p$	
$\mathfrak{M}'' =$	$- 0,0025$

Pro

	$\sin. p - q$
$\mathfrak{T}. - 0,0819. \sin. 4p$	
$T. - 0,0272.$ - - - -	$- 0,0544$
$T. + 0,0665. \cos. 4p$	
Ergo M''	$- 0,0544$

284.

rum valorum.

partes minores litterarum $\mathfrak{A}$, $\mathfrak{B}$; A, B, litteras $\mathfrak{M}''$ et M''.

$\mathfrak{M}''$.

cos. $p+q$	cos. $3p-q$	cos. $3p+q$
+ 0,0032	− 0,0003	+ 0,0000
+ 0,0004	− 0,0063	+ 0,0048
+ 0,0036	− 0,0066	+ 0,0048
− 0,0028	− 0,0025	− 0,0649
+ 0,0008	− 0,0091	− 0,0601

M''.

sin. $p+q$	sin. $3p-q$	sin. $3p+q$
+ 0,0003	− 0,0049	+ 0,0037
− 0,0017	− 0,0019	
− 0,0014	− 0,0068	
− 0,0023	− 0,0021	− 0,0527
− 0,0037	− 0,0089	− 0,0490

 § 285.

§. 285.

His igitur quaeramus litteras analogas $\mathfrak{M}''$ et N'' more consueto:

ω	$p-q$	$p+q$	$3p-q$	$3p+q$
L. M.''	$-8,7355989$	$-7,56820$	$-7,94939$	$-8,69019$
L. $\frac{2(m+1)}{\mu}$	$-1,4791532$	$+0,01846$	$+0,04962$	$+9,72501$
	$+0,2147521$	$-7,58666$	$-7,99901$	$-8,41520$
$\frac{2(m+1)M''}{\mu}$	$+1,6396$	$-0,0039$	$-0,0100$	$-0,0260$
$-\mathfrak{M}''$	$+0,0025$	$-0,0008$	$+0,0091$	$+0,0601$
Numer.	$+1,6421$	$-0,0047$	$-0,0009$	$+0,0341$
L.Num.	$+0,21540$	$-7,67210$	$-6,95424$	$+8,53275$
L. Den.	$+2,24659$	$-2,68071$	$-2,59287$	$-3,37276$
L. $\mathfrak{N}''$	$+7,96881$	$+4,99139$	$+4,36137$	$-5,15999$
L. $\frac{2(m+1)}{\mu}$	$-1,47915$	$+0,01846$	$+0,04962$	$+9,72501$
L. P. II.	$-9,44796$	$+5,00985$	$+4,41099$	$-4,88500$
L. M''.	$-8,73559$	$-7,56820$	$-7,94939$	$-8,69019$
L. μ^2	$+9,89594$	$+2,81733$	$+2,75500$	$+3,40421$
L. P. I.	$-8,83965$	$-4,75087$	$-5,19439$	$-5,28598$
P. I.	$-0,0691$	$-0,0000$	$-0,0000$	$-0,0000$
$-$P. II.	$+0,2805$	$-0,0000$	$-0,0000$	$+0,0000$
N''	$+0,2114$	$+0,0000$	$+0,0000$	$+0,0000$
at $\mathfrak{N}''$	$+0,0093$	$+0,0000$	$+0,0000$	$+0,0000$

§. 286.

§. 286.

Hos ergo valores primae columnae insuper adjici oportet ad litteras β et b in superioribus aequationibus §. 279. vnde duae priores erunt

$$\beta = -0,1261. + 0,3670.\gamma - 0,3146.\delta$$
$$+ 0,4692.c - 0,4466.d.$$
$$b = + 0,7489 + 8,5255.\gamma - 6,9460.\delta$$
$$+ 10,7356.c - 10,0536.d.$$

reliqui vero valores omnes manent, vt ante. Vnde substitutis pro δ et d valoribus, hae duae primae aequationes fient:

$$\beta = -0,1501 + 0,0004.\beta - 0,0025.b.$$
$$+ 0,3670.\gamma + 0,4692.c.$$

et

$$b = + 0,2075 + 0,0092.\beta - 0,0583.b$$
$$+ 8,5255.\gamma + 10,7356.c.$$

Tertia vero et quarta manent, scilicet

$$\gamma = + 0,1168 - 0,0053.\beta + 0,0017.b.$$
$$c = + 0,0711 + 0,0025.\beta - 0,0058.b.$$

§. 287.

Hi duo valores postremi in prioribus substituti praebent:

$$\beta = -0,0743 - 0,0004\beta - 0,0046.b.$$

siue

$$\beta = -0,0743 - 0,0046 . b . \text{ et}$$

$$b = +1,9666 - 0,0091 . \beta - 0,1061 . b .$$

hinc denique pro β substituto valore prodibit

$$b = +1,9672 - 0,1061 . b .$$

vel $1,1061 . b = +1,9672$ adeoque

$$b = +1,7785 \text{ ideoque } \beta = -0,0824$$

reliquae vero litterae erunt

$\gamma = +0,1202$	$c = +0,0608$
$\delta = -0,0088$	$d = +0,0702$
$\varepsilon = +0,0015$	$e = -0,0008$

§. 288.

En ergo valores correctos nostrarum litterarum $\mathfrak{T}$ et T.

$$\mathfrak{T} = -0,0824 . \text{cos.} p - q - 0,0088 . \text{cos.} 3p - q$$
$$+ 0,1202 . \text{cos.} p + q + 0,0015 . \text{cos.} 3p + q$$

$$T = +1,7785 . \text{sin.} p - q + 0,0702 . \text{sin.} 3p - q$$
$$+ 0,0608 . \text{sin.} p + q - 0,0008 . \text{sin.} 3p + q .$$

Quodsi hinc denuo correctionem petere velimus, pro prima columna reperiemus

$$\mathfrak{M}'' = -0,0022 \text{ et } M'' = -0,0610 .$$

hincque

$$\mathfrak{N}'' = +0,0105 \text{ et } N'' = +0,2377 .$$

vnde

vnde

$$\beta = -0,1489 + 0,0004.\beta - 0,0025.b$$
$$+ 0,3670.\gamma + 0,4692.c.$$
$$b = +0,2338 + 0,0092.\beta - 0,0583.b$$
$$+ 8,5255.\gamma + 10,7356.c.$$

§. 289.

Cum ergo fit, vt ante

$$\gamma = +0,1168 - 0,0053.\beta + 0,0017.b.$$
$$c = +0,0711 + 0,0025.\beta - 0,0058.b.$$

his fubftitutis fiet

$$\beta = -0,0731 - 0,0004.\beta - 0,0046.b.$$

fiue

$$\beta = -0,0731 - 0,0046.b. \text{ et}$$
$$b = +1,9929 - 0,0091.\beta - 0,1061.b$$

vnde tandem concluditur

$$b = +1,8023 \text{ ideoque } \beta = -0,0814.$$

Reliqui autem valores ita fe habebunt:

$\gamma = +0,1204$	$c = +0,0604$
$\delta = -0,0088$	$d = +0,0712$
$\varepsilon = +0,0015$	$e = -0,0008.$

 §. 290.

Ex hac igitur geminata correctione non difficulter concludere licet, quantas mutationes etiam plures correctiones essent producturae; atque hinc sequentes constituimus litterarum $\mathfrak{T}$ et T valores veros:

$$\mathfrak{T} = -0,0813.\, cos.(p-q) - 0,0088.\, cos.(3p-q) + 0,1205.\, cos.(p+q) + 0,0015.\, cos.(3p+q)$$

$$T = +1,8056.\, fin.(p-q) + 0,0720.\, fin.(3p-q) + 0,0603.\, fin.(p+q) - 0,0008.\, fin.(3p+q).$$

CAPVT VII.

CAPVT VII.
EVOLVTIO AEQVATIONVM ORDINIS VII, PRO LITTERIS $\mathfrak{U}$ ET U.

§. 290.

Partes annexae harum aequationum ita se habent:

$$\text{I. } 0 = \ldots + \mathfrak{U}.\mathfrak{A} + U.\mathfrak{B}$$
$$+\left(\tfrac{3}{2}\cos.t + \tfrac{21}{4}\cos(2p-t) - \tfrac{3}{4}\cos.(2p+t)\right)(1+\mathfrak{O})$$
$$+\left(-\tfrac{21}{4}\sin.(2p-t) + \tfrac{3}{4}\sin.(2p+t)\right)O$$

$$\text{II. } 0 = \ldots + \mathfrak{U}.A + U.B$$
$$+\left(\tfrac{3}{2}\cos\,t - \tfrac{21}{4}\cos.(2p-t) + \tfrac{3}{4}\cos.(2p+t)\right)O$$
$$+\left(-\tfrac{21}{4}\sin.(2p-t) + \tfrac{3}{4}\sin.(2p+t)\right)(1+\mathfrak{O})$$

vbi membra cognita ita breuitatis gratia referamus:

Pro priore aequatione

$$\mathfrak{P}'.(1+\mathfrak{O}) + P'\,O.$$

Pro posteriore aequatione

$$\mathfrak{P}''.O + P''.(1+\mathfrak{O});$$ ita, vt sit

$$\mathfrak{P}' = +\tfrac{3}{2}\cos.t + \tfrac{21}{4}\cos.(2p-t) - \tfrac{3}{4}\cos.(2p+t)$$
$$P' = -\tfrac{21}{4}\sin.(2p-t) + \tfrac{3}{4}\sin.(2p+t)$$
$$\mathfrak{P}'' = +\tfrac{3}{2}\cos.t - \tfrac{21}{4}\cos.(2p-t) + \tfrac{3}{4}\cos.(2p+t)$$
$$P'' = -\tfrac{21}{4}\sin.(2p-t) + \tfrac{3}{4}\sin.(2p+t)$$

hinc $P'' = P'$; tum vero

$$1+\mathfrak{O} = +1{,}000024 - 0{,}007180.\cos.2p + 0{,}000006.\cos.4p.$$
$$O = +0{,}010212.\sin.2p + 0{,}000006.\sin.4p.$$

§.

Ex partibus igitur cognitis quaeramus va-

Pro

	cos. t	cos. $2p - t$
$\mathfrak{P}'.\,(1,000024)$	$+1,500036$	$+5,250126$
$\mathfrak{P}'.-0,007180.\cos.2p$	$-0,018844$	$-0,005385$
	$+0,002692$	
	$+1,483884$	$+5,244741$
$\mathfrak{P}'.\,0,000006.\cos.4p$		$-0,000002$
$P'.+0,010212.\sin.2p$	$+0,003829$	
	$-0,026803$	
	$+1,460910$	$+5,244739$
$P'.+0,000006.\sin.4p$		$+0,000002$
Ergo $\mathfrak{M}$	$+1,460910$	$+5,244741$

Pro

	sin. t	sin. $2p - t$
$\mathfrak{P}''.\,0,01022.\sin.2p$	$-0,026803$	$+0,007659$
	$-0,003829$	
	$-0,030632$	
$\mathfrak{P}''.+0,00006.\sin.4p$		$+0,000002$
		$+0,007661$
$P''.\,1,000024.$		$-5,250126$
$P''.-0,007180.\cos.2p$	$-0,018844$	
$P''.\,0,000006.\cos.4p$	$-0,002692$	$-0,000002$
Ergo M	$-0,052168$	$-5,242467$

291.
lores litterarum nostrarum 𝔐 et M:

𝔐.

cos. $2p+t$	cos. $4p-t$	cos. $4p+t$
−0,750018		
−0,005385	−0,018844	+0,002692
−0,755403	−0,018844	+0,002692
+0,000015	+0,000005	+0,000005
	+0,026803	−0,003829
−0,755388	+0,007964	−0,001132
−0,000016		
−0,755404	+0,007964	−0,001132

M.

sin. $2p+t$	sin. $4p-t$	sin. $4p+t$
+0,007659	−0,027803	+0,003829
−0,000016	+0,000004	+0,000004
+0,007643	−0,027799	+0,003833
+0,750018	−0,018844	
		−0,002692
+0,000016		
+0,757677	−0,046643	+0,001141

§.

Constituamus iam elementa

ω	t	$2p-t$
$\mu =$	1.	$2m-1$
$\mu =$	1.	23, 73784
$L.\ 2(m+1) =$	1, 4271258	1, 4271258
$\text{Log.}\ \mu =$	0, 0000000	1, 3754411
$\text{Log.}\ \frac{2(m+1)}{\mu} =$	1, 4271258	0, 0516847
$\text{Log.}\ \mu^2 =$	0, 0000000	2, 7508822
$\lambda - 2 =$	177,228928	177,228928
$\mu^2 =$	− 1,000000	-563,485000
Denom. $=$	+176,228928	-386,256072
Log. $=$	+ 2,2460771	-2,5868753

292.

numerica pro his angulis:

$2p+t$	$4p-t$	$4p+t$
$2m+1$	$4m-1$	$4m+1$
25, 73784	48, 47568	50, 47568
1, 4271258	1, 4271258	1, 4271258
1, 4105719	1, 6855239	1, 7030823
0, 0165539	9, 7416019	9, 7240435
2, 8211438	3, 3710478	3, 4061646
177,228928	177,228928	177,228928
-662,435460	-2349,892000	-2547,790000
-485,206532	-2172,663072	-2370,561072
-2,6859265	-3,3369924	-3,3748511

§. 293.

Hic iterum primam columnam tantisper seponamus & pro reliquis litteras $\mathfrak{N}$ et N computemus:

	$2p-t$	$2p+t$
L. M	− 0,7195356	+ 9,8794842
L. $\frac{2(m+1)}{\mu}$	+ 0,0516847	+ 0,0165539
	− 0,7712203	+ 9,8960381
$\frac{2(m+1)M}{\mu}$	− 5,905006	+ 0,787114
− $\mathfrak{M}$	− 5,244741	+ 0,755404
Num.	− 11,149747	+ 1,542518
L. Num.	− 1,0472632	+ 0,1882302
L. Den.	− 2,5868753	− 2,6859265
∴ L. $\mathfrak{N}$	+ 8,4603879	− 7,5023037
L. $\frac{2(m+1)}{\mu}$	+ 0,0516847	+ 0,0165539
L. P. II.	+ 8,5120726	− 7,5188576
L. M	− 0,7195356	+ 9,8794842
L. μ^2	+ 2,7508822	+ 2,8211438
L. P. I.	− 7,9686534	+ 7,0583404
P. I.	− 0,009304	+ 0,001144
− P. II.	− 0,032514	+ 0,003303
N =	− 0,041818	+ 0,004447
at $\mathfrak{N}$ =	+ 0,028866	− 0,003179

Simili

Simili modo:

	$3p-t$	$3p+t$
Log. M	$-8,6687865$	$+7,0572856$
$L.\frac{2(m+1)}{\mu}$	$+9,7416019$	$+9,7240435$
	$-8,4103884$	$+6,7813291$
$\frac{2(m+1)M}{\mu}$	$-0,025727$	$+0,000604$
$-\mathfrak{M}$	$-0,007964$	$+0,001132$
Num.	$-0,033691$	$+0,001736$
Log.	$-8,5275139$	$+7,2395497$
L. den.	$-3,3369924$	$-3,3748511$
Log. $\mathfrak{N}$	$+5,1905215$	$-3,8546986$
$L.\frac{2(m+1)}{\mu}$	$+9,7416019$	$+9,7240435$
L. P. II.	$+4,9321234$	$-3,5787421$
Log. M	$-8,6687865$	$+7,0572856$
Log. μ^2	$+3,3710478$	$+3,4061646$
Log. P.I.	$-5,2977387$	$+3,6511210$
P. I.	$-0,000020$	$+0,000000$
$-$P. II.	$-0,000009$	$+0,000000$
N	$-0,000029$	$+0,000000$
$\mathfrak{N}$	$+0,000016$	$-0,000000$

§.

Pro partibus autem

$$\mathfrak{U} = \beta.\ \text{cos}.\ t + \gamma.\ \text{cos}.\ 2p - t + \delta.\ \text{cos}\ 2p + t.$$

$$U = b.\ \text{sin}.\ t + c.\ \text{sin}.\ 2p - t + d.\ \text{sin}.\ 2p + t.$$

vnde quaeramus

Pro

	$\text{cos}.\ t$	$\text{cos}.\ 2p - t$
$\mathfrak{U}.-9{,}221291.\ \text{cos}.\ 2p.$	$-\ 4{,}610645.\ \gamma$	$-\ 4{,}610645.\ \beta$
	$-\ 4{,}610645.\ \delta$	$-\ 4{,}610645.\ \varepsilon$
$U.-3{,}990696.\ \text{sin}.\ 2p.$	$-\ 1{,}995348.\ c$	$-\ 1{,}995348.\ b$
	$-\ 1{,}995348.\ d$	$-\ 1{,}995348.\ e$

Pro

	$\text{sin}.\ t$	$\text{sin}.\ 2p - t$
$\mathfrak{U}.-3{,}990696.\ \text{sin}.\ 2p.$	$-\ 1{,}995348.\ \gamma$	$-\ 1{,}995348.\ \beta$
	$+\ 1{,}995348.\ \delta$	$+\ 1{,}995348.\ \varepsilon$
$U.+5{,}360678.\ \text{cos}.\ 2p.$	$-\ 2{,}680322.\ c$	$-\ 2{,}680322.\ b$
	$+\ 2{,}680322.\ d$	$+\ 2{,}680322.\ e$

294.

incognitis statuamus:

$+ \varepsilon.\ \text{cos}.\ 4p - t + \zeta.\ \text{cos}.\ 4p + t.$

$+ e.\ \text{sin}.\ 4p - t + f.\ \text{sin}.\ 4p + t.$

numeros $\mathfrak{M}'$ et M'.

$\mathfrak{M}'$.

cos. $2p + t$	cos. $4p - t$	cos. $4p + t$
$-4,610645.\ \beta$	$-4,610645.\ \gamma$	$-4,610645.\ \delta$
$-4,610645.\ \zeta$		
$+1,995348.\ b$	$+1,995348.\ c$	$+1,995348.\ d$
$-1,995348.\ f$		

M'.

sin. $2p + t$	sin. $4p - t$	sin. $4p + t$
$-1,995348.\ \beta$	$-1,995348.\ \gamma$	$-1,995348.\ \delta$
$+1,995348.\ \zeta$		
$+2,680322.\ b$	$+2,680322.\ c$	$+2,680322.\ d$
$+2,680322.\ f$		

§. 295.

§. 295.

Seposita prima columna calculum pro secunda et quarta primum expediamus:

Pro $2p - t$.

	β	ε	b	e
L. M'	$-0{,}3000082$	+ - - -	$-0{,}4281834$	+ - - -
L. $\frac{2(m+1)}{\mu}$	$+0{,}0516847$	+ - - -	$+0{,}0516847$	+ - - -
	$-0{,}3516929$	+ - - -	$-0{,}4798681$	+ - - -
$\frac{2(m+1)M'}{\mu}$	$-2{,}247465$	$+2{,}247465$	$-3{,}019035$	$+3{,}019035$
$-\mathfrak{M}'$	$+4{,}610645$	$+2{,}610645$	$+1{,}995348$	$+1{,}995348$
Numer.	$+2{,}363180$	$+4{,}858110$	$-1{,}023687$	$+5{,}014383$
L.Num.	$+0{,}3735005$	$+0{,}6864664$	$-0{,}0101642$	$+0{,}7002190$
L. Den.	$-2{,}5868753$	$-2{,}5868753$	$-2{,}5868753$	$-2{,}5868753$
L. $\mathfrak{N}'$	$-7{,}7866252$	$-8{,}0995911$	$+7{,}4232889$	$-8{,}1133437$
L. $\frac{2(m+1)}{\mu}$	$+0{,}0516847$	$+0{,}0516847$	$+0{,}0516847$	$+0{,}0516847$
L. P. II.	$-7{,}8383099$	$-8{,}1512758$	$+7{,}4749736$	$-8{,}1650284$
L. M'	$-0{,}3000082$	+ - - -	$-0{,}4281834$	+ - - -
L. μ^2	$+2{,}7508822$	+ - - -	$+2{,}7508822$	+ - - -
L. P. I.	$-7{,}5491260$	+ - - -	$-7{,}6773012$	+ - - -
P. I.	$-0{,}003541$	$+0{,}003541$	$-0{,}004757$	$+0{,}004757$
$-$P. II.	$+0{,}006891$	$+0{,}014167$	$-0{,}002985$	$+0{,}014623$
N'	$+0{,}003350$	$+0{,}017708$	$-0{,}007742$	$+0{,}019380$
at $\mathfrak{N}'$	$-0{,}006118$	$-0{,}012578$	$+0{,}002650$	$-0{,}012982$

Simili

Simili modo:

Pro $4p-t$.

	γ	c
Log. M'	$-0{,}3000082$	$+0{,}4281834$
Log. $\frac{2(m+1)}{\mu}$	$+9{,}7416019$	$+9{,}7416019$
	$-0{,}0416101$	$+0{,}1697853$
$\frac{2(m+1)M'}{\mu}$	$-1{,}100550$	$+1{,}478370$
$-\mathfrak{M}'$	$+4{,}610645$	$+4{,}610645$
Numerat.	$+3{,}510095$	$+6{,}089015$
Log. Numerat.	$+0{,}5453195$	$+0{,}7845467$
Log. Denom.	$-3{,}3369924$	$-3{,}3369924$
L. $\mathfrak{N}'$	$-7{,}2083271$	$-7{,}4475543$
adde Log. $\frac{2(m+1)}{\mu}$	$+9{,}7416019$	$+9{,}7416019$
L. Pars II.	$-6{,}9499290$	$-7{,}1891562$
L. M'	$-0{,}3000082$	$+0{,}4281834$
L. μ^2	$+3{,}3710478$	$+3{,}3710478$
L. P. I.	$-6{,}9289604$	$+7{,}0571356$
P. I.	$-0{,}000849$	$+0{,}001141$
$-$ P. II.	$+0{,}000891$	$+0{,}001546$
N'	$+0{,}000042$	$+0{,}002687$
at $\mathfrak{N}'$	$-0{,}001616$	$-0{,}002802$

§. 296.

Hinc ergo colligimus quatuor sequentes determinationes:

$$\gamma = +0,028866 - 0,006118.\beta - 0,012578.\varepsilon + 0,002650.b - 0,012982.e$$

$$c = -0,041818 + 0,003350.\beta + 0,017708.\varepsilon - 0,007742.b + 0,019380.e$$

$$\varepsilon = +0,000016 - 0,001616.\gamma - 0,002802.c.$$

$$e = -0,000029 + 0,000042.\gamma + 0,002687.c.$$

qui bini posteriores valores in prioribus substituti praebent:

Pro prima

$$-0,012578.\varepsilon = +0,000020.\gamma + 0,000035.c.$$

$$-0,012982.e = \qquad\qquad -0,000035.c.$$

iunctim $= +0,000020.\gamma.$

Ergo

$$\gamma = +0,028866 - 0,006118.\beta + 0,002650.b + 0,000020.\gamma$$

atque hinc

$$\gamma = +0,028867 - 0,006118.\beta + 0,002650.b.$$

Pro secunda

$$+0,017708.\varepsilon = -0,000028.\gamma - 0,000050.c.$$

$$+0,019380.e = +0,000001.\gamma + 0,000052.c.$$

iunctim $= -0,000027.\gamma + 0,000002.c.$

Ergo

$$c = -0,041818 + 0,003350.\beta - 0,007742.b - 0,000027.\gamma + 0,000002.c.$$

ideoque

$$c = -0,041818 + 0,003350.\beta - 0,007742.b.$$

§. 297.

§. 297.

Simili modo pro angulis $2p+t$ et $4p+t$ calculum faciamus:

Pro $2p+t$.

	β	ζ	b	f
L. M′	− 0,3000082	+ - - -	+ 0,4281834	+ - - -
L. $\frac{2(m+1)}{\mu}$	+ 0,0165539	+ - - -	+ 0,0165539	+ - - -
	− 0,3165621	+ - - -	+ 0,4447373	+ - - -
$\frac{2(m+1)M'}{\mu}$	− 2,072824	+ 2,072824	+ 2,784433	+ 2,784433
− $\mathfrak{M}'$	+ 4,610645	+ 4,610645	− 1,995348	+ 1,995348
Numer.	+ 2,537821	+ 6,683469	+ 0,789085	+ 4,779781
L. Num	+ 0,4044574	+ 0,8250039	+ 9,8971210	+ 0,6794097
L. Den.	− 2,6859265	− 2,6859265	− 2,6859265	− 2,6859265
L. $\mathfrak{N}'$	− 7,7185309	− 8,1390774	− 7,2111945	− 7,9934832
L. $\frac{2(m+1)}{\mu}$	+ 0,0165539	+ 0,0165539	+ 0,0165539	+ 0,0165539
L. P. II	− 7,7350848	− 8,1556313	− 7,2277484	− 8,0100371
L. M′.	− 0,3000082	+ - - -	+ 0,4281834	+ - - -
L. μ^2	+ 2,8211438	+ - - -	+ 2,8211438	+ - - -
L. P. I.	− 7,4788644	+ - - -	+ 7,6070396	+ - - -
P. I.	− 0,003012	+ 0,003012	+ 0,004046	+ 0,004046
− P. II.	+ 0,005434	+ 0,014310	+ 0,001689	+ 0,010233
N′	+ 0,002422	+ 0,017322	+ 0,005735	+ 0,014279
at $\mathfrak{N}'$	− 0,005231	− 0,013774	− 0,001626	− 0,009850

Simili modo:

Pro $4p+t$.

	δ	d
Log. M′	− 0,3000082	+ 0,4281834
L. $\frac{2(m+1)}{\mu}$	+ 9,7240435	+ 9,7240435
	− 0,0240517	+ 0,1522269
$\frac{2(m+1)M}{\mu}$	− 1,056925	+ 1,419800
− 𝔐′	+ 4,610645	− 1,995348
Num.	+ 3,553720	− 0,575548
L. Num.	+ 0,5506808	− 9,7600831
L. Den.	− 3,3748511	− 3,3748511
L. 𝔑′	− 7,1758297	+ 6,3852320
L. $\frac{2(m+1)}{\mu}$	+ 9,7240435	+ 9,7240435
L. P. II.	− 6,8998732	+ 6,1092755
L. M′	− 0,3000082	+ 0,4281834
L. μ^2	+ 3,4061646	+ 3,4061646
L. P. I.	− 6,8938436	+ 7,0220188
P. I.	− 0,000782	+ 0,001052
− P. II.	+ 0,000792	− 0,000129
N′	+ 0,000010	+ 0,000923
at 𝔑′	− 0,001500	+ 0,000243

§. 298.

§. 298.

Quatuor ergo determinationes hinc deducuntur:

$$\delta = -0,003179 - 0,005231\,\beta - 0,013774.\zeta - 0,001626.b - 0,009850.f.$$

$$d = +0,004447 + 0,002422\,\beta + 0,017322.\zeta + 0,005735.b + 0,014279.f.$$

$$\zeta = * - 0,001500\,\delta + 0,000243.d.$$

$$f = * + 0,000010.\delta + 0,000923.d.$$

vnde manifesto sequitur, fore

$$\delta = -0,003179 - 0,005231.\beta - 0,001626.b.$$

$$d = +0,004447 + 0,002422\,\beta + 0,005785.b$$

$$\zeta = * - 0,001500\,\delta + 0,000243.d$$

$$f = * + 0,000010\,\delta + 0,000923.d.$$

§. 299.

Nunc tandem adgrediamur primam columnam anguli t et cum pro ea fuerit

$$\mathfrak{M}' = -4,610645(\gamma+\delta) - 1,995348(c+d) \text{ ob}$$

$$\gamma+\delta = +0,025688 - 0,011349.\beta + 0,001024.b$$

$$c+d = -0,037371 + 0,005772.\beta - 0,002007.b$$

habebimus

$$\mathfrak{M} + \mathfrak{M}' = +1,417034 + 0,040810.\beta - 0,000716.b.$$

Pari modo cum sit

$$M' = -1,995348(\gamma-\delta) - 2,6803227(c-d) \text{ ob}$$

$$\gamma-\delta = +0,032046 - 0,000887.\beta + 0,004276.b \text{ et}$$

$$c-d = -0,046265 + 0,000928.\beta - 0,013477.b.$$

inuenimus

$$M + M' = +0,007895 - 0,000717.\beta + 0,027592.b.$$

§. 300.

At vero breuitatis gratia loco $M + M'$ et $\mathfrak{M} + \mathfrak{M}'$ scribamus simpliciter M et $\mathfrak{M}$ et quaeramus litteras respondentes $\mathfrak{N}$ et N.

Pro angulo *t*.

	const.	β.	*b*.
L. M	+ 7,8973521	− 6,8555192	+ 8,4407832
L. $\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 1,4271258	+ 1,4271258
	+ 9,3244779	− 8,2826450	+ 9,8679090
$\frac{2(m+1)M'}{\mu}$	+ 0,211095	− 0,019171	+ 0,737750
− $\mathfrak{M}$	− 1,417034	− 0,040810	+ 0,000716
Numer.	− 1,205939	− 0,059981	+ 0,738466
L. Num.	− 0,0813258	− 8,7780137	+ 9,8683305
L. Den.	+ 2,2460771	+ 2,2460771	+ 2,2460771
Log. $\mathfrak{N}$	− 7,8352487	− 6,5319366	+ 7,6222534
L. $\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 1,4271258	+ 1,4271258
L. P. II.	− 9,2623745	− 7,9590624	+ 9,0493792
P. I.	+ 0,007895	− 0,000717	+ 0,027592
− P. II.	+ 0,182970	+ 0,009100	− 0,112050
N	+ 0,190865	+ 0,008383	− 0,084458
at $\mathfrak{N}$	− 0,006843	− 0,000340	+ 0,004191

§. 301.

Hinc vero fieri debet $\mathfrak{N} = \beta$ et $N = b$. ex quo habebimus

$\beta = -0,006843 - 0,000340.\ \beta + 0,004191.\ b.$

siue $\beta = -0,006841 + 0,004190\ b$. et

$b = +0,190865 + 0,008383.\ \beta - 0,084458.\ b$

hinc

$b = +0,190807 - 0,084424.\ b.$

consequenter

$b = +0,175950.$

hincque retro

$\beta = -0,006106.$ et

$\gamma = +0,029370.$	$c = -0,043201.$
$\delta = -0,003433.$	$d = +0,005443.$
$\varepsilon = +0,000046.$	$e = -0,000143.$
$\zeta = -0,000004.$	$f = +0,000005.$

sicque valores prope veri hactenus inuenti erunt:

$$\begin{aligned} \mathfrak{u} = -0,006106.\ \cos. t + 0,029370.\ \cos.\ 2p - t \\ - 0,003433.\ \cos.\ 2p + t \\ + 0,000046.\ \cos.\ 4p - t \\ - 0,000004.\ \cos.\ 4p + t. \end{aligned}$$

$$\begin{aligned} U = +0,175950.\ \sin. t - 0,043201.\ \sin.\ 2p - t \\ + 0,005443.\ \sin.\ 2p + t \\ - 0,000143.\ \sin.\ 4p - t \\ + 0,000005.\ \sin.\ 4p + t. \end{aligned}$$

Cor-

§.

Correctio ho-

Ducantur isti valores in partes minimas facto- litterae $\mathfrak{N}''$ et N''

Pro

	cos. t	cos. $2p-t$
U. $\mathfrak{A}^{(II)}$	$-0{,}000161$	$+0{,}000777$
U. $\mathfrak{A}^{(III)}$		$+0{,}000181$
		$+0{,}000958$
U. $\mathfrak{B}^{(II)}$		$-0{,}000223$
$\mathfrak{M}''$	$-0{,}000161$	$+0{,}000735$

Pro

	sin. t	sin. $2p-t$
U. $A^{(II)}$		$+0{,}000141$
U. $B^{(II)}$	$-0{,}004791$	$+0{,}001176$
		$+0{,}001317$
U. $B^{(III)}$		$-0{,}000181$
M''	$-0{,}004791$	$+0{,}001136$

302.

rum valorum.

rum 𝔄, A, 𝔅, B; vt inde obtineantur sequenti modo:

𝔐''.

cos. $2p+t$	cos. $4p-t$	cos. $4p+t$
− 0,000091		
− 0,001544	+ 0,000321	+ 0,000321
− 0,001635		
+ 0,001769	− 0,007203	+ 0,007203
+ 0,000134	− 0,006882	+ 0,007524

M''.

sin. $2p+t$	sin. $4p-t$	sin. $4p+t$
− 0,001202	+ 0,000250	+ 0,000250
− 0,000148		
− 0,001350	+ 0,000250	+ 0,000250
+ 0,001437	− 0,005854	+ 0,005854
+ 0,000087	− 0,005604	+ 0,006104

§. 303.

Seclusa iterum prima columna pro reliquis quaerantur litterae $\mathfrak{N}''$ et N''.

	$2p-t$	$2p+t$	$4p-t$	$4p+t$
L. M''	−7,0553783	+5,9395193	−7,7484981	+7,7856145
L. $\frac{2(m+1)}{\mu}$	+0,0516847	+0,0165539	+9,7416019	+9,7240435
	−7,1070630	+5,9560732	−7,4901000	+7,5096580
	−0,001279	+0,000090	−0,003091	+0,003233
$-\mathfrak{M}''$	+0,000735	−0,000134	+0,006892	−0,007524
Numer.	−0,000544	−0,000044	+0,003801	−0,004291
L Num.	−6,7355989	−5,6434527	+7,5798979	−7,6325585
L. den.	−2,5868753	−2,6859265	−3,3369924	−3,3748511
Log. $\mathfrak{N}''$	+4,1487236	+2,9575262	−4,2429055	+4,2577074
$\frac{2(m+1)}{\mu}$	+0,0516847	+0,0165539	+9,7416019	+9,7240435
L. P. II.	+4,2004083	+2,9740801	−3,9845074	+3,9817509
Log M''	−7,0553	+5,939	−7,748	+7,785
Log. μ^2	+2,7508	+2,821	+3,371	+3,406
L. P. I.	−4,3045	+3,118	−4,377	+4,379
P. I.	−0,000002	+0,000000	−0,000002	+0,000002
−P. II.	−0,000001	−0,000000	+0,000001	−0,000001
N''	−0,000003	+0,000000	−0,000001	+0,000001
$\mathfrak{N}''$	+0,000001	+0,000000	−0,000002	+0,000002

§. 304.

§. 304.

Quia hae correctiones non vltra paucas partes millionesimas quidquam produxerunt; eas tuto negligere poterimus: vnde in calculo quatuor posteriorum columnarum nihil mutabitur. Verum pro columna prima habebimus iam

$$\mathfrak{M} + \mathfrak{M}' + \mathfrak{M}'' = +1,417034 + 0,040810\,\beta.$$
$$-0,000716.\,b. - 0,000161.$$

$$M + M' + M'' = +0,007895 - 0,000717.\,\beta.$$
$$+0,027592.\,b - 0,004791.$$

vbi correctiones separatim subiunximus; vnde tantum opus est, vt ipsis respondentes litteras $\mathfrak{N}''$ et N'' inuestigemus Pro angulo t.

L. M''	− 7,68042
L. $\frac{2(m+1)}{\mu}$	+ 1,42712
	− 9,10754
$\frac{2(m+1)M''}{\mu}$	− 0,128100
− $\mathfrak{M}$	+ 0,000161
Numer.	− 0,127939
L.Num.	− 9,10700
L. Den.	+ 2,24607
L. $\mathfrak{N}''$	− 6,86093
L. $\frac{2(m+1)}{\mu}$	+ 1,42712
L. P. II.	− 8,28805
Pars I.	− 0,004791
− P. II.	+ 0,019411
N''	+ 0,014620
at $\mathfrak{N}''$	− 0,000726

§. 305.

His ergo correctionibus adhibitis pro §. 301. habebimus

$$\beta = -0,006843 - 0,000340.\beta + 0,004191.b. - 0,000726.$$

siue

$$\beta = -0,006843 + 0,004191.b - 0,000726. \text{ et}$$

$$b = +0,190865 + 0,008383.\beta - 0,084458.b + 0,014620.$$

vnde fit

$$b = +0,190807 - 0,084424.b + 0,015614.$$

hincque

$$b = +0,175950 + 0,014398. \text{ siue}$$

$$b = +0,190348.$$

Porro vero

$$\beta = -0,006106 - 0,000769 = -0,006875.$$

$$\gamma = +0,029370 + 0,000027 = +0,029397.$$

$$c = -0,043201 - 0,000111 = -0,043312.$$

$\delta =$

$\delta = -0,003433 - 0,000019$
$= -0,003452.$
$d = +0,005443 + 0,000082$
$= +0,005525.$
$\varepsilon = +0,000046;\ e = -0,000143.$
$\zeta = -0,000004;\ f = +0,000005.$

§. 306.

Ecce ergo valores correctos litterarum $\mathfrak{U}$ et U

$$\begin{aligned}
\mathfrak{U} = -0,006875.\cos. t &+ 0,029397.\cos. 2p - t\\
&+ 0,003452.\cos. 2p + t\\
&+ 0,000046.\cos. 4p - t\\
&- 0,000004.\cos. 4p + t\\
U = +0,190348.\sin. t &- 0,043312.\sin. 2p - t\\
&+ 0,005525.\sin. 2p + t\\
&- 0,000143.\sin. 4p - t\\
&+ 0,000005.\sin. 4p + t.
\end{aligned}$$

Determinatio succinctior.

§. 307.

Hac correctione inuenta nouam correctionem instituere oporteret; verum sequenti modo omni correctione supersedere poterimus:

Primo enim obſeruamus, duas poſtremas columnas vix quidquam priores adficere; ita, vt ſtatim licuiſſet, litteras ε et e; ζ et f negligere. Deinde etiam vidimus in correctione allata particulas poſtremas litterarum $\mathfrak{A}$, $\mathfrak{B}$; A, B; quae vel coſinum $4p$ vel ſinum $4p$ continent, nullius eſſe momenti in prima columna; vnde has particulas ſtatim praetermittere licebit, ſiquidem tota correctio ex particulis abſolutis litterarum $\mathfrak{A}$ et B eſt orta. Quocirca tantum opus erit, harum particularum rationem habere.

§. 308.

Hoc igitur obſeruato, particulas, quae hinc ad litteras $\mathfrak{M}$ et M accedunt, ſignis $\mathfrak{M}''$ et M'' indicemus, quas reiectis poſtremis columnis hoc modo reperiemus:

Pro $\mathfrak{M}''$.

	coſ. t.	coſ. $2p - t$.	coſ. $2p + t$.
$\mathfrak{U}.\ +0,026438$	$+0,026438\,\beta$	$+0,026438\,\gamma$	$+0,026438.\delta$

Pro M''.

	ſin. t.	ſin. $2p - t$.	ſin. $2p + t$.
U. $-0,027237$	$-0,027237.b$	$-0,027237.c$	$-0,027237.d$

§. 309.

§. 309.

Omissa iterum prima columna pro duabus sequentibus quaeramus respondentes litteras $\mathfrak{N}''$ et N''.

Pro $2p-t$.

	γ	c
L. M''		$-8,4351593$
L. $\frac{2(m+1)}{\mu}$		$+0,0516847$
		$-8,4868440$
Numer.	$-0,026438$	$-0,030679$
L. Num.	$-8,4222286$	$-8,4868440$
L. Den.	$-2,5868753$	$-2,5868753$
L. $\mathfrak{N}''$	$+5,8353533$	$+5,8999687$
L. $\frac{2(m+1)}{\mu}$	$+0,0516847$	$+0,0516847$
L. P. II.	$+5,8870380$	$+5,9516534$
L. M''		$-8,4351593$
L. μ^2		$+2,7508822$
L P. I.		$-5,6842771$
P. I.		$-0,000048$
- P. II.	$-0,000077$	$-0,000090$
N''	$-0,000077$	$-0,000138$
at $\mathfrak{N}''$	$+0,000069$	$+0,000079$

Simili

Simili modo

Pro $2p+t$.

	δ	d
L. M''		$-8,4351593$
L. $\frac{2(m+1)}{\mu}$		$+0,0165539$
		$-8,4517132$
Numer.	$-0,026438$	$-0,028295$
L Num.	$-8,4222286$	$-8,4517132$
L. Den.	$-2,6859265$	$-2,6859265$
Log $\mathfrak{N}''$	$+5,7363021$	$+5,7657867$
	$+0,0165539$	$+0,0165539$
L. P II.	$+5,7528560$	$+5,7823406$
L. M''		$-8,4351593$
L. μ^2		$+2,8211438$
L. P. I.		$-5,6140155$
P. I.		$-0,000041$
$-$ P. II.	$-0,000057$	$-0,000061$
N''	$-0,000057$	$-0,000102$
$\mathfrak{N}''$	$+0,000055$	$+0,000058$

§. 310.

§. 310.

Addantur igitur hi valores ad praecedentes $\mathfrak{N}$, $\mathfrak{N}'$; N, N′ et habebimus exacte

$$\gamma = +0,028867 - 0,006118\ \beta + 0,002650.\ b$$
$$+ 0,000069.\ \gamma + 0,000079.\ c.$$
$$c = -0,041818 + 0,003350.\ \beta - 0,007742.\ b$$
$$- 0,000077.\ \gamma - 0,000138.\ c.$$
$$\delta = -0,003179 - 0,005231.\ \beta - 0,001626\ b$$
$$+ 0,000955\ \delta + 0,000058.\ d.$$
$$d = +0,004447 + 0,002422\ \beta + 0,005735.\ b$$
$$- 0,000057.\ \delta - 0,000102.\ d.$$

vnde facile colligimus

$$\gamma = +0,028867 - 0,006118.\ \beta + 0,002650.\ b$$
$$- 0,000002.$$
$$c = -0,041818 + 0,003350.\ \beta - 0,007742.\ b$$
$$+ 0,000004.$$
$$\delta = -0,003179 - 0,005231.\ \beta - 0,001626.\ b.$$
$$d = +0,004447 + 0,002422.\ \beta + 0,005735.\ b.$$

§. 311.

Nunc igitur pro prima columna obtinebimus

$$\mathfrak{M} + \mathfrak{M}' = +1,417034 + 0,040810.\ \beta - 0,000716.\ b$$
$$+ 0,000001.\ \text{et}$$
$$M + M' = +0,007895 - 0,000717.\ \beta + 0,027592.\ b$$
$$- 0,000008.$$

huc igitur addamus $\mathfrak{M}''$ et M'', vt obtineamus valores completos $\mathfrak{M}$ et M;

 $\mathfrak{M} =$

$\mathfrak{M} = + 1,417035 + 0,067248.\beta - 0,000716.b.$
$M = + 0,007887 - 0,000717\ \beta + 0,000355.b.$
his ergo quaeramus respondentes litteras $\mathfrak{N}$ et N.

	conſt.	β.	b.
L. M	+ 7,8969118	− 6,8555192	+ 6,5502284
$L.\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 1,4271258	+ 1,4271258
	+ 9,3240376	− 8,2826450	+ 7,9773542
$\frac{2(m+1)M'}{\mu}$	+ 0,210881	− 0,019171	+ 0,009492
$-\mathfrak{M}$	− 1,417035	− 0,067248	+ 0,000716
Numer.	− 1,206154	− 0,086419	+ 0,010208
L. Num.	− 0,0814014	− 8,9366092	+ 8,0089407
L. Den.	+ 2,2460771	+ 2,2460771	+ 2,2460771
Log. $\mathfrak{N}$	− 7,8353243	− 6,6905321	+ 5,7628636
$L.\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 1,4271258	+ 1,4271258
L. P. II.	− 9,2624501	− 8,1176579	+ 7,1899894
P. I.	+ 0,007887	− 0,000717	+ 0,000355
− P. II.	+ 0,182999	+ 0,013111	− 0,001549
N	+ 0,190886	+ 0,012394	− 0,001194
at $\mathfrak{N}$	− 0,006844	− 0,000490	+ 0,000058

§. 312.

Quare cum ſit $\mathfrak{N} = \beta$ et $N = b$. fiet

$\beta = - 0,006844 - 0,000490.\ \beta + 0,000058.b.$

ſiue $\beta = - 0,006840 + 0,000058\ b.$ et

$b = + 0,190886 + 0,012394\ \beta - 0,001194.b.$

ſiue

siue

$$b = +0,190804 - 0,001194.\, b.$$

hincque

$$b = +0,190587. \text{ et } \beta = -0,006829.$$

reliquae vero litterae γ, c; δ, d. manent, vt ante.

§. 313.

Consequenter valores veri hoc capite quaesiti ita se habebunt:

$$\begin{aligned} u = -0,006829.\, \text{cos.}\, t + 0,029397.\, \text{cos.}\, 2p - t \\ - 0,003452.\, \text{cos.}\, 2p + t \\ + 0,000046.\, \text{cos.}\, 4p - t \\ - 0,000004.\, \text{cos.}\, 4p + t. \end{aligned}$$

$$\begin{aligned} U = +0,190587.\, \text{sin.}\, t - 0,043312.\, \text{sin.}\, 2p - t \\ + 0,005525.\, \text{sin.}\, 2p + t \\ - 0,000143.\, \text{sin.}\, 4p - t \\ + 0,000005.\, \text{sin.}\, 4p + t. \end{aligned}$$

ex quo intelligi potest, hoc compendium in sequentibus ordinibus eo tutius adhiberi posse, quod ibi sufficiat valores tantum ad quinque vel et pauciores figuras decimales exprimi.

CAPVT VIII.

EVOLVTIO AEQVATIONVM ORDINIS VIII, PRO LITTERIS $\mathfrak{V}$ ET V.

§. 314.

Partes annexae nostrarum aequationum sunt

$$\begin{aligned}
\text{I.}\quad 0 = \ldots &+ \mathfrak{V}\,\mathfrak{A} + V.\,\mathfrak{B}.\\
&+ 2.\,\mathfrak{P}\,\mathfrak{U}.\,\mathfrak{C} + (\mathfrak{P}\,U + P\,\mathfrak{U})\,\mathfrak{D} + 2\,PU.\,\mathfrak{E}\\
&+ (\tfrac{5}{2}\cos. t + \tfrac{21}{4}\cos. 2p-t - \tfrac{3}{4}\cos\, 2p+t).\,\mathfrak{P}\\
&+ (-\tfrac{21}{4}\sin. 2p-t + \tfrac{3}{4}\sin. 2p+t)\,P.\\
\text{II.}\quad 0 = \ldots &+ \mathfrak{V}.\,A + V.\,B\\
&+ 2.\,\mathfrak{P}.\,\mathfrak{U}.\,C + (\mathfrak{P}\,U + P\,\mathfrak{U})\,D + 2\,PU.\,E\\
&+ (-\tfrac{21}{4}\sin. 2p-t + \tfrac{3}{4}\sin. 2p+t)\,\mathfrak{P}\\
&+ (\tfrac{5}{2}\cos. t - \tfrac{21}{4}\cos. 2p-t + \tfrac{3}{4}\cos. 2p+t)\,P.
\end{aligned}$$

vbi breuitatis gratia pro postremis membris scribamus in priore aequatione $\mathfrak{P}.\,\mathfrak{T}'. + P.\,T'.$ in altera vero $\mathfrak{P}.\,T'' + P.\,\mathfrak{T}''.$ ita, vt sit

$\mathfrak{T}' =$

$\mathfrak{T}' = \frac{3}{2}\,\text{cof.}\,t + \frac{21}{4}\,\text{cof.}\,2p - t - \frac{3}{4}\,\text{cof.}\,2p + t$

$T' = -\frac{21}{4}\,\text{fin.}\,2p - t + \frac{3}{4}\,\text{fin.}\,2p + t$

$\mathfrak{T}'' = \frac{3}{2}\,\text{cof.}\,t - \frac{21}{4}\,\text{cof.}\,2p - t + \frac{3}{4}\,\text{cof.}\,2p + t.$

$T'' = T'.$

§. 315.

In hoc calculo ante omnia terminos, qui complectuntur angulum $4p$, prorfus omittemus, quoniam fatis vidimus, eum nullius effe momenti; tum vero etiam in multiplicatoribus particulas fiue cofinus $4p$ fiue finus $4p$ continentes tuto omittere licebit; quo obferuato euolutio productorum hic occurrentium fequenti modo facilius inftituetur:

$$\begin{aligned} 2.\mathfrak{P}.\mathfrak{U} = & -0{,}00130.\,\text{cof.}\,q-t - 0{,}00473.\,\text{cof.}\,2p-q+t \\ & + 0{,}02942.\,\text{cof.}\,2p+q-t \\ & - 0{,}00756.\,\text{cof.}\,q+t + 0{,}02812.\,\text{cof.}\,2p-q-t \\ & - 0{,}00343.\,\text{cof.}\,2p+q+t \end{aligned}$$

cuius multiplicator pro aequatione

$$\begin{matrix} \text{prima} \\ \text{fecunda} \end{matrix} \left\{ \begin{matrix} +537{,}6336. + 15{,}4426.\,\text{cof.}\,2p. \\ +10{,}9814.\,\text{fin.}\,2p. \end{matrix} \right.$$

$$\begin{aligned} \mathfrak{P}U + P\mathfrak{U} = \pi = & -0{,}08643.\,\text{fin.}(q-t) + 0{,}01858.\,\text{fin.}(2p-q+t) \\ & - 0{,}05095.\,\text{fin.}(2p+q-t) \\ & + 0{,}10187.\,\text{fin.}\,q+t - 0{,}00855\,\text{fin.}\,2p-q-t \\ & + 0{,}00600.\,\text{fin.}\,2p+q+t \end{aligned}$$

cuius multiplicator pro aequatione

$$\begin{matrix} \text{prima} \\ \text{fecunda} \end{matrix} \left\{ \begin{matrix} +21{,}9628.\,\text{fin.}\,2p \\ -537{,}5636 - 15{,}4426.\,\text{cof.}\,2p. \end{matrix} \right.$$

$$2PU = -0,36580.\cos. q-t + 0,06727.\cos. 2p-q+t$$
$$-0,08778.\cos. 2p+q-t$$
$$+0,38146.\cos. q+t + 0,00879.\cos. 2p-q-t$$
$$+0,01172.\cos. 2p+q+t$$

cuius multiplicator pro aequatione

$$\begin{matrix} \text{prima} \\ \text{secunda} \end{matrix} \left\{ \begin{matrix} -268,7817 - 7,7213.\cos. 2p \\ -8,2360.\sin 2p \end{matrix} \right.$$

$$\mathfrak{P}.\mathfrak{T}' = +1,24367.\cos. q-t - 0,23424.\cos. 2p-q+t$$
$$+2,62298.\cos. 2p+q-t$$
$$+0,67255.\cos. q+t + 2,76576.\cos. 2p-q-t$$
$$-0,37702.\cos. 2p+q+t$$

cuius multiplicator pro prima aequatione $= 1$.

$$PT' = +1,07827.\cos. q-t - 0,75479.\cos. 2p-q+t$$
$$-5,28353.\cos. 2p+q-t$$
$$-0,14581.\cos. q+t + 5,28353.\cos. 2p-q-t$$
$$+0,75479.\cos. 2p+q+t$$

cuius multiplicator pro aequatione prima $= 1$.

$$\mathfrak{P}.T'' = -0,49165.\sin. q-t + 0,37500.\sin. 2p-q+t$$
$$-2,62500.\sin. 2p+q-t$$
$$+0,06331.\sin. q+t - 2,62500.\sin. 2p-q-t$$
$$+0,37500.\sin. 2p+q+t$$

cuius multiplicator pro secunda aequatione $= 1$.

$$P\mathfrak{T}'' = -2,59006\ \sin. q-t + 0,44636.\sin. 2p-q+t$$
$$+5,28151.\sin. 2p+q-t$$
$$-1,34830.\sin.(q+t) - 5,59196.\sin. 2p-q-t$$
$$-0,75681.\sin. 2p+q+t$$

cuius multiplicator pro secunda aequatione $= 1$.

Prima

Prima euolutio, terminorum angulum $q-t$ continentium.

§. 316.

Ex partibus cognitis primae aequationis colligamus litteras $\mathfrak{M}$ sequenti modo:

	cos. $q-t$	cos. $2p-q+t$	cos. $2p+q-t$
$2\mathfrak{PU}$ 537,6336.	− 0,69892	− 2,54300	+ 15,81700
$2\mathfrak{PU}$ 15,4426. cos. $2p$	+ 0,19064	− 0,01004	− 0,01004
	− 0,50828	− 2,55304	+ 15,80696
π. 21,9628. sin. $2p$	− 0,35547	− 0,94912	+ 0,94912
	− 0,86375	− 3,50216	+ 16,75608
2 PU. − 268,7817.	+ 98,32000	− 18,08100	+ 23,59400
	+ 97,45625	− 21,58316	+ 40,35008
	+ 0,07929	+ 1,41220	+ 1,41220
	+ 97,53554	− 20,17096	+ 41,76228
$\mathfrak{P}$ T''	+ 1,24367	− 0,23424	+ 2,62298
	+ 98,77921	− 20,40520	+ 44,38526
P $\mathfrak{T}''$	+ 1,07827	− 0,75479	− 5,28353
$\mathfrak{M}$ =	+ 99,85748	− 21,15999	+ 39,10173

§. 317.

§. 317.

Similiter modo ex altera aequatione eruemus valorem M.

	sin. $q-t$	sin. $2p-q+t$	sin. $2p+q-t$
$2\mathfrak{P}U.\, 10{,}9814.\, \text{sin.}\, 2p$	$-\ 0{,}02597$	$-\ 0{,}00714$	$-\ 0{,}00714$
	$-\ 0{,}16153$		
	$-\ 0{,}18750$		
$\pi.\ -537{,}5636.$	$+\ 46{,}46700$	$-\ 9{,}98900$	$+\ 27{,}39200$
	$+\ 46{,}27950$	$-\ 9{,}99614$	$+\ 27{,}38486$
$\pi.\ -15{,}4426.\, \text{cos.}\, 2p$	$+\ 0{,}14346$	$-\ 0{,}66735$	$+\ 0{,}66735$
	$+\ 0{,}39340$		
	$+\ 46{,}81636$	$-\ 10{,}66349$	$+\ 28{,}05221$
$2PU.\ -8{,}2360.\, \text{sin.}\, 2p$	$-\ 0{,}27701$	$+\ 1{,}50630$	$+\ 1{,}50630$
	$+\ 46{,}53935$		
	$-\ 0{,}36147$		
	$+\ 46{,}17788$	$-\ 9{,}15719$	$+\ 29{,}55851$
$\mathfrak{P}\, T''$	$-\ 0{,}49165$	$+\ 0{,}37500$	$-\ 2{,}62500$
	$+\ 45{,}68623$	$-\ 8{,}78219$	$+\ 26{,}93351$
$P\, \mathfrak{T}''$	$-\ 2{,}59006$	$+\ 0{,}44636$	$+\ 5{,}28151$
$M =$	$+\ 43{,}09617$	$-\ 8{,}33583$	$+\ 32{,}21502$

§. 318.

§. 318.

Antequam vlterius progredi liceat, elementa numerica pro his angulis constituamus.

$\omega =$	$q - t$	$2p - q + t$	$2p + q - t$
$\mu =$	$n - 1$	$2m - n + 1$	$2m + n - 1$
$\mu =$	12,25604	12,48180	36,99384
$L.\,2(m+1) =$	1,4271258	1,4271258	1,4271258
$Log.\,\mu =$	1,0883502	1,0962675	1,5681293
$Log.\,\frac{2(m+1)}{\mu} =$	0,3387756	0,3308583	9,8589965
$Log.\,\mu^2 =$	2,1767004	2,1925350	3,1362586
$\lambda - 2 =$	177,22893	177,22893	177,22893
$-\mu^2 =$	− 150,21051	−155,79000	−1368,50000
Den. $=$	+ 27,01842	+21,43893	−1191,27107
Log. $=$	+1,4316599	+1,3312032	−3,0760211

§. 319.

Etiam hic primam columnam commode vltimo loco referuamus; pro duabus ergo reliquis quaeramus litteras $\mathfrak{N}$ et N:

	$2p-q+t$	$2p+q-t$
Log. M	− 0,9209489	+ 1,5080581
Log. $\frac{2(m+1)}{\mu}$	+ 0,3308583	+ 9,8589965
	− 1,2518072	+ 1,3670546
$\frac{2(m+1)M}{\mu}$	− 17,85700	+ 23,28400
− $\mathfrak{M}$	+ 21,15999	− 39,10173
Numerat.	+ 3,30299	− 15,81773
Log. Numerat.	+ 0,5189074	− 1,1991441
Log. Denom.	+ 1,3312032	− 3,0760211
L. $\mathfrak{N}$	+ 9,1877042	+ 8,1231230
adde Log. $\frac{2(m+1)}{\mu}$	+ 0,3308583	+ 9,8589965
L. Pars II.	+ 9,5185625	+ 7,9821195
L. M	− 0,9209489	+ 1,5080581
L. μ^2	+ 2,1925350	+ 3,1362586
L. P. I.	− 8,7284139	+ 8,3717995
P. I.	− 0,05351	+ 0,02354
− P. II.	− 0.33004	− 0,00960
N	− 0,38355	+ 0,01394
at $\mathfrak{N}$	+ 0,15406	+ 0,01328

§. 320.

Nunc ipsas quantitates quaesitas ponamus.

$$\mathfrak{V} = \beta.\cos.q-t+\gamma.\cos.2p-q+t+\delta.\cos.2p+q-t$$

$$V = b.\sin.q-t+c.\sin.2p-q+t+d.\sin.2p+q-t$$

vnde tam $\mathfrak{M}'$, quam M' deriuemus:

Pro $\mathfrak{M}'$.

	$\cos.q-t$	$\cos.2p-q+t$	$\cos.2p+q-t$
$\mathfrak{V}.-9,22139.\cos.2p$	$-4,61064.\gamma$	$-4,61064.\beta$	$-4,61064.\beta$
	$-4,61064.\delta$		
$\mathfrak{V}.+0,02644.$ -	$+0,02644.\beta$	$+0,02644.\gamma$	$+0,02644.\delta$
$V.-3,99069.\sin.2p$	$-1,99524.c$	$-1,99524.b$	$+1,99524.b$
	$-1,99524.d$		

Pro M'.

	$\sin.q-t$	$\sin.2p-q+t$	$\sin.2p+q-t$
$\mathfrak{V}.-3,99069.\sin.2p$	$-1,99524.\gamma$	$-1,99524.\beta$	$-1,99524\ \beta$
	$+1,99524.\delta$		
$V.+5,36064.\cos.2p$	$-2,68032.c$	$-2,68032.b$	$+2,68032.b$
	$+2,68032.d$		
$V.-0,02724.$ -	$-0,02724.b$	$-0,02724.c$	$-0,02724.d$

§. 321.

Reseruata iterum prima columna, calculum litterarum $\mathfrak{N}'$ et N' incipiamus a columna secunda:

Pro angulo $2p-q+t$.

	β	b	γ	c
L. M.′	− 0,2999864	− 0,4281834		− 8,4352071
L. $\frac{2(m+1)}{\mu}$	+ 0,3308583	+ 0,3308583		+ 0,3308583
	− 0,6308447	− 0,7590417		− 8,7660654
$\frac{2(m+1)M'}{\mu}$	− 4,27410	− 5,74170		− 0,0584
− $\mathfrak{M}'$	+ 4,61064	+ 1,99524	− 0,02644	
Numer.	+ 0,33654	− 3,74646	− 0,02644	− 0,0584
L. Num	+ 9,5270367	− 0,5736211	− 8,4222614	− 8,7660654
L. Den.	+ 1,3312032	+ 1,3312032	+ 1,3312032	+ 1,3312032
L. $\mathfrak{N}'$	+ 8,1958335	− 9,2424179	− 7,0910582	− 7,4348622
L. $\frac{2(m+1)}{\mu}$	+ 0,3308583	+ 0,3308583	+ 0,3308583	+ 0,3308583
L. P. II	+ 8,5266918	− 9,5732762	− 7,4219165	− 7,7657205
L. M′.	− 0,2999864	− 0,4281834		− 8,4352071
L. μ^2	2,1925350	2,1925350		2,1925350
L. P. I.	− 8,1074514	− 8,2356484		− 6,2426721
P. I.	− 0,01280	− 0,01720		− 0,00017
− P. II.	− 0,03360	+ 0,37435	+ 0,00264	+ 0,00583
N′	− 0,04640	+ 0,35715	+ 0,00264	+ 0,00566
at $\mathfrak{N}'$	+ 0,01570	− 0,17475	− 0,00123	− 0,00272

hinc ergo sequentes determinationes adipiscimur

$\gamma = +0,15406 + 0,01570.\beta - 0,17475.b - 0,00123.\gamma - 0,00272.c.$

$c = -0,38355 - 0,04640.\beta + 0,35715.b + 0,00264.\gamma + 0,00566.c.$

hinc

hinc ergo fit

$$\gamma = +0,15491 + 0,01580.\beta - 0,17550.b$$
$$c = -0,38531 - 0,04660.\beta + 0,35870.b$$

§. 322.

Simili modo faciamus calculum

Pro angulo $2p + q - t$.

	β	b	δ	d
L. M′	− 0,3000082	+ 0,4281834		− 8,4352071
L. $\frac{2(m+1)}{\mu}$	+ 9,8589965	+ 9,8589965		+ 9,8589965
	− 0,1590047	+ 0,2871799		− 8,2942036
$\frac{2(m+1)M'}{\mu}$	− 1,44220	+ 1,93725		− 0,01969
− 𝔐′	+ 4,61064	− 1,99524	− 0,02644	
Numer.	+ 3,16844	− 0,05799	− 0,02644	− 0,01969
L. Num.	+ 0,5008455	− 8,7633531	− 8,4222614	− 8,2942457
L. Den.	− 3,0760211	− 3,0760211	− 3,0760211	− 3,0760211
L. 𝔑′	− 7,4248244	+ 5,6873320	+ 5,3462403	+ 5,2182246
L. $\frac{2(m+1)}{\mu}$	+ 9,8589965	+ 9,8589965	+ 9,8589965	+ 9,8589965
L. P. II.	− 7,2838209	+ 5,5463285	+ 5,2052368	+ 5,0772211
L. M′	− 0,3000082	+ 0,4281834		− 8,4352071
L. μ^2	3,1362586	3,1362586		3,1362586
L. P. I.	− 7,1637496	+ 7,2919248		− 5,2980485
P. I.	− 0,00145	+ 0,00196		− 0,00002
− P. II.	+ 0,00192	− 0,00003	− 0,00002	− 0,00001
N′	+ 0,00047	+ 0,00193	− 0,00002	− 0,00003
at 𝔑′	− 0,00266	+ 0,00005	+ 0,00002	+ 0,00002

 vnde

Vnde

$$\delta = +0,01328 - 0,00266.\beta + 0,00005.b.$$
$$+ 0,00002.\delta + 0,00002.d.$$
$$d = +0,01394 + 0,00047.\beta + 0,00193.b$$
$$- 0,00002.\delta - 0,00003.d.$$

ideoque

$$\delta = +0,01328 - 0,00266.\beta + 0,00005.b.$$
$$d = +0,01394 + 0,00047.\beta + 0,00193.b.$$

§. 323.

Iam pro angulo primo $q-t$ propter

$$\gamma + \delta = +0,16819 + 0,01314.\beta - 0,17545.b.$$
$$c + d = -0,37137 - 0,04613.\beta + 0,36063.b.$$

reperiemus

$$\mathfrak{M} + \mathfrak{M}' = +99,82300 + 0,05790.\beta + 0,08937.b.$$
$$= \mathfrak{M} \text{ ſimpliciter}$$

ſimili modo propter

$$\gamma - \delta = +0,14163 + 0,01846.\beta - 0,17555.b.$$
$$c - d = -0,39925 - 0,04707.\beta + 0,35677.b$$

reperiemus

$$M + M' = +43,88373 + 0,08933.\beta - 0,63322.b.$$
$$= M. \text{ ſimpliciter.}$$

§. 324.

§. 324.

His ergo litteris $\mathfrak{M}$ et M quaeramus respondentes $\mathfrak{N}$ et N.

Pro angulo $q - t$.

	Ter. absol.	β	b
L. M	+ 1,6423062	+ 8,9509973	− 9,8015546
L. $\frac{2(m+1)}{\mu}$	+ 0,3387756	+ 0,3387756	+ 0,3387756
	+ 1,9810818	+ 9,2897729	− 0,1403302
$\frac{2(m+1)M}{\mu}$	+ 95,73744	+ 0,19488	− 1,38144
− $\mathfrak{M}$	− 99,82300	− 0,05790	− 0,08937
Num.	− 4,08556	+ 0,13698	− 1,47081
L. Num.	− 0,6112558	+ 9,1366572	− 0,1675565
L. Den.	+ 1,4316599	+ 1,4316599	+ 1,4316599
L. $\mathfrak{N}$	− 9,1795959	+ 7,7049973	− 8,7358966
L. $\frac{2(m+1)}{\mu}$	+ 0,3387756	+ 0,3387756	+ 0,3387756
L. P. II.	− 9,5183715	+ 8,0437729	− 9,0746722
L. M	+ 1,6423062	+ 8,9509973	− 9,8015546
L. μ^2	+ 2,1767004	+ 2,1767004	+ 2,1767004
L. P. I.	+ 9,4656058	+ 6,7742969	− 7,6248542
P. I.	+ 0,29215	+ 0,00060	− 0,00421
− P. II.	+ 0,32989	− 0,01106	+ 0,11876
N =	+ 0,62204	− 0,01046	+ 0,11455
at $\mathfrak{N}$ =	− 0,15122	+ 0,00507	− 0,04324

hinc

hinc

$$\beta = -0,15122 + 0,00507.\beta - 0,04324.b.$$

$$b = +0,62204 - 0,01046.\beta + 0,11455.b.$$

Ex hac colligitur

$$b = +0,70251 - 0,01181.\beta.$$

qui valor in illa ſubſtitutus dat

$$\beta = -0,18160 + 0,00558.\beta$$

conſequenter

$$\beta = -0,18182 \text{ et}$$

$$b = +0,68575.$$

§. 325.

Ex his deducuntur ſequentes coefficientes

$$\begin{array}{l|l} \gamma = +0,03084 & c = -0,13086 \\ \delta = +0,01379 & d = +0,01518. \end{array}$$

vnde valores hac euolutione ſuppeditati ſunt:

$$\mathfrak{W} = -0,18182.\text{coſ.}\, q-t + 0,03084.\text{coſ.}\, 2p-q+t$$
$$+ 0,01379.\text{coſ.}\, 2p+q-t.$$

$$V = +0,68575.\text{ſin.}\, q-t - 0,13086.\text{ſin.}\, 2p-q+t$$
$$+ 0,01518.\text{ſin.}\, 2p+q-t.$$

Secunda euolutio terminorum angulum $q+t$ continentium.

§. 326.

Ex partibus cognitis primae aequationis colligamus litteras 𝔐 sequenti modo:

	cos. $q+t$	cos. $2p-q-t$	cos. $2p+q+t$
2 𝔓 𝔘. + 537, 6336.	− 4,06450	+ 15,11800	− 1,84400
2 𝔓 𝔘. + 15,44. cos. $2p$	+ 0,19065	− 0,05837	− 0,05837
	− 3,87385	+ 15,05963	− 1,90237
π. 21, 96. sin. $2p$	− 0,02800	+ 1,11868	− 1,11868
	− 3,90185	+ 16,17831	− 3,02105
2 P U. − 268, 7817.	− 102,52950	− 2,36260	− 3,15012
	− 106,43135	+ 13,81571	− 6,17117
2 P U. − 7,7213. cos. $2p$	− 0,03393	− 1,47260	− 1,47260
	− 0,04525		
	− 106,51053	+ 12,34311	− 7,64377
𝔓 𝔗′	+ 0,67255	+ 2,76576	− 0,37702
	− 105,83798	+ 15,10887	− 8,02079
P T′	− 0,14581	+ 5,28353	+ 0,75479
𝔐	− 105,98379	+ 20,39240	− 7,26600

§. 327.

§. 327.

Simili modo ex altera aequatione eruamus valorem litterarum M.

	sin. $q+t$	sin. $2p-q-t$	sin. $2p+q+t$
$2\mathfrak{P}\mathfrak{U}.+10,9814.\text{sin}.2p$	+ 0,17322	− 0,04151	− 0,04151
$\pi.-537,5636.$	− 54,76900	+ 4,59680	− 3,22580
	− 54,59578	+ 4,55529	− 3,26731
$\pi.-15,4426.\text{cos}.2p$	+ 0,01969	+ 0,78658	− 0,78658
	− 54,57609	+ 5,34187	− 4,05389
$2PU.-8,2360.\text{sin}.2p$	+ 0,01206	− 1,57080	− 1,57080
	− 54,56403	+ 3,77107	− 5,62469
$\mathfrak{P}T''$	+ 0,06331	− 2,62500	+ 0,37500
	− 54,50072	+ 1,14607	− 5,24969
$P\mathfrak{T}''$	− 1,34830	− 5,59196	− 0,75681
Ergo M	− 55,84902	− 4,44589	− 6,00650

§. 328.

Ante quam vlterius progredi liceat, elementa numerica pro his angulis constitui oportet.

ω	$q+t$	$2p-q-t$	$2p+q+t$
$\mu =$	$n+1$	$2m-n-1$	$2m+n+1$
$\mu =$	14,25604	10,48180	38,99384
$L. 2(m+1) =$	1,4271258	1,4271258	1,4271258
$Log. \mu =$	1,1539989	1,0204360	1,5909955
$Log. \frac{2(m+1)}{\mu} =$	0,2731269	0,4066898	9,8361303
$Log. \mu^2 =$	2,3079978	2,0408720	3,1819910
$\lambda - 2 =$	177,22893	177,22893	177,22893
$\mu^2 =$	−203,23470	−109,87000	−1520,50000
Denom. =	− 26,00577	+ 67,35893	−1343,27107
Log. =	− 1,4150686	+ 1,8283952	− 3,1281629

 §. 329.

§. 329.

Etiam hic primam columnam vltimo loco referuamus; pro duabus ergo reliquis quaeramus litteras 𝔑 et N.

	$2p-q-t$	$2p+q+t$
L. M	− 0,6479597	− 0,7786215
L. $\frac{2(m+1)}{\mu}$	+ 0,4066898	+ 9,8361303
	− 1,0546495	− 0,6147518
$\frac{2(m+1)M}{\mu}$	− 11,34090	− 4,11860
− 𝔐	− 18,15504	+ 5,02864
Numer.	− 29,49594	+ 0,91004
Log.	− 1,4697631	+ 9,9590605
L. Den.	+ 1,8283952	− 3,1281629
Log. 𝔑	− 9,6413679	− 6,8308976
	+ 0,4066898	+ 9,8361303
L. P. II.	− 0,0480577	− 6,6670279
Log. M	− 0,6479597	− 0,7786215
Log. μ^2	2,0408720	3,1819910
L. P. I.	− 8,6070877	− 7,5966305
P. I.	− 0,04047	− 0,00395
− P. II.	+ 1,11701	+ 0,00046
N	+ 1,07654	− 0,00349
𝔑	− 0,43789	− 0,00068

§. 330.

§. 330.

Nunc ipsas quantitates incognitas ponamus

$\mathfrak{V} = \beta.\cos.\,q+t + \gamma.\cos.\,2p-q-t + \delta.\cos.\,2p+q+t.$

$V = b.\sin.\,q+t + c.\sin.\,2p-q-t + d.\sin.\,2p+q+t.$

vnde tam 𝔐′, quam M′ deriuemus:

Pro 𝔐′.

	$\cos.\,q+t$	$\cos.\,2p-q-t$	$\cos.\,2p+q+t$
$\mathfrak{V}.-9,22.\cos.2p$	$-4,61064.\gamma$	$-4,61064.\beta$	$-4,61064\ \beta$
	$-4,61064.\delta$		
$\mathfrak{V}.+0,02644.$	$+0,02644.\beta$	$+0,02644.\gamma$	$+0,02644.\delta$
$V.-3,99\sin.2p$	$-1,99524.c$	$-1,99524.b$	$+1,99524.b$
	$-1,99524.d$		

Pro M′.

	$\sin.\,q+t$	$\sin.\,2p-q-t$	$\sin.\,2p+q+t$
$\mathfrak{V}.-3,99.\sin.2p$	$-1,99524\gamma$	$-1,99524.\beta$	$-1,99524.\beta$
	$+1,99524.\delta$		
$V.+5,36.\cos.2p$	$-2,68032.c$	$-2,68032.b$	$+2,68032.b$
	$+2,68032.d$		
$V.-0,027.$	$-0,02724.b$	$-0,02724.c$	$-0,02724.d$

§. 331.

Reseruata prima columna, calculum litterarum $\mathfrak{N}'$ et N' incipiamus a columna secunda:

Pro angulo $2p-q-t$.

	β	b	γ	c
Log. M′	− 0,2999864	− 0,4281834		− 8,4352071
L. $\frac{2(m+1)}{\mu}$	+ 0,4066898	+ 0,4066898		+ 0,4066898
	− 0,7066762	− 0,8348732		− 8,8418969
$\frac{2(m+1)M'}{\mu}$	− 5,98951	− 6,83712		− 0,06949
− $\mathfrak{M}'$	+ 4,61064	+ 1,99524	− 0,02644	
Numer.	− 0,47887	− 4,84188	− 0,02644	− 0,06949
L. Num.	− 9,6802176	− 0,6850140	− 8,4222614	− 8,8419223
L. den.	+ 1,8283952	+ 1,8283952	+ 1,8283952	+ 1,8283952
Log. $\mathfrak{N}$	− 7,8518224	− 8,8566188	− 6,5938662	− 7,0135271
	+ 0,4066898	+ 0,4066898	+ 0,4066898	+ 0,4066898
L. P. II.	− 8,2585122	− 9,2633086	− 7,0005560	− 7,4202169
Log. M′	− 0,2999864	+ 0,4281834		− 8,4352071
Log. μ^2	2,0408720	2,0408720		+ 2,0408720
L. P. I.	− 8,2591144	− 8,3873114		− 6,3943351
P. I.	− 0,01816	− 0,02440		− 0,00025
− P. II.	+ 0,01814	+ 0,18336	+ 0,00100	+ 0,00263
N	− 0,00002	+ 0,15896	+ 0,00100	+ 0,00238
$\mathfrak{N}$	− 0,00711	− 0,07188	− 0,00039	− 0,00103

hinc

$\gamma = -0,43789 - 0,00711.\beta - 0,07188\,b - 0,00039.\gamma - 0,00103.c.$

$c = +1,07654 + 0,00002.\beta + 0,15896.b + 0,00100.\gamma + 0,00238.c.$

vnde

vnde concludimus

$$\gamma = -0,43883 - 0,00711.\beta - 0,07201.b$$
$$c = +1,07866 - 0,00003.\beta + 0,15926.b.$$

§. 332.

Simili modo faciamus calculum

Pro angulo $2p+q+r$.

	β	b	δ	d
L. M′	− 0,3000082	+ 0,4281834		− 8,4352071
L.$\frac{2(m+1)}{\mu}$	+ 9,8361303	+ 9,8361303		+ 9,8361303
	− 0,1361385	+ 0,2643137		− 8,2713374
$\frac{2(m+1)M'}{\mu}$	− 1,36813	+ 1,83785		− 0,01867
− 𝔐′	+ 4,61064	− 1,99524	− 0,02644	
Numer.	+ 3,24251	− 0,15739	− 0,02644	− 0,01867
L.Num.	+ 0,5108890	− 9,1969771	− 8,4222614	− 8,27
L. Den.	− 3,1281629	− 3,1281629	− 3,1281629	− 3,12
L. 𝔑	− 7,3827171	+ 6,0688142	+ 5,2940985	+ 5,15
	+ 9,8361303	+ 9,8361303	+ 9,8361303	+ 9,83
L. P. II.	− 7,2188474	+ 5,9049445	+ 5,1302288	+ 4,98
L. M′	− 0,3000082	+ 0,4281834		− 8,43520
L. μ^2	+ 3,1819910	+ 3,1819910		+ 3,18199
L. P. I.	− 7,1180172	+ 7,2461924		− 5,25321
P. I.	− 0,00131	+ 0,00176		− 0,00001
−P. II.	+ 0,00165	− 0,00008	− 0,00001	− 0,00001
N′	+ 0,00034	+ 0,00168	− 0,00001	− 0,00002
at 𝔑′	− 0,00241	+ 0,00012	+ 0,00002	+ 0,00001

hinc

hinc

$$\delta = -0,00068 - 0,00241.\beta + 0,00012.b + 0,00002.\delta + 0,00001.d.$$

$$d = -0,00349 + 0,00034.\beta + 0,00168.b - 0,00001.\delta - 0,00002.d.$$

vbi bina postrema membra tuto reiicere licet.

§. 333.

Iam pro angulo primo $q+t$ ob

$$\gamma+\delta = -0,43951 - 0,00952.\beta - 0,07189.b \text{ et}$$

$$c+d = +1,07517 + 0,00031.\beta + 0,16094.b.$$

reperitur

$$\mathfrak{M}+\mathfrak{M}' = -106,10274 + 0,06971.\beta + 0,01033.b.$$

$$= \mathfrak{M} \text{ simpliciter.}$$

Simili modo propter

$$\gamma-\delta = -0,43815 - 0,00470.\beta - 0,07213.b.$$

$$c-d = +1,08215 - 0,00037.\beta + 0,15758.b.$$

reperimus

$$M+M' = -57,87518 + 0,01036.\beta - 0,30568.b.$$

$$= M \text{ simpliciter.}$$

§. 334.

His ergo litteris $\mathfrak{M}$ et M quaeramus respondentes $\mathfrak{N}$ et N.

Pro angulo $q + t$.

	I.	β.	b.
L. M	−1,7624925	+8,0153598	−9,4852670
$L\frac{2(m+1)}{\mu}$	+0,2731269	+0,2731269	+0,2731269
	−2,0356194	+8,2884867	−9,7583939
$\frac{2(m+1)M}{\mu}$	−108,54800	+0,01943	−0,57332
$-\mathfrak{M}$	+106,10274	−0,06971	−0,01033
Numer.	−2,44526	−0,05028	−0,58365
L Num.	−0,3883251	−8,7013957	−9,7661525
L den.	−1,4150686	−1,4150686	−1,4150686
Log. $\mathfrak{N}$	+8,9732565	+7,2863267	+8,3510839
	+0,2731269	+0,2731269	+0,2731269
L. P. II.	+9,2463834	+7,5594536	+8,6242108
Log. M	−1,7624925	+8,0153598	−9,4852670
Log. μ^2	+2,3079978	+2,3079978	+2,3079978
L. P. I.	−9,4544947	+5,7073620	−7,1772692
P. I.	−0,28478	+0,00005	−0,00150
P. II.	−0,17635	−0,00363	−0,04209
N	−0,46113	−0,00358	−0,04359
$\mathfrak{N}$	+0,09403	+0,00193	+0,02244

hinc $\beta = +0,09403 + 0,00193\,\beta + 0,02244\,b.$

$b = -0,46113 - 0,00358.\beta - 0,04359.b.$

 §. 335.

§. 335.

Ex posteriore aequatione deducimus

$$b = -0,44187 - 0,00343.\beta$$

qui valor in prima praebet

$$\beta = +0,08412 + 0,00185.\beta$$

hoc est

$\beta = +0,08427.$ ideoque $b = -0,44216.$

vnde sequentes obtinemus valores

$\gamma = -0,40759$	$c = +1,00824$
$\delta = -0,00093$	$d = -0,00422.$

Conclusio.

§. 336.

Ecce ergo completos valores hoc capite inuestigatos:

$$\begin{aligned}\mathfrak{V} = &-0,18182.\cos.(q-t) + 0,03084.\cos.2p-q+t\\ &\qquad\qquad\qquad\qquad + 0,01379.\cos.2p+q-t\\ &+0,08427.\cos.(q+t) - 0,40759.\cos.2p-q-t\\ &\qquad\qquad\qquad\qquad - 0,00093.\cos.2p+q+t\end{aligned}$$

$$\begin{aligned}V = &+0,68575.\sin.(q-t) - 0,13086.\sin.2p-q+t\\ &\qquad\qquad\qquad\qquad + 0,01518.\sin.2p+q-t\\ &-0,44216.\sin.(q+t) + 1,00824.\sin.2p-q-t\\ &\qquad\qquad\qquad\qquad - 0,00422.\sin.2p+q+t.\end{aligned}$$

CAPUT IX.

EVOLUTIO AEQUATIONUM ORDINIS IX. CHARACTERIS κ K K PRO LITTERIS $\mathfrak{W}$ ET W.

§. 337.

Partes annexae nostrarum aequationum ita exhibeantur:

$$\text{I. } 0 = \ldots + \mathfrak{W}.\mathfrak{A} + W.\mathfrak{B}$$
$$+ (2\mathfrak{P}\mathfrak{V} + 2\mathfrak{Q}\mathfrak{U})\mathfrak{C} + (\mathfrak{P}V + \mathfrak{Q}U + P\mathfrak{V} + Q\mathfrak{U})\mathfrak{D}$$
$$+ (2PV + 2QU)\mathfrak{E} + 3.\mathfrak{P}^2\mathfrak{U}.\mathfrak{F}$$
$$+ (\mathfrak{P}^2U + 2\mathfrak{P}P\mathfrak{U})\mathfrak{G} + (P^2\mathfrak{U} + 2\mathfrak{P}PU)\mathfrak{H}$$
$$+ 3.P^2U.\mathfrak{I}$$
$$+ (\tfrac{3}{2}\cos. t + \tfrac{21}{4}\cos. 2p-t - \tfrac{3}{4}\cos. 2p+t)\mathfrak{Q}$$
$$+ (-\tfrac{21}{4}\sin. 2p-t + \tfrac{3}{4}\sin\ 2p+t)Q.$$

$$\text{II. } 0 = \ldots + \mathfrak{W}\,A + W.B$$
$$+ (2\mathfrak{P}\mathfrak{V} + 2\mathfrak{Q}\mathfrak{U})C + (\mathfrak{P}V + \mathfrak{Q}U + P\mathfrak{V} + Q\mathfrak{U})D$$
$$+ (2PV + 2QU)E + 3.\mathfrak{P}^2.\mathfrak{U}.F$$
$$+ (\mathfrak{P}^2U + 2\mathfrak{P}P\mathfrak{U})G + (P^2\mathfrak{U} + 2\mathfrak{P}PU)H$$
$$+ 3.P^2U.I.$$
$$+ (-\tfrac{21}{4}\sin. 2p-t + \tfrac{3}{4}\sin. 2p+t)\mathfrak{Q}$$
$$+ (\tfrac{3}{2}\cos. t - \tfrac{21}{4}\cos. 2p-t + \tfrac{3}{4}\cos. 2p+t)Q.$$

§. 388.

Ante omnia igitur singula producta, quae hic occurrunt, debite euolui oportet:

I^{mo}. $2\,\mathfrak{P}\mathfrak{V} + 2\,\mathfrak{Q}\mathfrak{U}$ praebet

$$-0,1552.\cos. t - 0,4613.\cos. 2p - t$$
$$+ 0,0485.\cos. 2p + t$$
$$-0,1886.\cos. 2q - t - 0,0037.\cos. 2p - 2q + t$$
$$+ 0,0294.\cos. 2p + 2q - t$$
$$+0,0826.\cos. 2q + t - 0,3754.\cos. 2p - 2q - t$$
$$-0,0029.\cos. 2p + 2q + t$$

Eius multiplicator pro

$$\begin{matrix} \text{prima} \\ \text{secunda} \end{matrix} \left\{ \begin{matrix} +537,6336 + 15,4425.\cos. 2p \\ +10,9814.\sin. 2p \end{matrix} \right.$$

II^{do}. $\mathfrak{P}V + \mathfrak{Q}U + P\mathfrak{V} + Q\mathfrak{U}$ dat

$$-0,4087.\sin. t + 1,0390.\sin. 2p - t$$
$$-0,1395.\sin. 2p + t$$
$$+0,4802.\sin. 2q - t - 0,0886.\sin. 2p - 2q + t$$
$$-0,0142.\sin. 2p + 2q - t$$
$$-0,2566.\sin. 2q + t + 0,1215.\sin. 2p - 2q - t$$
$$+0,0002.\sin. 2p + 2q + t$$

Cuius multiplicator pro

$$\begin{matrix} \text{prima} \\ \text{secunda} \end{matrix} \left\{ \begin{matrix} +21,9628.\sin. 2p \\ -537,5635 - 15,4425.\cos. 2p. \end{matrix} \right.$$

III.

III^{tio}. $2 PV + 2 QU$ praebet

$-0,8546.\cos t + 2,3005.\cos 2p - t$
$-0,4574.\cos 2p + t$
$+1,4087.\cos 2q - t - 0,0766.\cos 2p - 2q + t$
$+0,0446.\cos 2p + 2q - t$
$-0,9379.\cos 2q + t - 1,7990.\cos 2p - 2q - t$
$-0,0103.\cos 2p + 2q + t.$

Cuius multiplicator pro

prima $\left\{ -268,7817 - 7,7213.\cos 2p \right.$
secunda $\left\{ -8,2360.\sin 2p. \right.$

IV^{to}: $3 \mathfrak{P}\mathfrak{U}$ praebet

$-0,0034.\cos t + 0,0436.\cos 2p - t$
$-0,0073.\cos 2p + t$
$+0,0031.\cos 2q - t - 0,0045.\cos 2p - 2q + t$
$+0,0220.\cos 2p + 2q - t$
$-0,0063.\cos 2q + t + 0,0201.\cos 2p - 2q - t$
$-0,0025.\cos 2p + 2q + t.$

Cuius multiplicator pro

prima $\left\{ -716,8266 - 25,7376.\cos 2p \right.$
secunda $\left\{ -18,3023.\sin 2p. \right.$

V^{to}. $\mathfrak{P}^2 U + 2 \mathfrak{P} P. \mathfrak{U}$. praebet

$$+ 0,0900.\ \sin. t - 0,0374.\ \sin. 2p - t$$
$$+ 0,0229.\ \sin. 2p + t$$
$$- 0,0447.\ \sin. 2q - t + 0,0158.\ \sin. 2p - 2q + t$$
$$- 0,0404.\ \sin. 2p + 2q - t$$
$$+ 0,0550.\ \sin. 2q + t + 0,0009.\ \sin. 2p - 2q - t$$
$$+ 0,0048.\ \sin. 2p + 2q + t.$$

Cuius multiplicator pro

$$\begin{matrix} \text{prima} \\ \text{secunda} \end{matrix} \left\{ \begin{matrix} - 54,9069.\ \sin. 2p \\ + 1075,0302 + 38,6064.\ \cos. 2p. \end{matrix} \right.$$

VI^{to}. $P^2 \mathfrak{U} + 2 \mathfrak{P} P. U$. dat

$$- 0,0100.\ \cos. t - 0,0113.\ \cos. 2p - t$$
$$+ 0,0716.\ \cos. 2p + t$$
$$- 0,1720.\ \cos. 2q - t - 0,0017.\ \cos. 2p - 2q + t$$
$$- 0,0733.\ \cos. 2p + 2q - t$$
$$+ 0,1973.\ \cos. 2q + t + 0,0079.\ \cos. 2p - 2q - t$$
$$+ 0,0091.\ \cos. 2p + 2q + t.$$

Cuius multiplicator pro

$$\begin{matrix} \text{prima} \\ \text{secunda} \end{matrix} \left\{ \begin{matrix} + 1075,0302 + 38,6064.\ \cos. 2p. \\ + 41,1802.\ \sin. 2p. \end{matrix} \right.$$

VII.

VII^{mo}. 3. $P^2 U$ dat

$+1,1464.\sin. t - 0,0393.\sin. 2p - t$

$- 0,1998.\sin. 2p + t$

$+0,5250.\sin. 2q - t + 0,2197.\sin. 2p - 2q + t$

$+0,1314.\sin. 2p + 2q - t$

$-0,5718.\sin. 2q + t - 0,1051.\sin. 2p - 2q - t$

$-0,0168.\sin. 2p + 2q + t.$

Eius multiplicatores pro

$$\begin{matrix} \text{prima} \\ \text{secunda} \end{matrix} \left\{ \begin{matrix} +13,7267.\sin. 2p \\ -268,7399 - 9,6516.\cos. 2p. \end{matrix} \right.$$

Postremae partes prioris aequationis dant

$-0,5360.\cos. t - 2,6651.\cos. 2p - t$

$+0,5684.\cos. 2p + t$

$-0,9654.\cos. 2q - t - 0,2479.\cos. 2p - 2q + t$

$+2,0028.\cos. 2p + 2q - t$

$+0,5761.\cos. 2q + t + 0,5248.\cos. 2p - 2q - t$

$-0,2820.\cos. 2p + 2q + t.$

Eius multiplicator pro prima est 1.

Postre-

Poſtremae partes poſterioris aequationis dant

$$+0,3632.\,\text{ſin.}\,t + 2,9029.\,\text{ſin.}\,2p - t$$
$$-0,3307.\,\text{ſin.}\,2p + t$$
$$+1,5363.\,\text{ſin.}\,2q - t + 0,3303.\,\text{ſin.}\,2p - 2q + t$$
$$-1,9959.\,\text{ſin.}\,2p + 2q - t$$
$$-0,0020.\,\text{ſin.}\,2q + t + 2,2329.\,\text{ſin.}\,2p - 2q - t$$
$$+0,2888.\,\text{ſin.}\,2p + 2q + t.$$

Eius multiplicator pro ſecunda aequatione eſt 1.

Cuncta haec producta ſponte in terna membra diſtinguuntur, ſeorſim pertractanda.

Prima

Prima euolutio terminorum angulum simplicem t continentium.

§. 339.

Ex partibus cognitis primae aequationis valores litterae 𝔐 colligamus:

	cos. t	cos. $2p - t$	cos. $2p + t$
N. I. + 537,6336.	− 83,4400	− 248,0100	+ 26,0750
N. I. + 15,4426. cos. $2p$	− 3,1873	− 1,1983	− 1,1983
	− 86,6273	− 249,2083	+ 24,8767
N. II. + 21,9628. sin. $2p$	+ 9,8772	− 4,4879	+ 4,4879
	− 76,7501	− 253,6962	+ 29,3646
N. III. − 268, 7817.	+ 229,7000	− 618,3200	+ 122,9300
	+ 152,9499	− 872,0162	+ 152,2946
N. III. − 7,7213. cos. $2p$	− 7,1155	+ 3,2992	+ 3,2992
	+ 145,8344	− 868,7170	+ 155,5938
N. IV. − 716, 82	+ 2,4372	− 31,2540	+ 5,2329
	+ 148,2716	− 899,9710	+ 160,8267
N. IV. − 25,7376. cos. $2p$	− 0,4672	+ 0,0438	+ 0,0438
	+ 147,8044	− 899,9272	+ 160,8705
N. V. − 54,9069. sin. $2p$	+ 0,3980	− 2,4708	+ 2,4708
	+ 148,2024	− 902,3980	+ 163,3413
N. VI. + 1075,0302.	− 10,7503	− 12,1470	+ 76,9700
	+ 137,4521	− 914,5450	+ 240,3113
N. VI. + 38,6064 cos. $2p$	+ 1,1640	− 0,1930	− 0,1930
	+ 138,6161	− 914,7380	+ 240,1183
N. VII. 13, 72. sin. $2p$	− 1,6409	+ 7,8680	− 7,8680
	+ 136,9752	− 906,8700	+ 232,2503
Poster. Pars - - -	− 0,5360	− 2,6651	+ 0,5684
𝔐	+ 30,4392	− 909,5351	+ 232,8187

 §. 340.

§. 340.

Eodem modo colligemus valores litterae M.

	fin. t	fin. $2p - t$	fin. $2p + t$
N. I. + 10,9814. fin. $2p$	− 2,7993	− 0,8522	− 0,8522
N. II. − 537,5635.	+ 219,7000	− 558,5300	+ 74,9900
	+ 216,9007	− 559,3822	+ 74,1378
N. II. − 15,4425. cof. $2p$	+ 9,0994	− 3,1556	+ 3,1556
	+ 226,0001	− 562,5378	+ 77,2934
N. III. − 8,2360. fin. $2p$	− 11,3571	+ 3,5193	+ 3,5193
	+ 214,6430	− 559,0185	+ 80,8127
N. IV. − 18,3023. fin. $2p$	− 0,4658	+ 0,0311	+ 0,0311
	+ 214,1772	− 558,9874	+ 80,8438
N. V. 1075,0302.	+ 96,7500	− 40,2050	+ 24,6170
	+ 310,9272	− 599,1924	+ 105,4608
N. V. 38,6064. cof. $2p$	+ 0,7219	− 1,7372	+ 1,7372
	+ 0,4420		
	+ 312,0911	− 600,9296	+ 107,1980
N. VI. + 41,1802. fin. $2p$	− 1,7068	− 0,2059	− 0,2059
	+ 310,3843	− 601,1355	+ 106,9921
N. VII. − 268,7399.	− 308,0800	+ 10,5610	+ 53,6940
	+ 2,3043	− 590,5745	+ 160,6861
N. VII. − 9,6516. cof. $2p$	+ 0,7746	+ 5,5323	− 5,5323
	+ 3,0789	− 585,0422	+ 155,1538
Part. poster. − − −	+ 0,3632	+ 2,9029	− 0,3307
M	+ 3,4421	− 582,1393	+ 154,8231

§. 341.

§. 341.

Nunc ergo ex elementis numericis Cap. VII. §. 292. computemus litteras $\mathfrak{N}$ et N:

ω	t	$2p-t$	$2p+t$
L. M	+ 0,5368235	− 2,7650274	+ 2,1898271
L. $\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 0,0516847	+ 0,0165539
	+ 1,9639493	− 2,8167121	+ 2,2063810
$\frac{2(m+1)M}{\mu}$	+ 92,0340	− 655,7100	+ 160,8400
	− 136,4392	+ 909,5351	− 232,8187
Numer.	− 44,4052	+ 253,8251	− 71,9787
L.Num.	− 1,6474319	+ 2,4045258	− $\bar{1}$,8572058
L. Den.	+ 2,2460771	− 2,5868753	− 2,6859265
L. $\mathfrak{N}$	− 9,4013548	− 9,8176505	+ 9,1712793
L. $\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 0,0516847	+ 0,0165539
L. P. II.	− 0,8284806	− 9,8693352	+ 9,1878332
Log. M	+ 0,5368235	− 2,7650274	+ 2,1898271
Log μ^2	+ 0,0000000	+ 2,7508822	+ 2,8211438
L. P. I.	+ 0,5368235	− 0,0141452	+ 9,3686833
Pars I.	+ 3,4421	− 1,0331	+ 0,2337
− P. II.	+ 6,7373	+ 0,7402	− 0,1541
N	+ 10,1794	− 0,2929	+ 0,0796
et $\mathfrak{N}$	− 0,2519	− 0,6571	+ 0,1484

§. 342.

Pro partibus autem incognitis statuamus

$$\mathfrak{W} = \beta.\ \text{cof.}\ t + \gamma.\ \text{cof.}\ 2p - t + \delta.\ \text{cof.}\ 2p + t$$
$$W = b.\ \text{fin.}\ t + c.\ \text{fin.}\ 2p - t + d.\ \text{fin.}\ 2p + t.$$

hincque deducamus valores $\mathfrak{M}'$ et M':

Pro litteris $\mathfrak{M}'$.

	cof. t	cof. $2p-t$	cof. $2p+t$
$\mathfrak{W}.-9{,}2213.\text{cof.}\,2p$	$-4{,}6106.\gamma$ $-4{,}6106.\delta$	$-4{,}6106.\beta$	$-4{,}6106\,\beta$
$\mathfrak{W}.+0{,}02644.$	$+0{,}0264.\beta$	$+0{,}0264.\gamma$	$+0{,}0264.\delta$
$W.-3{,}9907\ \text{fin.}\,2p$	$-1{,}9952.c$ $-1{,}9952.d$	$+1{,}9952.b$	$+1{,}9952.b$

Pro litteris M'.

	fin. t	fin. $2p-t$	fin. $2p+t$
$\mathfrak{W}.-3{,}9907.\text{fin.}\,2p$	$-1{,}9952.\gamma$ $+1{,}9952.\delta$	$-1{,}9952.\beta$	$-1{,}9952.\beta$
$W.+5{,}3606.\text{cof.}\,2p$	$-2{,}6803.c$ $+2{,}6803.d$	$-2{,}6803.b$	$+2{,}6803.b$
$W.-0{,}0276.$	$-0{,}0272.b$	$-0{,}0272.c$	$-0{,}0272.d$

§. 343.

§. 343.

Sumamus nunc primo fecundam columnam, et quaeramus litteras $\mathfrak{N}'$ et N'.

Pro angulo $2p - t$.

	β	b	γ	c
Log. M'	−0,3000082	−0,4281834		−8,4352071
$L.\frac{2(m+1)}{\mu}$	+0,0516847	+0,0516847		+0,0516847
	−0,3516929	−0,4798681		−8,4868918
$\frac{2(m+1)M'}{\mu}$	−2,2474	−3,0190		−0,0307
$-\mathfrak{M}'$	+4,6106	+1,9952	−0,0264	
Num.	+2,3632	−1,0238	−0,0264	−0,0307
Log.	+0,3735005	−0,0092151	−8,4216039	−8,4871384
L. den.	−2,5868753	−2,5868753	−2,5868753	−2,5868753
Log. $\mathfrak{N}'$	+7,7866252	+7,4223398	+5,8347286	+5,9002631
$L.\frac{2(m+1)}{\mu}$	+0,0516847	+0,0516847	+0,0516847	+0,0516847
L. P. II.	−7,8383099	+7,4740245	+5,8864133	+5,9519478
Log. M'	−0,3000082	−0,4281834		−8,4352071
Log. μ^2	+2,7508820	+2,7508820		+2,7508820
Log. P. I.	−7,5491260	−7,6773014		−5,6843151
P. I.	−0,0035	−0,0048		−0,0000
−P. II.	+0,0069	−0,0030	−0,0001	−0,0001
N'	+0,0034	−0,0078	−0,0001	−0,0001
$\mathfrak{N}'$	−0,0061	+0,0026	+0,0001	+0,0001

hinc $\gamma = -0,6571 - 0,0061\,\beta + 0,0026.b + 0,0001\,\gamma + 0,0001.c.$

$c = -0,2929 + 0,0034.\beta - 0,0078.b - 0,0001\,\gamma - 0,0001.c.$

vbi manifesto bina vltima membra reiici possunt.

§. 344.

Eodem modo calculus se habet

Pro angulo $2p+t$.

	β	b	δ	d
Log. M′	− 0,3000082	+ 0,4281834		− 8,4352071
$L.\frac{2(m+1)}{\mu}$	+ 0,0165539	+ 0,0165539		+ 0,0165539
	− 0,3165621	+ 0,4447373		− 8,4517610
$\frac{2(m+1)M'}{\mu}$	− 2,0728	+ 2,7844		− 0,0283
− $\mathfrak{M}'$	+ 4,6106	− 1,9952	− 0,0264	
Num.	+ 2,5378	+ 0,7892	− 0,0264	− 0,0283
L. Num.	+ 0,4044574	+ 9,8971871	− 8,4216039	− 8,4517864
L. Den.	− 2,6859265	− 2,6859265	− 2,6859265	− 2,6859265
L. $\mathfrak{N}'$	− 7,7185309	− 7,2112606	+ 5,7356774	+ 5,7658599
$L.\frac{2(m+1)}{\mu}$	+ 0,0165539	+ 0,0165539	+ 0,0165539	+ 0,0165539
L. P. II.	− 7,7350848	− 7,2278145	+ 5,7522313	+ 5,7824138
L. M′	− 0,3000082	+ 0,4281834		− 8,4352071
L. μ^2	+ 2,8211438	+ 2,8211438		+ 2,8211438
L. P. I.	− 7,4788644	+ 7,6070396		− 5,6140633
P. I.	− 0,0030	+ 0,0041		− 0,0000
− P. II	+ 0,0054	+ 0,0017	− 0,0001	− 0,0001
N′	+ 0,0024	+ 0,0058	− 0,0001	− 0,0001
a: $\mathfrak{N}'$	− 0,0052	− 0,0016	+ 0,0001	+ 0,0001

hinc

$$\delta = +0{,}1484 - 0{,}0052\,\beta - 0{,}0016\,b$$
$$d = +0{,}0796 + 0{,}0024\,\beta + 0{,}0058.b.$$

§. 345.

§. 345.

Cum nunc hinc habeamus

$$\gamma+\delta=-0,5087-0,0113.\beta+0,0010.b.$$
$$\gamma-\delta=-0,8055-0,0009.\beta+0,0042.b.$$
$$c+d=-0,2133+0,0058.\beta-0,0020\ b.$$
$$c-d=-0,3725+0,0010.\beta-0,0136.b.$$

reperiemus

$$\mathfrak{M}+\mathfrak{M}'=+139,2101+0,0670.\beta-0,0006.b.$$
$$=\mathfrak{M} \text{ simpliciter. et}$$
$$M+M'=+6,0475-0,0009.\beta+0,0008.b.$$

vnde sequens calculus instituatur.

Pro angulo t.

	1	β	b.
L. M	+ 0,78157	− 6,95424	+ 6,90309
$L.\frac{2(m+1)}{\mu}$	+ 1,42712	+ 1,42712	+ 1,42712
	+ 2,20869	− 8,38136	+ 8,33021
$\frac{2(m+1)M}{\mu}$	+ 161,6900	− 0,0241	+ 0,0214
− $\mathfrak{M}$	− 139,2101	− 0,0670	+ 0,0006
Numer.	+ 22,4799	− 0,0911	+ 0.0220
L. Num.	+ 1,35177	− 8,95951	+ 8,34242
L. Den.	+ 2,24607	+ 2,24607	+ 2,24607
Log. $\mathfrak{N}$	+ 9,10570	− 6,71344	+ 6,09635
$L.\frac{2(m+1)}{\mu}$	+ 1,42712	+ 1,42712	+ 1,42712
L. P. II.	+ 0,53282	− 8,14056	+ 7,52347
P. I.	+ 6,0475	− 0,0009	+ 0,0008
− P. II.	− 3,4105	+ 0,0138	− 0,0033
N	+ 2,6370	+ 0,0129	− 0,0025
at $\mathfrak{N}$	+ 0,1276	− 0,0005	+ 0,0001

§ 346.

§. 346.

Hinc ergo consequimur

$$\beta = +0,1276 - 0,0005.\beta + 0,0001.b.$$

$$b = +2,6370 + 0,0129.\beta - 0,0025.b.$$

ex illa colligitur

$$\beta = +0,1275 + 0,0001.b.$$

qui valor in altera substitutus praebet

$$b = +2,6386 - 0,0025.b.$$

siue

$$b = +2,6319. \text{ et } \beta = +0,1278.$$

vnde reliquae litterae euadent

$$\begin{array}{l|l} \gamma = -0,6509 & c = -0,3130 \\ \delta = +0,1436 & d = +0,0952. \end{array}$$

sicque haec euolutio producit

$$\mathfrak{W} = +0,1278.\operatorname{cos}.t - 0,6509 \operatorname{cos}.2p - t$$
$$+0,1436.\operatorname{cos}.2p + t$$

$$W = +2,6319.\operatorname{sin}.t - 0,3130.\operatorname{sin}.2p - t$$
$$+0,0952.\operatorname{sin}.2p + t.$$

Secunda

Secunda euolutio, terminorum angulum $2q-t$ continentium.

§. 347.

Primo igitur ex aequatione priore colligamus litteram 𝔐:

	$\cos. 2q-t$	$\cos. 2p-2q+t$	$\cos 2p+2q-t$
N.I. $+537,6336$. -	$-101,40000$	$-1,9892$	$+15,8060$
	$-0,0286$	$-1,4562$	$-1,4562$
	$-101,4286$		
N.I. $+15,4426.\cos. 2p$	$+0,2270$		
	$-101,2016$	$-3,4454$	$+14,3498$
N.II. $+21,9628.\sin. 2p$	$-0,9180$	$+5,2731$	$-5,2731$
	$-0,1559$		
	$-102,2755$	$+1,8277$	$+9,0767$
N.III. $-268,7817$.	$-378,6300$	$+20,5880$	$-11,9870$
	$-480,9055$	$+22,4157$	$-2,9103$
N.III. $-7,7213.\cos. 2p$	$+0,2957$	$-5,4384$	$-5,4384$
	$-480,6098$		
	$-0,1722$		
	$-480,7820$	$+16,9773$	$-8,3487$
N.IV. $-716,82$.	$-2,2222$	$+3,2183$	$-15,7700$
	$-483,0042$	$+20,1956$	$-24,1187$

	cof. $2q-t$	cof. $2p-2q+t$	cof. $2p+2q-t$
	− 483,0042	+ 20,1956	− 24,1187
N. IV. − 25,7376. cof. $2p$	− 0,2831	− 0,0399	− 0,0399
	− 483,2873		
	+ 0,0579		
	− 483,2294	+ 20,1557	− 24,1586
N. V. − 54,9069. fin. $2p$	− 0,4338	+ 1,2272	− 1,2272
	− 483,6632		
	+ 1,1091		
	− 482,5541	+ 21,3829	− 25,3858
N. VI. + 1075,0302.	− 184,9000	− 1,8275	− 78,7980
	− 667,4541	+ 19,5554	− 104,1838
N. VI. + 38,6064. cof. $2p$	− 1,4149	− 3,3201	− 3,3201
	− 668,8690		
	− 0,0328		
	− 668,9018	+ 16,2353	− 107,5039
N. VII. 13, 72. fin. $2p$	+ 1,5078	+ 3,6032	− 3,6032
	− 667,3940	+ 19,8385	− 111,1071
	+ 0,9018	− 0,2479	+ 2,0028
	− 666,4922		
Pars pofter.	− 0,9654		
𝔐	− 667,4576	+ 19,5906	− 109,1043

§. 348.

§. 348.

Simili modo ex altera aequatione litteram M eliciamus.

	ſin. $2q - t$	ſin. $2p - 2q + t$	ſin. $2p + 2q - t$
N. I. + 10,9814. ſin. $2p$	− 0,0203	− 1,0355	− 1,0355
	− 0,1614		
	− 0,1817		
N. II. − 537, 5635.	− 258,1400	+ 44,9410	+ 7,6334
	− 258,3217	+ 43,9055	+ 6,5979
N. II. − 15,44 coſ. $2p$	− 0,5359	+ 3,7077	− 3,7075
	− 258,8576	+ 47,6132	+ 2,8904
N. III. − 8, 23. ſin. $2p$	+ 0,4991	− 5,8011	− 5,8011
	− 258,3585	+ 41,8121	− 2,9107
N. IV. − 18,3023. ſin. $2p$	+ 0,2425	− 0,0284	− 0,0284
	− 258,1160	+ 41,7837	− 2,9391
N. V. 1075, 0302.	− 48,0530	+ 16,9850	− 43,4310
	− 306,1690	+ 58,7687	− 46,3701
N. V. 38, 60. coſ. $2p$	− 0,3050	+ 0,8628	− 0,8628
	− 0,7798		
	− 307,2538	+ 59,6315	− 47,2329
N. VI. 41, 1802. ſin. $2p$	+ 1,4742	− 3,5415	− 3,5415
	− 305,7796	+ 56,0900	− 50,7744
N. VII. − 268, 7399.	− 141,0800	− 59,0420	− 35,3120
	− 446,8596	− 2,9520	− 86,0864
N. VII. − 9,6516. coſ. $2p$	+ 0,4261	+ 2,5335	− 2,5335
	− 446,4335	− 0,4185	− 88,6199
Pars poſter.	+ 1,5363	+ 0,3303	− 1,9959
M	− 444,8972	− 0,0882	− 90,6158

§. 349.

Constituamus igitur pro his angulis elementa numerica.

$\omega =$	$2q - t$	$2p - 2q + t$	$2p + 2q - t$
$\mu =$	$2n - 1$	$2m - 2n + 1$	$2m + 2n - 1$
$\mu =$	$+ 25,5121$	$- 0,7743$	$+ 50,2499$
$L\, 2(m+1) =$	$+ 1,42712$	$+ 1,42712$	$+ 1,42712$
$Log.\ \mu =$	$+ 1,40674$	$- 9,88891$	$+ 1,70114$
$Log.\ \frac{2(m+1)}{\mu} =$	$+ 0,02038$	$- 1,53821$	$+ 9,72598$
$Log.\ \mu^2 =$	$+ 2,81348$	$+ 9,77782$	$+ 3.40228$
$\lambda - 2 =$	$+ 177,2289$	$+ 177,2289$	$+ 177,2289$
$- \mu^2 =$	$- 650,8500$	$- 0,5996$	$- 2525,2000$
$Den. =$	$- 473\,6211$	$+ 176,6293$	$- 2347,9711$
$Log. =$	$- 2,67543$	$+ 2,24706$	$- 3,37069$

§. 350.

§. 350.

Hic iterum omittamus primam columnam, et calculum pro duabus sequentibus expediamus.

	$2p-2q+t$	$2p+2q-t$
Log. M	$-8,94546$	$-1,95720$
Log. $\frac{2(m+1)}{\mu}$	$-1,53821$	$+9,72598$
	$+0,48367$	$-1,68318$
$\frac{2(m+1)M}{\mu}$	$+3,0456$	$-48,2150$
$-\mathfrak{M}$	$-19,5906$	$+109,1043$
Numerat.	$-16,5450$	$+60,8893$
Log. Numerat.	$-1,21866$	$+1,78454$
Log. Denom.	$+2,24706$	$-3,37069$
L. $\mathfrak{N}$	$-8,97160$	$-8,41385$
adde Log. $\frac{2(m+1)}{\mu}$	$-1,53821$	$+9,72598$
L. Pars II.	$+0,50981$	$-8,13983$
L. M	$-8,94546$	$-1,95720$
L. μ^2	$+9,77782$	$+3,40228$
L. P. I.	$-9,16764$	$-8,55492$
P. I.	$-0,1471$	$-0,0359$
$-$P. II.	$-3,2345$	$+0,0138$
N	$-3,3816$	$-0,0221$
at $\mathfrak{N}$	$-0,0935$	$-0,0259$

§. 350.

Pro partibus incognitis ponamus

$\mathfrak{W} = \beta . \cos. 2q - t + \gamma \cos. 2p - 2q + t + \delta . \cos. 2p + 2q - t.$

$W = b . \sin. 2q - t + c . \sin. 2p - 2q + t + d . \sin. 2p + 2q - t.$

hincque deducimus

Pro angulo $2q - t$

$\mathfrak{M}' = -4,6106(\gamma + \delta) - 1,9952(c + d) + 0,0264.\beta.$

$M' = -1,9952(\gamma - \delta) - 2,6803(c - d) - 0,0272.b.$

Pro angulo $2p - 2q + t$

$\mathfrak{M}' = -4,6106.\beta - 1,9952.b + 0,0264.\gamma.$

$M' = -1,9952.\beta - 2,6803.b - 0,0272.c.$

Pro angulo $2p + 2q - t$

$\mathfrak{M}' = -4,6106.\beta + 1,9952.b + 0,0264.\delta.$

$M' = -1,9952.\beta + 2,6803.b - 0,0272.d.$

§. 351.

Incipiamus ergo a secundo angulo $2p-2q+r$ et calculus sequens instituatur.

Pro angulo $2p-2q+r$.

	β	b	γ	c
L. M′	− 0,30001	− 0,42818		− 8,43521
L. $\frac{2(m+1)}{\mu}$	− 1,53821	− 1,53821		− 1,53821
	+ 1,83822	+ 1,96639		+ 9,97342
$\frac{2(m+1)M'}{\mu}$	+ 68,9010	+ 92,5530		+ 0,9406
− 𝔐′	+ 4,6106	+ 1,9952	− 0,0264	
Numer.	+ 73,5116	+ 94,5482	− 0,0264	+ 0,9406
L. Num.	+ 1,86635	+ 1,97565	− 8,42160	+ 9,97340
L. Den.	+ 2,24706	+ 2,24706	+ 2,24706	+ 2,24706
L. 𝔑′	+ 9,61929	+ 9,72859	− 6,17454	+ 7,72634
L. $\frac{2(m+1)}{\mu}$	− 1,53821	− 1,53821	− 1,53821	− 1,53821
L. P. II.	− 1,15750	− 1,26680	+ 7,71275	− 9,26455
L. M′	− 0,30001	− 0,42818		− 8,43521
L. μ^2	+ 9,77782	+ 9,77782		+ 9,77782
L. P. I.	− 0,52219	− 0,65036		− 8,65739
P. I.	− 3,3281	− 4,4706		− 0,0454
− P. II.	+ 14,3690	+ 18,4840	− 0,0052	+ 0,1839
N′	+ 11,0409	+ 14,0134	− 0,0052	+ 0,1385
at 𝔑′	+ 0,4161	+ 0,5353	− 0,0001	+ 0,0053

hinc

hinc

$$\gamma = -0,0935 + 0,4161.\beta + 0,5353.b.$$
$$-0,0001.\gamma + 0,0053.c.$$
$$c = -3,3816 + 11,0409.\beta + 14,0134.b.$$
$$-0,0052.\gamma + 0,1385.c.$$

prior statim dat

$$\gamma = -0,0935 + 0,4161.\beta + 0,5352.b$$
$$+ 0,0053.c.$$

vnde fit

$$c = -3,3812 + 11,0388.\beta + 14,0106.b$$
$$+ 0,1385.c.$$

siue

$$c = -3,9252 + 12,8150.\beta + 16,2650.b.$$

et

$$\gamma = -0,1138 + 0,4841.\beta + 0,6214.b.$$

§. 352.

§. 352.

Pro tertio angulo bina postrema membra litteris δ et d adfecta praetermittere licet; vnde calculus erit sequens:

Pro angulo $2p + 2q - t$.

	β.	b.
L. M′	− 0,30001	+ 0,42818
L. $\frac{2(m+1)}{\mu}$	+ 9,72598	+ 9,72598
	− 0,02599	+ 0,15416
$\frac{2(m+1)M'}{\mu}$	− 1,0617	+ 1,4262
− $\mathfrak{M}'$	+ 4,6106	− 1,9952
Numer.	+ 3,5489	− 0,5690
L. Num.	+ 0,55009	− 9,75511
L. Den.	− 3,37069	− 3,37069
L. $\mathfrak{N}'$	− 7,17940	+ 6,38442
L. $\frac{2(m+1)}{\mu}$	+ 9,72598	+ 9,72598
L. P. II.	− 6,90538	+ 6,11040
L. M′.	− 0,30001	+ 0,42818
L. μ^2	+ 3,40228	+ 3,40228
L. P. I.	− 6,89773	+ 7,02590
P. I.	− 0,0008	+ 0,0011
− P. II.	+ 0,0008	− 0,0001
N′	+ 0,0000	+ 0,0010
at $\mathfrak{N}'$	− 0,0015	+ 0,0002

 hinc

hinc

$$\delta = -0,0259 - 0,0015.\beta + 0,0002.b.$$
$$d = -0,0221 + * \qquad + 0,0010.b.$$

§. 353.

Quia ergo hinc deducimus

$$\gamma + \delta = -0,1397 + 0,4826.\beta + 0,6216.b.$$
$$\gamma - \delta = -0,0879 + 0,4856.\beta + 0,6212.b.$$
$$c + d = -3,9473 + 12,8150.\beta + 16,2660.b.$$
$$c - d = -3,9031 + 12,8150.\beta + 16,2640.b.$$

obtinebimus pro primo angulo $2q - t$

$$\mathfrak{M} + \mathfrak{M}' = -658,9374 - 27,7686.\beta - 35,3220.b.$$
$$= \mathfrak{M} \text{ ſimpliciter}$$

$$M + M' = -434,2599 - 35,3169.\beta - 44,8655.b.$$
$$= M'. \text{ ſimpliciter.}$$

§. 354.

His ergo valoribus sequens calculus superstruatur

Pro angulo $2q - t$.

	I.	β	b
L. M	$-2,63775$	$-1,54798$	$-1,65191$
L. $\frac{2(m+1)}{\mu}$	$+0,02038$	$+0,02038$	$+0,02038$
	$-2,65813$	$-1,56836$	$-1,67229$
$\frac{2(m+1)M}{\mu}$	$-455,1300$	$-37,0140$	$-47,0210$
$-\mathfrak{M}$	$+658,9374$	$+27,7686$	$+35,3220$
Num.	$+203,8074$	$-9,2454$	$-11,6990$
L. Num.	$+2,30922$	$-0,96592$	$-1,06815$
L. Den.	$-2,67543$	$-2,67543$	$-2,67543$
L. $\mathfrak{N}$	$-9,63379$	$+8,29049$	$+8,39272$
L. $\frac{2(m+1)}{\mu}$	$+0,02038$	$+0,02038$	$+0,02038$
L. P. II.	$-9,65417$	$+8,31087$	$+8,41310$
L. M	$-2,63775$	$-1,54798$	$-1,65191$
L. μ^2	$2,81348$	$2,81348$	$2,81348$
L. P. I.	$-9,82427$	$-8,73450$	$-8,83843$
P. I.	$-0,6672$	$-0,0543$	$-0,0689$
$-$ P. II.	$+0,4510$	$-0,0204$	$-0,0259$
N $=$	$-0,2162$	$-0,0747$	$-0,0948$
at $\mathfrak{N} =$	$-0,4303$	$+0,0195$	$+0,0247$

 hinc

hinc

$$\beta = -0,4303 + 0,0195.\beta + 0,0247.b.$$
$$b = -0,2162 - 0,0747.\beta - 0,0948.b.$$

§. 355.

Ex posteriore colligimus

$b = -0,1975 - 0,0682.\beta.$ hincque

$\beta = -0,4352 + 0,0178.\beta.$

vnde concludimus

$$\beta = -0,4431. \text{ et } b = -0,1673.$$
$$\gamma = -0,4322. \text{ et } c = -11,3541.$$
$$\delta = -0,0253. \text{ et } d = -0,0223.$$

hic autem merito praegrandis valor litterae c maxime nobis suspectus videri debet; cum hinc in loco Lunae inaequalitas ad duo fere minuta prima assurgens nasceretur; qualem tamen obseruationes neutiquam ostendunt; neque tamen calculo repetito vllum errorem nobis deprehendere licuit.

§. 356.

Verum hic probe perpendendum est, nos non solum in hoc capite, sed etiam in praecedente terminos angulum $4p$ complexos plane neglexisse; quandoquidem hinc nullum errorem in reliquas inaequalitates irrepere animaduertimus. Verum hoc loco, vbi

vbi valor litterae M fere in nihilum abiit, iste neglectus insignis momenti esse poterit; manifestum autem est, leuissimam mutationem in hac littera factam plurimum calculum nostrum immutare potuisse ita, vt valor litterae c multo minor fuisset proditurus; quamobrem si eius valorem verum cognoscere vellemus, non solum in hoc capite, sed iam in praecedente necesse foret, cunctos calculos etiam ad terminos angulum $4p$ continentes extendere; quem laborem tam prolixis ac taediosis calculis iam plurimum defessi in aliud tempus differre cogimur; idque eo magis, quod reliquae inaequalitates hinc nullam sensibilem immutationem essent accepturae, omneque discrimen in solum hunc angulum $2p - 2q + t$ sit redundaturum; quin etiam inprimis formulas principales I, II, III etc. non solum ad quatuor, sed vsque ad sex figuras decimales computari oporteret. Quam ob caussam hic potius valorem litterae c tanquam incertum spectabimus, atque indefinitum relinquemus; interim tamen satis tuto colligere licet, eius valorem esse negatiuum; multo autem minorem, quam hic elicuimus.

§. 357.

Hoc igitur constituto valores in hac euolutione repertos ita exhibeamus:

$$\mathfrak{W} = -0,4431 . \cos. 2q - t - 0,4322 . \cos. 2p - 2q + t$$
$$- 0,0253 . \cos. 2p + 2q - t.$$
$$W = -0,1673 . \sin. 2q - t - \;.\;.\;.\;.\; \sin. 2p - 2q + t$$
$$- 0,0233 . \sin. 2p + 2q - t.$$

vbi notasse iuuabit, integram vnitatem in littera W tantum 10. secunda in loco Lunae producere; in littera $\mathfrak{W}$ vero vix vnum minutum secundum; vnde solus angulus medius fortasse ad vnum minutum secundum assurgere posset, sed praestat eius valorem tantisper tanquam incognitum spectare.

Tertia

Tertia Euolutio terminorum angulum $2q+t$ continentium.

§. 358.

Primo igitur ex aequatione priore colligamus litteram 𝔐.

	cos. $2q+t$	cos. $2p-2q-t$	cos. $2p+2q+t$
N. I. + 537, 6336.	+ 44,4080	− 201,8200	− 1,5591
N. I. + 15,4426. cos. $2p$	− 2,9210	+ 0,6378	+ 0,6378
	+ 41,4870	− 201,1822	− 0,9213
N. II. + 21,9628. sin. $2p$	+ 1,3341	− 2,8177	+ 2,8177
	+ 0,0022		
	+ 42,8233	− 203,9999	+ 1,8964
N. III. − 268, 7817.	+ 252,0800	+ 483,5300	+ 2,7684
	+ 294,9033	+ 279,5301	+ 4,6648
N. III. − 7,7213. cos. $2p$	+ 6,9452	+ 3,6208	+ 3,6208
	+ 0,0398		
	+ 301,8883	+ 283,1509	+ 8,2856
N. IV. − 716, 82.	+ 4,5161	− 14,4080	+ 1,7921
	+ 306,4044	+ 268,7429	+ 10,0777
N. IV. − 25,7376. cos. $2p$	− 0,2266	+ 0,0811	+ 0,0811
	+ 306,1778	+ 268,8240	+ 10,1588
N. V. − 54,9069. sin. $2p$	− 0,1565	− 1,5099	+ 1,5099
	+ 306,0213	+ 267,3141	+ 11,6687
N. VI. + 1075, 0302.	+ 21,2100	+ 0,8492	+ 0,9782
	+ 327,2313	+ 268,1633	+ 12,6469
N. VI. + 38,6064 cos. $2p$	+ 0,1525	+ 3,8087	+ 3,8087
	+ 0,1757		
	+ 327,5595	+ 271,9720	+ 16,4556
N. VII. 13, 72. sin. $2p$	− 0,8362	− 3,9225	+ 3,9225
	+ 326,7233	+ 268,0495	+ 20,3781
Pars poster.	+ 0,5761	+ 0,5248	− 0,2820
𝔐	+ 327,2994	+ 268,5743	+ 20,0961

§. 359.

Simili modo ex altera aequatione litteram M eliciamus.

	sin. $2q+t$	sin $2p-2q-t$	sin. $2p+2q+t$
N. I. + 10,9814. sin. $2p$	− 2,0612	+ 0,4535	+ 0,4535
	+ 0,0159		
	− 2,0453		
N. II. − 537, 5635.	+ 137,9300	− 65,3140	− 0,1074
	+ 135,8847	− 64,8605	+ 0,3461
N. II. − 15, 44. cos. $2p$	+ 0,9366	− 1,9812	+ 1,9812
	+ 136,8213	− 66,8417	+ 2,3273
N. III. − 8, 23. sin. $2p$	+ 7,4084	+ 3,8623	+ 3,8623
	− 0,0424		
	+ 143,3873	− 62,9794	+ 6,1896
N. IV. − 18, 30. sin. $2p$	− 0.2068	+ 0,0576	+ 0,0576
	+ 143,1805	− 62,9218	+ 6,2472
N. V. + 1075. 0302.	+ 0,0550	+ 0,9675	+ 5,1601
	+ 143,2355	− 61,9543	+ 11,4073
N. V. 38, 60. cos. $2p$	+ 0,0752	− 1,0616	+ 1,0616
	+ 143,3107	− 63,0159	+ 12,4689
N. VI. 41, 18. sin. $2p$	+ 0,1626	+ 4,0624	+ 4,0624
	− 0,1873		
	+ 143,2860	− 58,9535	+ 16,5313
N. VII. − 268, 7399.	+ 153,6600	+ 28,2440	+ 4,5149
	+ 296,9460	− 30,7095	+ 21,0462
N. VII. − 9, 6 cos. $2p$	− 0,5072	− 2,7594	+ 2,7594
	+ 0,0811		
	+ 296,5199	− 33,4689	+ 23,8056
Pars poster.	− 0,0020	+ 2,2329	+ 0,2888
M	+ 296,5179	− 31,2360	+ 24,0944

§. 360.

§. 360.

Nunc ergo pro his angulis conſtituamus elementa numerica.

ω	$2q+t$	$2p-2q-t$	$2p+2q+t$
$\mu =$	$2n+1$	$2m-2n-1$	$2m+2n+1$
$\mu =$	$+27,5121$	$-2,7743$	$+52,2499$
$L.\, 2(m+1) =$	$+1,42712$	$+1,42712$	$+1,42712$
$Log.\, \mu =$	$+1,43952$	$-0,44315$	$+1,71808$
$Log.\, \frac{2(m+1)}{\mu} =$	$+9,98760$	$-0,98397$	$+9,70904$
$Log.\, \mu^2 =$	$+2,87904$	$+0,88630$	$+3,43616$
$\lambda - 2 =$	$+177,2289$	$+177,2289$	$+177,2289$
$\mu^2 =$	$-756,9100$	$-7,6967$	$-2730,0000$
Num. $=$	$-579,6811$	$+169,5322$	$-2552,7711$
Log. $=$	$-2,76319$	$+2,22925$	$-3,40702$

§. 361.

Omisso primo angulo pro binis reliquis calculus ita se habebit.

	Pro $2p-2q-t$.	Pro $2p+2q+t$.
Log. M	$-1,49465$	$+1,38191$
L. $\frac{2(m+1)}{\mu}$	$-0,98397$	$+9,70904$
	$+2,47862$	$+1,09095$
	$+301,0400$	$+12,3291$
$-\mathfrak{M}$	$-268,5743$	$-20,0961$
Numer.	$+32,4657$	$-7,7670$
L. Num.	$+1,51143$	$-0,89025$
L. den.	$+2,22925$	$-3,40702$
Log. $\mathfrak{N}$	$+9,28218$	$+7,48323$
$\frac{2(m+1)}{\mu}$	$-0,98397$	$+9,70904$
L. P. II	$-0,26615$	$+7,19227$
Log. M	$-1,49465$	$+1,38191$
Log. μ^2	$+0,88630$	$+3,43616$
L. P. I.	$-0,60835$	$+7,94575$
P. I.	$-4,0584$	$+0,0088$
$-$ P. II.	$+1,8456$	$-0,0015$
N	$-2,2128$	$+0,0073$
$\mathfrak{N}$	$+0,1915$	$+0,0030$

§. 362.

§. 362.

Ponendo nunc pro partibus incognitis

$$\mathfrak{W} = \beta.\ \text{cof.}\ 2q + t + \gamma.\ \text{cof.}\ 2p - 2q - t$$
$$+ \delta.\ \text{cof.}\ 2p + 2q + t$$
$$W = b.\ \text{fin.}\ 2q + t + c.\ \text{fin.}\ 2p - 2q - t$$
$$+ d.\ \text{fin.}\ 2p + 2q + t.$$

ftatim colligimus

Pro angulo $2q + t$

$$\mathfrak{M}' = -4{,}6106(\gamma + \delta) - 1{,}9952\ (c + d) + 0{,}0264.\ \beta$$
$$M' = -1{,}9952(\gamma - \delta) - 2{,}6803\ (c - d) - 0{,}0272.\ b.$$

Pro angulo $2p - 2q - t$

$$\mathfrak{M}' = -4{,}6106.\ \beta - 1{,}9952.\ b + 0{,}0264.\ \gamma$$
$$M' = -1{,}9952.\ \beta - 2{,}6803.\ b - 0{,}0272.\ c.$$

Pro angulo $2p + 2q + t$

$$\mathfrak{M}' = -4{,}6106.\ \beta + 1{,}9952.\ b + 0{,}0264.\ \delta$$
$$M' = -1{,}9952.\ \beta + 2{,}6803.\ b - 0{,}0272.\ d.$$

§. 363.

Hinc faciamus calculum sequentem

Pro angulo $2p - 2q - t$.

	β	b	γ	c
L. M'	$-0,30001$	$-0,42818$		$-8,43521$
L. $\frac{2(m+1)}{\mu}$	$-0,98397$	$-0,98397$		$-0,98397$
	$+1,28398$	$+1,41215$		$+9,41918$
$\frac{2(m+1)M'}{\mu}$	$+19,2300$	$+25,8320$		$+0,2625$
$-\mathfrak{M}'$	$+4,6106$	$+1,9952$	$-0,0264$	
Numer.	$+23,8406$	$+27,8272$	$-0,0264$	$+0,2625$
L.Num.	$+1,37732$	$+1,44447$	$-8,42160$	$+9,41918$
L. Den.	$+2,22925$	$+2,22925$	$+2,22925$	$+2,22925$
L. $\mathfrak{N}'$	$+9,14807$	$+9,21522$	$-6,19235$	$+7,18993$
L. $\frac{2(m+1)}{\mu}$	$-0,98397$	$-0,98397$	$-0,98397$	$-0,98397$
L. P. II.	$-0,13204$	$-0,19919$	$+7,17632$	$-8,17390$
L. M'	$-0,30001$	$-0,42818$		$-8,43521$
L. μ^2	$+0,88630$	$+0,88630$		$+0,88630$
L. P. I.	$-9,41371$	$-9,54188$		$-7,54891$
P. I.	$-0,2592$	$-0,3483$		$-0,0035$
- P. II.	$+1,3553$	$+1,5820$	$-0,0015$	$+0,0149$
N'	$+1,0961$	$+1,2337$	$-0,0015$	$+0,0114$
at $\mathfrak{N}'$	$+0,1406$	$+0,1641$	$-0,0001$	$+0,0015$

hinc $\gamma = +0,1915 + 0,1406.\beta + 0,1641.b - 0,0001.\gamma + 0,0015.c.$

$c = -2,2128 + 1,0961.\beta + 1,2337.b - 0,0015.\gamma + 0,0114.c.$

vnde concluditur

$$\gamma = +0,1883 + 0,1424.\beta + 0,1659\ b. \text{ et}$$

$$c = -2,2382 + 1,1084.\beta + 1,2474.b.$$

§. 364.

§. 364.

Pro litteris autem δ et d sequens calculus sufficiet

Pro angulo $2p + 2q + t$.

	β	b.
Log. M′	− 0,30001	+ 0,42818
L. $\frac{2(m+1)}{\mu}$	+ 9,70904	+ 9,70904
	− 0,00905	+ 0,13722
	− 1,0211	+ 1,3716
− 𝔐′	+ 4,6106	− 1,9952
Num.	+ 3,5895	− 0,6236
L.Num.	+ 0,55505	− 9,79491
L. Den.	− 3,40702	− 3,40702
L. 𝔑	− 7,14803	+ 6,38789
	+ 9,70904	+ 9,70904
L. P. II.	− 6,85707	+ 6,09693
L. M′	− 0,30001	+ 0,42818
L. μ^2	+ 3,43616	+ 3,43616
L. P. I.	− 6,86385	+ 6,99202
P. I.	− 0,0007	+ 0,0010
− P. II.	+ 0,0007	− 0,0001
N	+ 0,0000	+ 0,0009
et 𝔑	− 0,0014	+ 0,0002

hinc

$$\delta = + 0,0030 - 0,0014.\beta + 0,0002.b.$$
$$d = + 0,0073 + * \qquad + 0,0009.b.$$

§. 365.

Cum nunc hinc fiat

$$\gamma + \delta = + 0,1913 + 0,1410.\beta + 0,1661.b.$$
$$\gamma - \delta = + 0,1853 + 0,1438.\beta + 0,1657.b.$$
$$c + d = - 2,2309 + 1,1084.\beta + 1,2483.b.$$
$$c - d = - 2,2455 + 1,1084.\beta + 1,2465.b.$$

pro hoc angulo $2q + t$ conficietur

$$\mathfrak{M} + \mathfrak{M}' = + 330,8690 - 2,8353.\beta - 3,2566.b.$$
$$= \mathfrak{M} \text{ simpliciter. et}$$

$$M + M' = + 302,1668 - 3,2577.\beta - 3,6988.b.$$
$$= M \text{ simpliciter.}$$

quibus

quibus valoribus superstruatur sequens calculus

Pro angulo $2q+t$.

	1.	β	b
L. M	+ 2,48025	− 0,51291	− 0,56806
L. $\frac{2(\pi+1)}{\mu}$	+ 9,98760	+ 9,98760	+ 9,98760
	+ 2,46785	− 0,50051	− 0,55566
	+ 293,7400	− 3,1660	− 3,5947
− 𝔐	− 330,8690	+ 2,8353	+ 3,2566
Numer.	− 37,1290	− 0,3307	− 0,3381
L. Num.	− 1,56971	− 9.51943	− 9,52904
L. Den.	− 2,76319	− 2,76319	− 2,76319
Log. 𝔑	+ 8,80652	+ 6,75624	+ 6,76585
	+ 9,98760	+ 9,98760	+ 9,98760
L. P. II	+ 8,79412	+ 6,74384	+ 6,75345
L. M	+ 2,48025	− 0,51291	− 0,56806
Log. μ^2	2,87904	2,87904	2,87904
L. P. I.	+ 9,60121	− 7,63387	− 7,68902
P. I.	+ 0,3992	− 0,0044	− 0,0049
− P. II.	− 0,0622	− 0,0005	− 0,0005
N	+ 0,3370	− 0,0049	− 0,0054
at 𝔑	+ 0,0640	+ 0,0005	+ 0,0005

§. 366.

Hinc ergo deducimus istas determinationes:

$$\beta = +0,0640 + 0,0005.\beta + 0,0005.b.$$
$$b = +0,3370 - 0,0049.\beta - 0,0054.b.$$

ex

ex priore ſequitur

$$\beta = +0,0640 + 0,0005 . b.$$

qui valor in altera ſubſtitutus dat

$$b = +0,3367 - 0,0054 . b.$$

atque hinc

$$b = +0,3349.$$

atque hinc reliqui coefficientes ita erunt comparati:

$\beta = +0,0642$	$b = +0,3349$
$\gamma = +0,2528$	$c = -1,7240$
$\delta = +0,0030$	$d = +0,0076.$

Concluſio.

Omnino ergo valores, quos in hoc capite inueſtigauimus, ſunt ſequentes:

$$\begin{aligned}\mathfrak{W} = &+0,1278 . \operatorname{coſ.} t - 0,6509 . \operatorname{coſ.} 2p - t \\ &+0,1436 . \operatorname{coſ.} 2p + t \\ &-0,4431 . \operatorname{coſ.} 2q - t - 0,4322 . \operatorname{coſ.} 2p - 2q + t \\ &-0,0253 . \operatorname{coſ.} 2p + 2q - t \\ &+0,0642 . \operatorname{coſ.} 2q + t + 0,2528 . \operatorname{coſ.} 2p - 2q - t \\ &+0,0030 . \operatorname{coſ.} 2p + 2q + t.\end{aligned}$$

$$\begin{aligned}W = &+2,6319 . \operatorname{ſin.} t - 0,3130 . \operatorname{ſin.} 2p - t \\ &+0,0952 . \operatorname{ſin.} 2p + t \\ &-0,1673 . \operatorname{ſin.} 2q - t - \ldots\ldots \operatorname{ſin.} 2p - 2q + t \\ &-0,0233 . \operatorname{ſin.} 2p + 2q - t \\ &+0,3349 . \operatorname{ſin.} 2q + t - 1,7240 . \operatorname{ſin.} 2p - 2q - t \\ &+0,0076 . \operatorname{ſin} 2p + 2q + t.\end{aligned}$$

CAPVT X.

CAPUT X.

EVOLUTIO AEQUATIONUM ORDINIS X, CHARACTERE $a\,\varkappa$ PRO LITTERIS $\mathfrak{w}$ et w.

§. 367.

Partes annexae harum aequationum ita exhibentur:

$$
\begin{aligned}
\text{I. } 0 = \ldots & \mathfrak{w}.\mathfrak{A} + w.\mathfrak{B} \\
& + 2\mathfrak{S}\mathfrak{U}.\mathfrak{C} + (\mathfrak{S}U + S\mathfrak{U})\mathfrak{D} + 2SU\mathfrak{E} \\
& + (-\tfrac{9}{4}\cos. p - \tfrac{15}{4}\cos. 3p)\mathfrak{U}(1+\mathfrak{D}) \\
& + (\tfrac{3}{4}\sin. p + \tfrac{15}{4}\sin. 3p)U(1+\mathfrak{D}) \\
& + (\tfrac{3}{4}\sin. p + \tfrac{15}{4}\sin. 3p)\mathfrak{U}.O \\
& + (-\tfrac{3}{4}\cos. p + \tfrac{15}{4}\cos. 3p)U.O \\
& + (\tfrac{3}{2}\cos. t + \tfrac{21}{4}\cos.(2p-t) - \tfrac{3}{4}\cos.(2p+t))\,\mathfrak{S} \\
& + (-\tfrac{21}{4}\sin.(2p-t) + \tfrac{3}{4}\sin.(2p+t))\,S. \\
& + (\tfrac{27}{8}\cos(p-t) + \tfrac{9}{8}\cos(p+t) + \tfrac{75}{8}\cos.(3p-t) - \tfrac{15}{8}\cos.(3p+t))(1+2\mathfrak{D}+\mathfrak{D}^2) \\
& + (-\tfrac{9}{4}\sin.(p-t) - \tfrac{3}{4}\sin.(p+t) - \tfrac{75}{4}\sin.(3p-t) + \tfrac{15}{4}\sin.(3p+t))(O+\mathfrak{D}O) \\
& + (\tfrac{9}{8}\cos.(p-t) + \tfrac{3}{8}\cos.(p+t) - \tfrac{75}{8}\cos. 3p-t + \tfrac{15}{8}\cos. 3p+t)\,O^2.
\end{aligned}
$$

 II. $0 =$

$$\text{II. } 0 = \ldots\ldots \mathfrak{w}.A + w.B.$$
$$+2\mathfrak{S}\mathfrak{U}.C + (\mathfrak{S}U + S\mathfrak{U})D + 2SU.E$$
$$+(\tfrac{3}{4}\sin.p + \tfrac{15}{4}\sin.3p)\mathfrak{U}(1+\mathfrak{O})$$
$$+(-\tfrac{3}{4}\cos p + \tfrac{15}{4}\cos.3p)U(1+\mathfrak{O})$$
$$+(-\tfrac{3}{4}\cos.p + \tfrac{15}{4}\cos.3p)\mathfrak{U}O.$$
$$+(\tfrac{9}{4}\sin.p - \tfrac{15}{4}\sin.3p)UO.$$
$$+(-\tfrac{21}{4}\sin.(2p-t) + \tfrac{3}{4}\sin.(2p+t))\mathfrak{S}$$
$$+(\tfrac{6}{4}\cos.t - \tfrac{21}{4}\cos.2p-t + \tfrac{3}{4}\cos.2p+t)S$$
$$+(-\tfrac{9}{8}\sin.(p-t) - \tfrac{3}{8}\sin.p+t) - \tfrac{75}{8}\sin.(3p-t) + \tfrac{15}{8}\sin.(3p+t))'1 + 2\mathfrak{O} + \mathfrak{O}')$$
$$+(\tfrac{9}{4}\cos p-t + \tfrac{3}{4}\cos.p+t - \tfrac{75}{4}\cos.3p-t + \tfrac{15}{4}\cos.3p+t))O + \mathfrak{O}O)$$
$$+(-\tfrac{27}{8}\sin.p-t - \tfrac{9}{8}\sin.p+t + \tfrac{75}{8}\sin.3p-t - \tfrac{15}{8}\sin.3p+t)O^2.$$

§. 368.

Hic ante omnia distinguere debemus ea producta, quae vtrique aequationi sunt communia, ab iis, quae in alterutra seorsim occurrunt: quocirca primum euoluamus membra vtrique aequationi communia:

I. $2\mathfrak{S}\mathfrak{U}$ dat

$$+0{,}0025.\cos.p-t + 0{,}0033.\cos.3p-t$$
$$-0{,}0012.\cos.p+t - 0{,}0004.\cos.3p+t$$

cuius multiplicatores pro

$$\begin{array}{l} \text{prima} \\ \text{secunda} \end{array}\left\{\begin{array}{l} +537{,}6336 + 15{,}4426.\cos.2p. \\ +10{,}9814.\sin.2p. \end{array}\right.$$

II.

II. $\mathfrak{S}U + S\mathfrak{U}$ dat

$$- 0{,}0161.\ \text{fin.}\ p - t - 0{,}0060.\ \text{fin.}\ 3p - t$$
$$+ 0{,}0116.\ \text{fin.}\ p + t + 0{,}0007.\ \text{fin.}\ 3p + t.$$

cuius multiplicator pro

prima $\left\{ \begin{array}{l} + 21{,}9628.\ \text{fin.}\ 2p \\ - 537{,}5635 - 15{,}4425.\ \text{cof.}\ 2p. \end{array} \right.$
fecunda

III. $2SU$ dat

$$- 0{,}0354.\ \text{cof.}\ p - t - 0{,}0104.\ \text{cof.}\ 3p - t$$
$$+ 0{,}0445.\ \text{cof.}\ p + t + 0{,}0013.\ \text{cof.}\ 3p + t.$$

cuius multiplicatores pro

prima $\left\{ \begin{array}{l} - 268{,}7817 - 7{,}7213.\ \text{cof.}\ 2p \\ - 8{,}2360.\ \text{fin.}\ 2p. \end{array} \right.$
fecunda

§. 369.

Producta vero, quae primae aequationi funt propria, ita fe habent euoluta:

$$\text{IV}^{\text{L}}.\ (-\tfrac{9}{4}\ \text{cof.}\ p - \tfrac{15}{4}\ \text{cof.}\ 3p)\ \mathfrak{U}$$
$$+ (+\tfrac{3}{4}\ \text{fin.}\ p + \tfrac{15}{4}\ \text{fin.}\ 3p)\ U$$

coniunctim praebent

$$+ 0,0470 . \cos . p - t + 0,3534 . \cos . 3p - t$$
$$- 0,1943 . \cos . p + t - 0,3427 . \cos . 3p + t$$

cuius multiplicator est

$$1 - 0,0072 . \cos . 2p .$$

$\mathrm{V}^{\mathrm{I}}.$ $\left(+ \frac{3}{4} \sin . p + \frac{15}{4} \sin . 3p\right) \mathfrak{U}$
$$+ \left(- \frac{3}{4} \cos . p + \frac{15}{4} \cos . 3p\right) \mathrm{U}$$

praebet

$$+ 0,0573 . \sin . p - t - 0,3429 . \sin . 3p - t$$
$$+ 0,0615 . \sin . p + t + 0,3412 . \sin . 3p + t$$

cuius multiplicator est

$$+ 0,0102 . \sin . 2p .$$

$\mathrm{VI}^{\mathrm{I}}.$ $\left(\frac{3}{2} \cos . t + \frac{21}{4} \cos . 2p - t - \frac{3}{4} \cos . 2p + t\right) \mathfrak{S}$
$+ \left(- \frac{21}{4} \sin . 2p - t + \frac{3}{4} \sin . 2p + t\right) \mathrm{S}.$ dat

$$+ 1,0209 . \cos . p - t - 0,3310 . \cos . 3p - t$$
$$- 0,0621 . \cos . p + t + 0,0448 . \cos . 3p + t$$

cuius multiplicator $= 1$.

$\mathrm{VII}^{\mathrm{I}}.$ $\left(\frac{27}{8} \cos . p - t + \frac{9}{8} \cos . p + t + \frac{75}{8} \cos . 3p - t - \frac{15}{8} \cos . 3p + t\right)$
dat

+ 3,

$+ 3,3750. \cos. p - t + 9,3750. \cos. 3p - t$

$+ 1,1250. \cos. p + t - 1,8750. \cos. 3p + t.$

eius multiplicator est

$1 - 0,0143. \cos. 2p.$

VIII^I. $(-\frac{9}{4} \sin. p - t - \frac{3}{4} \sin. p + t - \frac{75}{4} \sin. 3p - t + \frac{15}{4} \sin. 3p + t)$
dat

$- 2,2500. \sin. p - t - 18,7500. \sin. 3p - t$

$- 0,7500. \sin. p + t + 3,7500. \sin. 3p + t$

eius multiplicator est $+ 0,0102. \sin. 2p.$

§. 370.

Partes vero posteriori aequationi propriae ita definiuntur:

IV^II. $(+\frac{3}{4} \sin. p + \frac{15}{4} \sin. 3p)\, u$
$+ (-\frac{5}{4} \cos. p + \frac{15}{4} \cos. 3p)\, U$

praebet

$+ 0,0574. \sin. p - t - 0,3429. \sin. 3p - t$

$+ 0,0615. \sin. p + t + 0,3412. \sin. 3p + t$

eius multiplicator est

$1 - 0,0072. \cos. 2p.$

$\text{V}^{\text{II}}.\ (-\tfrac{8}{4}\,\text{cos}.\,p + \tfrac{15}{4}\,\text{cos}.\,3\,p)\,\mathfrak{U}$
$\qquad + (\tfrac{9}{4}\,\text{sin}.\,p - \tfrac{15}{4}\,\text{sin}.\,3\,p)\,U$

dat

$$+\,0,1469.\,\text{cos}.\,p - t - 0,3325.\,\text{cos}.\,3\,p - t$$
$$-\,0,1232.\,\text{cos}.\,p + t + 0,3396.\,\text{cos}.\,3\,p + t$$

cuius multiplicator est

$$+\,0,0102.\,\text{sin}.\,2\,p.$$

$\text{VI}^{\text{II}}.\ (-\tfrac{21}{4}\,\text{sin}.\,2\,p - t + \tfrac{3}{4}\,\text{sin}.\,2\,p + t)\,\mathfrak{S}$
$\qquad + (\tfrac{3}{8}\,\text{cos}.\,t - \tfrac{21}{4}\,\text{cos}.\,2\,p - t + \tfrac{3}{4}\,\text{cos}.\,2\,p + t)\,S$

dat

$$-\,1,1105.\,\text{sin}.\,p - t + 0,3309.\,\text{sin}.\,3\,p - t$$
$$-\,0,2446.\,\text{sin}.\,p + t - 0,0448.\,\text{sin}.\,3\,p + t$$

eius multiplicator $= 1$.

$\text{VII}^{\text{II}}.\ (-\tfrac{9}{8}\,\text{sin}.\,p - t - \tfrac{3}{8}\,\text{sin}.\,p + t - \tfrac{75}{8}\,\text{sin}.\,3\,p - t + \tfrac{15}{8}\,\text{sin}.\,3\,p + t)$

dat

$$-\,1,1250.\,\text{sin}.\,p - t - 9,3750.\,\text{sin}.\,3\,p - t$$
$$-\,0,3750.\,\text{sin}.\,p + t + 1,8750.\,\text{sin}.\,3\,p + t$$

eius multiplicator

$$= 1 - 0,0143.\,\text{cos}.\,2\,p.$$

VIII.

VIII^II. $(+\frac{9}{4}\,\text{cof.}\,p-t+\frac{3}{4}\,\text{cof.}\,p+t-\frac{75}{4}\,\text{cof.}\,3p-t+\frac{15}{4}\,\text{cof.}\,3p+t)$
dat

$$+2,2500.\ \text{cof.}\ p-t-18,7500.\ \text{cof.}\ 3p-t$$
$$+0,7500.\ \text{cof.}\ p+t+3,7500.\ \text{cof.}\ 3p+t.$$

eius multiplicator eſt $+0,0102.$ ſin. $2p.$

§. 371.

Membra haec vtrique aequationi propria ad minorem numerum redigere licet, quandoquidem in iis tantum tres multiplicatores, vel vnitas vel ♎ vel 0, occurrunt; quae idcirco ſequenti modo repraeſentari poſſunt:

Pro priore aequatione.

I'. $+4,4429.\ \text{cof.}\ p-t+9,3974.\ \text{cof.}\ 3p-t$
$+0,8686.\ \text{cof.}\ p+t-2,1729.\ \text{cof.}\ 3p+t$
eius multiplicator $=1.$

II'. $+6,7970.\ \text{cof.}\ p-t+19,1034.\ \text{cof.}\ 3p-t$
$+2,0557.\ \text{cof.}\ p+t-4,0927.\ \text{cof.}\ 3p+t$
eius multiplicator $=-0,0072.$ cof. $2p.$

III'.

III¹. $- 2,1927 . \sin . p - t - 19,0929 . \sin . 3p - t$
$- 0,6885 . \sin . p + t + 4,0912 . \sin . 3p + t$
eius multiplicator $= + 0,0102 . \sin . 2p$.

Pro aequatione posteriore.

I². $- 2,1781 . \sin . p - t - 9,3870 . \sin . 3p - t$
$- 0,5581 . \sin . p + t + 2,1714 . \sin . 3p + t$
eius multiplicator $= 1$.

II². $- 2,1926 . \sin . p - t - 19,0929 . \sin . 3p - t$
$- 0,6885 . \sin . p + t + 4,0912 . \sin . 3p + t$
eius multiplicator $= - 0,0072 . \cos . 2p$.

III². $+ 2,3969 . \cos . p - t - 19,0825 . \cos . 3p - t$
$+ 0,6268 . \cos . p + t + 4,0890 . \cos . 3p + t$
eius multiplicator $= + 0,0102 . \sin . 2p$.

§. 372.

Nunc ergo ex membris cognitis prioris aequationis colligamus litteram 𝔐.

	cos. $p-t$.	cos. $p+t$.	cos. $3p-t$.	cos. $3p+t$.
I. + 537, 6336.	+ 1,3440	− 0,6452	+ 1,7740	− 0,2150
I. + 15,4426. cos. $2p$	− 0,0093	+ 0,0193	+ 0,0193	− 0,0093
	+ 1,3347	− 0,6259		
	+ 0,0255	− 0,0031		
	+ 1,3602	− 0,6290	+ 1,7933	− 0,2243
II. + 21,9628 sin. $2p$	+ 0,1274	− 0,1768	+ 0,1768	− 0,1274
	+ 1,4876	− 0,8058		
	− 0,0659	+ 0,0076		
	+ 1,4217	− 0,7982	+ 1,9701	− 0,3517
III. − 268, 7817.	+ 9,5150	− 11,9600	+ 2,7952	− 0,3494
	+ 10,9367	− 12,7582	+ 4,7653	− 0,7011
III. − 7,7213. cos $2p$	− 0,1718	+ 0,1366	+ 0,1366	− 0,1718
	+ 10,7649	− 12,6216	+ 4,9019	− 0,8729
	+ 0,0401	− 0,0050		
	+ 10,8050	− 12,6266		
I'.	+ 4,4429	+ 0,8686	+ 9,3974	− 2,1729
	+ 15,2479	− 11,7580	+ 14,2993	− 3,0458
I'. − 0,0072. cos $2p$	− 0.0762	− 0,0097	− 0,0244	− 0,0074
	+ 15,1717	− 11,7677	+ 14,2749	− 3,0532
II'. + 0,0102. sin. $2p$	+ 0,0097	− 0,1009	+ 0,0112	+ 0,0035
𝔐	+ 15,1814	− 11,8686	+ 14,2861	− 3,0497

§. 373.

Eodem modo ex posteriore aequatione computemus litteram M.

	sin. $p-t$	sin. $p+t$	sin. $3p-t$	sin. $3p+t$
I. + 10, 9814. sin. 2 p	− 0,0066	+ 0,0137	+ 0,0137	− 0,0066
	− 0,0181	+ 0,0022		
	− 0,0247	+ 0,0159		
II. − 537, 5635.	+ 8,6540	− 6,2350	+ 3,2250	− 0,3762
	+ 8,6293	− 6,2191	+ 3,2387	− 0,3828
II. − 15, 4425. cos. 2 p	+ 0,0895	− 0,1243	+ 0,1243	− 0,0895
	+ 8,7188	− 6,3434		
	+ 0,0463	− 0,0054		
	+ 8,7651	− 6,3488	+ 3,3630	− 0,4723
III. − 8, 2360. sin. 2 p	− 0,1833	+ 0,1458	+ 0,1458	− 0,1833
	+ 8,5818	− 6,2030		
	− 0,0428	+ 0,0053		
	+ 8,5390	− 6,1977	+ 3,5088	− 0,6556
I². − 1.	− 2,1781	− 0,5581	− 9,3870	+ 2,1714
	+ 6,3609	− 6,7558	− 5,8782	+ 1,5158
II². − 0, 0072. cos. 2 p	− 0,0025	− 0,0079	+ 0,0079	+ 0,0025
	+ 6,3584	− 6,7637		
	+ 0,0688	− 0,0147		
	+ 6,4272	− 6,7784	− 5,8703	+ 1,5183
III². + 0, 0102. sin. 2 p	+ 0,0032	+ 0,0121	+ 0,0121	+ 0,0032
	+ 0,0973	− 0,0208		
M	+ 6,5277	− 6,7871	− 5,8582	+ 1,5215

§. 374.

§. 374.

Nunc ergo constituenda sunt elementa numerica

$\omega =$	$p - t$	$p + t$	$3p - t$	$3p + t$
$\mu =$	$m - 1$	$m + 1$	$3m - 1$	$3m + 1$
$\mu =$	11,36892	13,36892	36,10676	38,10676
$l.2(m+1) =$	1,4271	1,4271	1,4271	1,4271
$Log.\ \mu =$	1,0557	1,1260	1,5576	1,5809
$Log.\ \frac{2(m+1)}{\mu} =$	0,3714	0,3011	9,8695	9,8462
$Log.\ \mu^2 =$	2,1114	2,2520	3,1152	3,1618
$\lambda - 2 =$	177,2289	177,2289	177,2289	177,2289
$-\mu^2 =$	−129,2500	− 178,7275	−1304,0000	−1451,5000
Den. $=$	+ 47,9789	− 1,4986	−1126,7711	−1274,2711
Log. $=$	+ 1,68105	− 0,17568	− 3,05184	− 3,10527

 §. 375.

§. 375.

Hinc ergo quaeramus litteras $\mathfrak{N}$ et N.

	$p-t$	$p+t$	$3p-t$	$3p+t$
L. M	+ 0,81476	− 0,83168	− 0,76813	+ 0,18227
L. $\frac{2(m+1)}{\mu}$	+ 0,37140	+ 0,30100	+ 9,86950	+ 9,84620
	+ 1,18616	− 1,13268	− 0,63763	+ 0,02847
$\frac{2(m+1)M}{\mu}$	+ 15,3520	− 13,5730	− 4,3414	+ 1,0677
$-\mathfrak{M}$	− 15,1814	+ 11,8686	− 14,2861	+ 3,0497
Numer.	+ 0,1706	− 1,7044	− 18,6275	+ 4,1174
L Num.	+ 9,23198	− 0,23157	− 1,27014	+ 0,61462
L. den.	+ 1,68105	− 0,17568	− 3,05184	− 3,10527
Log. $\mathfrak{N}$	+ 7,55093	+ 0,05589	+ 8,21830	− 7,50935
	+ 0,37140	+ 0,30100	+ 9,86950	+ 9,84620
L. P. II.	+ 7,92233	+ 0,35689	+ 8,08780	− 7,35555
Log. M	+ 0,81476	− 0,83168	− 0,76813	+ 0,18227
Log. μ^2	+ 2,11140	+ 2,25220	+ 3,11520	+ 3,16180
L. P. I.	+ 8,70336	− 8,57948	− 7,65293	+ 7,02047
P. I.	+ 0,0505	− 0,0380	− 0,0044	+ 0,0010
− P. II.	− 0,0084	− 2,2745	− 0.0122	+ 0,0023
N	+ 0,0421	− 2,3125	− 0,0166	+ 0,0033
$\mathfrak{N}$	+ 0,0036	+ 1,1373	+ 0,0165	− 0,0032

§. 376.

§. 376.

Pro partibus autem incognitis ponamus

$$\mathfrak{w} = \beta.\ \text{cof.}\ p - t + \gamma.\ \text{cof.}\ p + t + \delta.\ \text{cof.}\ 3p - t + \varepsilon.\ \text{cof.}\ 3p + t$$

$$w = b.\ \text{fin.}\ p - t + c.\ \text{fin.}\ p + t + d.\ \text{fin.}\ 3p - t + e.\ \text{fin.}\ 3p + t.$$

hincque calculum noftrum profequamur:

Pro $\mathfrak{M}'$.

	cof. $p - t$	cof. $p + t$	cof. $3p - t$	cof. $3p + t$
$\mathfrak{w}. -9{,}2213.\text{cof.}\ 2p$	$-4{,}6106.\gamma$ $-4{,}6106.\delta$	$-4{,}6106.\beta$ $-4{,}6106.\varepsilon$	$-4{,}6106.\beta$	$-4{,}6106.\gamma$
$\mathfrak{w}. +0{,}0264.$	$+0{,}0264.\beta$	$+0{,}0264.\gamma$	$-0{,}0264.\delta$	$+0{,}0264.\varepsilon$
$w. -3{,}99.\ \text{fin.}\ 2p$	$-1{,}9952.c$ $-1{,}9952.d$	$-1{,}9952.b$ $-1{,}9952.e$	$+1{,}9952.b$	$+1{,}9952.c$

Pro M'.

	fin. $p - t$	fin. $p + t$	fin. $3p - t$	fin. $3p + t$
$\mathfrak{w}. -3{,}99\ \text{fin.}\ 2p$	$-1{,}9952.\gamma$ $+1{,}9952.\delta$	$-1{,}9952.\beta$ $+1{,}9952.\varepsilon$	$-1{,}9952.\beta$	$-1{,}9952.\gamma$
$w. +5{,}36.\ \text{cof.}\ 2p$	$-2{,}6803.c$ $+2{,}6803.d$	$-2{,}6803.b$ $+2{,}6803.e$	$+2{,}6803.b$	$+2{,}6803.c$
$w. -0{,}027.$	$-0{,}0272.b$	$-0{,}0272.c$	$-0{,}0272.d$	$-0{,}0272.e$

§. 377.

Hic incipiamus ab angulo tertio $3p - t$; pro eo numeros $\mathfrak{N}'$ et N' quaerentes:

Pro angulo $3p - t$.

	β	b	δ	d
Log. M'	$-0,2999$	$+0,4281$		$-8,4345$
$L.\frac{2(m+1)}{\mu}$	$+9,8695$	$+9,8695$		$+9,8695$
	$-0,1694$	$+0,2976$		$-8,3040$
$\frac{2(m+1)M'}{\mu}$	$-1,4771$	$+1,9843$		$-0,0201$
$-\mathfrak{M}'$	$+4,6106$	$-1,9952$	$-0,0264$	
Num.	$+3,1335$	$-0,0109$	$-0,0264$	$-0,0201$
Log.	$+0,4960$	$-8,0374$	$-8,4216$	$-8,3032$
L. den.	$-3,0518$	$-3,0518$	$-3,0518$	$-3,0518$
Log. $\mathfrak{N}'$	$-7,4442$	$+4,9856$	$+5,3698$	$+5,2514$
$L.\frac{2(m+1)}{\mu}$	$+9,8695$	$+9,8695$	$+9,8695$	$+9,8695$
L. P. II.	$-7,3137$	$+4,8551$	$+5,$	$+5,$
Log. M'	$-0,2999$	$+0,4281$		$-8,4345$
Log. μ^2	$3,1152$	$3,1152$		$3,1152$
Log. P. I.	$-7,1847$	$+7,3129$		$-5,3$
P. I.	$-0,0015$	$+0,0021$		$-0,0000$
$-$ P. II.	$+0,0021$	$-0,0000$	$-0,0000$	$-0,0000$
N'	$+0,0006$	$+0,0021$	$+0,0000$	$+0,0000$
$\mathfrak{N}'$	$-0,0028$	$+0,0000$	$+0,0000$	$+0,0000$

hinc $\delta = +0,0165 - 0,0028.\beta$

$d = -0,0166 - 0,0006.\beta + 0,0021.b.$

§. 378.

§. 378.

Simili modo quartum angulum tractemus, vbi liquet, binas partes litteris s et e adfectas omitti posse

Pro angulo $3p+t$.

	γ	ϵ
L. M′	−0,2999	+0,4281
L. $\frac{2(m+1)}{\mu}$	+9,8461	+9,8461
	−0,1460	+0,2742
$\frac{2(m+1)M}{\mu}$	−1,3983	+1,8803
$-\mathfrak{M}$	+4,6106	−1,9952
Numer.	+3,2123	−0,1149
L. Num.	+0,5068	−9,0603
L. Den.	−3,1053	−3,1053
L. $\mathfrak{N}'$	−7,4015	+5,9550
L. $\frac{2(m+1)}{\mu}$	+9,8461	+9,8461
L. P. II.	−7,2476	+5,8011
Log. M′	−0,2999	+0,4281
Log. μ^2	+3,1618	+3,1618
L. P. I.	−7,1381	+7,2663
Pars I.	−0,0014	+0,0018
−P. II.	+0,0017	+0,0001
N′	+0,0003	+0,0017
$\mathfrak{N}'$	−0,0025	+0,0001

hinc

hinc

$$\varepsilon = -0,0032 - 0,0025.\,\gamma + 0,0001.\,c.$$
$$e = +0,0033 + 0,0003.\,\gamma + 0,0017.\,c.$$

§. 379.

Hos igitur valores in formulis prioribus substituentes reperiemus

Pro angulo $p-t$

$$\mathfrak{M}' = -0,0428 + 0,0380.\,\beta - 0,0042.\,b$$
$$-4,6106.\,\gamma - 1,9952.\,c.$$
$$M' = -0,0129 - 0,0039.\,\beta - 0,0215.\,b.$$
$$-1,9952.\,\gamma - 2,6803.\,c.$$

Pro angulo $p+t$

$$\mathfrak{M}' = +0,0082 + 0,0374.\,\gamma - 0,0039.\,c.$$
$$-4,6106.\,\beta - 1,9952.\,b.$$
$$M' = +0,0021 - 0,0043.\,\gamma - 0,0228.\,c.$$
$$-1,9952.\,\beta - 2,6803.\,b.$$

§. 380.

§. 380.

Faciamus igitur sequentem calculum.

Pro angulo $p = t$.

	1	β	b	γ	c
L. M′	−8,1106	−7,5911	−8,3324	−0,3000	−0,4282
L. $\frac{2(m+1)}{\mu}$	+0,3714	+0,3714	+0,3714	+0,3714	+0,3714
	−8,4820	−7,9625	−8,7038	−0,6714	−0,7996
$\frac{2(m+1)M'}{\mu}$	−0,0303	−0,0092	−0,0505	−4,6930	−6,3055
−𝔐′	+0,0428	+0,0380	+0,0042	+4,6106	+1,9952
Numer.	+0,0125	−0,0472	−0,0463	−0,0824	−4,3103
L.Num.	+8,0969	−8,6739	−8,6656	−8,9159	−0,6345
L. Den.	+1,6810	+1,6810	+1,6810	+1,6810	+1,6810
L. 𝔑′	+6,4159	−6,9919	−6,9846	−7,2349	−8,9539
L. $\frac{2(m+1)}{\mu}$	0,3714	0,3714	0,3714	0,3714	0,3714
L. P. II.	+6,7873	−7,3633	−7,3560	−7,6063	−9,3253
L. M′	−8,1106	−7,5911	−8,3324	−0,3000	−0,4282
L. μ^2	2,1114	2,1114	2,1114	2,1114	2,1114
L. P. I.	−5,9992	−5,4797	−6,2210	−8,1886	−8,3168
P. I.	−0,0001	−0,0000	−0,0001	−0,0154	−0,0207
−P. II.	−0,0006	+0,0023	+0,0022	+0,0040	+0,2115
N′	−0,0007	+0,0023	+0,0021	−0,0114	+0,1908
at 𝔑′	+0,0002	−0,0010	−0,0010	−0,0017	−0,0899

hinc

$$\beta = +0,0038 - 0,0010.\beta - 0,0010.b - 0,0017.\gamma - 0,0899.c.$$

$$b = +0,0414 + 0,0023.\beta + 0,0021.b - 0,0114.\gamma + 0,1908.c.$$

vnde concludimus

$$\beta = +0,0038 - 0,0017.\gamma - 0,0900.c.$$

$$b = +0,0415 - 0,0114.\gamma + 0,1911.c.$$

§. 381.

His valoribus ſubſtituendis eliciemus

Pro angulo $p+t$

$$\mathfrak{M}' = -0,0943 + 0,0680.\gamma + 0,0288.c.$$

$$M' = -0,1167 + 0,0296.\gamma - 0,3585.c.$$

vnde ſequens calculus inſtituatur.

Pro

Pro angulo $p + t$.

	1.	γ.	c
L. M′	− 9,0671	+ 8,4713	− 9,5484
$L.\frac{2(m+1)}{\mu}$	+ 0,3010	+ 0,3010	+ 0,3010
	− 9,3681	+ 8,7723	− 9,8494
	− 0,2334	+ 0,0592	− 0,7070
− 𝔐′	+ 0,0943	− 0,0680	− 0,0288
Numer.	− 0,1391	− 0,0088	− 0,7358
L. Num.	− 9,1433	− 7,9445	− 9,8667
L. Den.	− 0,1757	− 0,1757	− 0,1757
Log. 𝔑′	+ 8,9676	+ 7,7688	+ 9,6910
	+ 0,3010	+ 0,3010	+ 0,3010
L. P. II.	+ 9,2686	+ 8,0698	+ 9,9920
L. M	− 9,0671	+ 8,4713	− 9,5484
Log. μ^2	2,2522	2,2522	2,2522
L. P. I.	− 6,8149	+ 6,2191	− 7,2962
P. I.	− 0,0006	+ 0,0001	− 0,0020
− P. II.	− 0,1856	− 0,0117	− 0,9818
N′	− 0,1862	− 0,0116	− 0,9838
at 𝔑′	+ 0,0928	+ 0,0058	+ 0,4909

hinc

$$\gamma = + 1,2301 + 0,0058.\gamma + 0,4909.c.$$

$$c = - 2,4987 - 0,0116.\gamma - 0,9838.c.$$

§. 382.

Ex priore fit

$$\gamma = + 1,2371 + 0,4937 . c.$$

qui valor in altera substitutus praebet

$$c = - 2,5124 - 0,9892 . c.$$

hincque

$$c = - 1,2630 \text{ et } \gamma = + 0,6135.$$

omnes autem nostri coefficientes ita se habebunt:

$\beta = + 0,1164$	$b = - 0,1093$
$\gamma = + 0,6135$	$c = - 1,2630$
$\delta = + 0,0162$	$d = - 0,0167$
$\varepsilon = - 0,0048$	$e = + 0,0015.$

§. 383.

En ergo plenos valores litterarum, quas hic quaesiuimus:

$$\mathfrak{w} = + 0,1164 . \text{cos.} \, p - t + 0,6135 . \text{cos.} \, p + t$$
$$+ 0,0162 \, \text{cos.} \, 3p - t - 0,0048 . \text{cos.} \, 3p + t.$$

$$w = - 0,1093 . \text{fin.} \, p - t - 1,2630 . \text{fin.} \, p + t$$
$$- 0,0167 . \text{fin.} \, 3p - t + 0,0015 . \text{fin.} \, 3p + t.$$

NOVAE

NOVAE THEORIAE MOTVVM LVNAE

LIBER PRIMVS

CONTINENS IPSAM LVNAE THEORIAM.

PARS TERTIA.

EVOLVTIO NVMERICA TERTIAE AEQVATIONIS PRO COORDINATA z.

CAPVT I.

EVOLVTIO AEQVATIONIS (I), SEV ORDINIS I. PRO LITTERA p.

§. 384.

Aequatio ſpecialis, vnde valorem litterae p deduci oportet, ita ſupra reperitur expoſita:

$$o = \frac{ddp}{dt^2} + (\lambda + 1)p, + p(-3\lambda\mho + 6.\lambda\mho^2 - \tfrac{3}{2}\lambda O^2).$$

At

At ex §. 152. eſt

$$-3\lambda\Omega + 6.\lambda\Omega^2 - \tfrac{3}{2}\lambda O^2 = +0,0007980$$
$$+3,8606451 \text{ coſ. } 2p + 0,0385110. \text{ coſ. } 4p.$$

Hunc autem coefficientem breuitatis gratia littera a deſignemus.

§. 385.

Quia hic pars annexa tota eſt incognita, cuius terminus principalis, vti ſupra iam oſtendimus, eſt ſin. r

exiſtente $\frac{dr}{dt} = l = 13,42263$,

ſtatuamus ergo

$$\mathfrak{p} = \text{ſin. } r + c. \text{ ſin. } (2p - r) + d. \text{ ſin. } (2p + r)$$
$$+ e. \text{ ſin. } (4p - r) + f. \text{ ſin. } (4p + r),$$

atque hanc formulam multiplicari oportet in a, vt inde eliciantur litterae M, ex quibus deinceps litterae N deriuantur. Supra enim iam eſt demonſtratum, ſi pars annexa contineat terminum formae M. ſin. ω, exiſtente $\frac{d\omega}{dt} = \mu$, tum quantitatem quaeſitam $\mathfrak{p}$ complecti terminum N. ſin. ω, ita vt ſit $N = \frac{M}{\mu^2 - \lambda - 1}$.

§. 386.

§. 386.

Primum autem ipſam formam $\mathfrak{p}$ tantum per partem principalem formulae $\mathfrak{a}$ multiplicemus et quod inde naſcitur littera M' denotemus, cui vi regulae modo datae reſpondeat N', hocque modo ſaltem proxime valorem litterae $\mathfrak{p}$ nanciſcemur. Hic ergo calculus ita ſe habebit:

	ſin. r	ſin. $(2p - r)$	ſin. $(2p + r)$
$+3,860645 . p\cos . 2p$	$-1,930322 . c$	$-1,9303225$	$+1,9303225$
	$+1,930322 . d$	$+1,9303225 . e$	$+1,9303225 . f$

	ſin. $(4p - r)$	ſin. $(4p + r)$
$+3,860645 . p\cos . 2p$	$+1,9303225 . c$	$+1,9303225 . d$

§. 387.

§.

Euoluamus nunc pro his quinque $\mu^2 - \lambda - 1$

$\omega =$	r	$2p - r$
$\mu =$	l	$2m - l$
seu $\mu =$	13,42263	11,31521
Log. $\mu =$	1,1278376	1,0536627
Log. $\mu^2 =$	2,2556752	2,1073254
$\mu^2 =$	180,166958	128,03403
$-(1 + \lambda) =$	−180,228928	−180,22893
Denom. $=$	−0,061970	− 52,19490
Log. Den. $=$	−8,7921815	−1,7176281

387.

angulis valores denominatoris.
hoc modo:

$2p+r$	$4p-r$	$4p+r$
$2m+l$	$4m-l$	$4m+l$
38,16047	36,05305	62,89831
1,5816138	1,5569420	1,7986390
3,1632276	3,1138840	3,5972780
1456,22222	1299,82239	+3956,19818
−180,22893	−180,22893	− 180,22893
1275,99329	1119,59346	+3775,96925
+3,1058484	+3,0490604	+3,5770284

§. 388.

Quia prima columna peculiare iudicium postulat, eius euolutionem in vltimum locum referuemus. Vlteriorem vero calculum pro secunda et quarta columna, quoniam sibi sunt affines, simul expediamus:

	$(2p-r)$		$(2p+r)$
	1	e	c
Log. M =	−0,2856299	+0,2856299	+0,2856299
L. Den. =	−1,7176281	−1,7176281	+3,0490604
Log. N. =	+8,5680018	−8,5680018	+7,2365695
N =	+0,036983	−0,036983. e	+0,001724. c

hincque iam proxime consequimur:

$c = +0,036983 - 0,036983.e$ et $e = +0,001724.c$,

qui vltimus valor in praecedente substitutus producit:

$$c = +0,036981 \text{ et } e = +0,000064.$$

§. 389.

Simili modo columnam tertiam et quintam simul expediamus:

	$(2p+r)$		$(4p+r)$
	1	f	d
Log. M =	+0,2856299		+0,2856299
L. Den. =	+3,1058484		+3,5770284
Log. N. =	+7,1797815	+7,1797815	+6,7086015
N =	+0,001512	+0,001512. f	+0,000511

hinc

hinc ergo

$$d = +0,001512 + 0,001512.f \text{ et } f = +0,000511.d,$$

vnde

$$d = +0,001512 \text{ et } f = +0,000001.$$

Pro prima igitur columna hinc obtinemus

$$M = -1,9303225(c-d) = -0,068341,$$

qui numerus ipsi denominatori pro hac columna $-0,061970$ deberet esse aequalis, quod si secus euenit, caussa est primo, quod valores inuenti tantum proxime sint veri, deinde vero imprimis in eo est sita, quod hic terminus sin. r non solum a suis similibus in eodem ordine, sed adeo in omnibus ordinibus coniunctim debeat deprimi, quandoquidem ex hoc ipso principio valor litterae l definitus esse censetur.

§. 390.

Valor igitur vero proximus, quem hactenus pro littera p elicuimus est

$$p = \text{sin.}\, r + 0,036981.\text{sin.}(2p-r) + 0,000964.\text{sin.}(4p-r)$$
$$+ 0,001512.\text{sin.}(2p+r) + 0,000001.\text{sin.}(4p+r)$$

in hunc ergo valorem iam ducamus partes exiguas litterae a et formulas inde natas littera M indicemus, cui deinceps litteram respondentem N inuestigemus:

	sin. r	sin. $(2p-r)$	sin. $(2p+r)$
$p(+0,000798)$	$+0,000798$	$+0,000029$	$+0,000001$
$p(+0,038511.\cos.4p)$	$-0,000002$	$-0,000029$	$-0,000712$
ergo $M' =$	$+0,000796$	$+0,000000$	$-0,000711$

	sin. $(4p-r)$	sin. $(4p+r)$
$p(+0,000798)$		
$p(+0,038511.\cos 4p)$	$-0,019256$	$+0,019256$
ergo $M' =$	$-0,019256$	$+0,019256$

Seposita iterum prima columna calculus pro reliquis, ita instituatur:

	$2p-r$	$2p+r$	$4p-r$	$4p+r$
L. $M' =$	$+0,$	$-6,8518696$	$-8,2845661$	$+8,2845661$
L.Den $=$		$+3,1058484$	$+3,0490604$	$+3,5770284$
Log. N $=$		$-3,7460212$	$-5,2355057$	$+4,7075377$
ergo N $=$		$-0,000000$	$-0,000017$	$+0,000005$

hos ergo valores litteris c, d, e, f supra exhibitis insuper adiungi oportet, vt earum veros valores consequamur, erit ergo

I°. $c = +0,036983 - 0,036983.e.$

II°. $d = +0,001512 + 0,001512.f.$

III°. $e = +0,001724.c - 0,000017.$

IV°. $f = +0,000511.d + 0,000005.$

vnde

vnde colligimus hos valores

$$c = +0,036982; \quad d = +0,001513;$$
$$e = +0,000047; \quad f = +0,000006.$$

vnde pro prima columna oritur $M + M' = -0,967545$, qui numerus iam propius ad denominatorem accedit, ita vt sequentibus ordinibus minus destruendum relinquatur.

§. 391.

En ergo verum valorem litterae p, quam euolutio huius capitis nobis suppeditauit:

$$p = \sin. r + 0,036982. \sin.(2p - r) + 0,000047. \sin.(4p - r) + 0,001513. \sin.(2p + r) + 0,000006 \sin.(4p + r).$$

CAPVT II.

CAPVT II.

EVOLVTIO ORDINIS (II.) ik, PRO VALORE LITTERAE $\mathfrak{q}$.

§. 392.

Aequatio differentialis, vnde hanc determinationem peti oportet, in §. 142, ita expoſita reperitur:

$$0 = \frac{dd\mathfrak{q}}{dt^2} + (\lambda + 1)\mathfrak{q} + \mathfrak{a}\mathfrak{q} + \mathfrak{b}\mathfrak{P}\mathfrak{p} + \mathfrak{c}\mathrm{P}\mathfrak{p}.$$

vbi ſcilicet $\mathfrak{a}$ eundem habet valorem, quem capite praecedente, binae reliquae vero $\mathfrak{b}$ et $\mathfrak{c}$, breuitatis ergo ſunt introductae, quarum valores ex §. 152, ſequenti modo determinantur:

$$\mathfrak{b} = -3\lambda + 12\lambda\mathfrak{O} - 30\lambda\mathfrak{O}^2 + \tfrac{15}{2}\lambda\mathrm{O}^2$$

ſiue

$$\mathfrak{b} = -537,7036780 - 15,4425816.\ \mathrm{cof.}\ 2p - 0,1957813.\ \mathrm{cof.}\ 4p.$$

$$\mathfrak{c} = -3\lambda\mathrm{O} + 15\lambda\mathfrak{O}\mathrm{O}\ \text{ſiue}$$

$$\mathfrak{c} = -5,4906964.\ \mathrm{fin.}\ 2p - 0,1016218.\ \mathrm{fin.}\ 4p.$$

§. 393.

Ante omnia igitur ambo producta $\mathfrak{P}\mathfrak{p}$ et $P\mathfrak{p}$ euolui conuenit, quae sequenti modo expressa repe-riuntur:

$$\begin{aligned}
\mathfrak{P}\mathfrak{p} = &-0,49653.\sin.(q-r) + 0,09461.\sin.(2p-q+r) \\
&\qquad\qquad\qquad\qquad + 0,01985.\sin.(2p+q-r) \\
&\qquad\qquad\qquad\qquad - 0,00011.\sin.(4p-q+r) \\
&\qquad\qquad\qquad\qquad - 0,00002.\sin.(4p+q-r) \\
&+0,50019.\sin.(q+r) - 0,07536.\sin.(2p-q-r) \\
&\qquad\qquad\qquad\qquad - 0,00059.\sin.(2p+q+r) \\
&\qquad\qquad\qquad\qquad + 0,00377.\sin.(4p-q-r) \\
&\qquad\qquad\qquad\qquad - 0,00001.\sin.(4p+q+r)
\end{aligned}$$

$$\begin{aligned}
P\mathfrak{p} = &-1,01393.\cos.(q-r) + 0,20409.\cos.(2p-q+r) \\
&\qquad\qquad\qquad\qquad + 0,03561.\cos.(2p+q-r) \\
&\qquad\qquad\qquad\qquad + 0,00067\ \cos.(4p-q+r) \\
&\qquad\qquad\qquad\qquad + 0,00010.\cos.(4p+q-r) \\
&+1,00595.\cos.(q+r) - 0,24284.\cos.(2p-q-r) \\
&\qquad\qquad\qquad\qquad + 0,00313\ \cos.(2p+q+r) \\
&\qquad\qquad\qquad\qquad + 0,00719.\cos.(4p-q-r) \\
&\qquad\qquad\qquad\qquad + 0,00002.\cos(4p+q+r).
\end{aligned}$$

Hae expressiones sponte se manifesto diuidunt in duas classes, quarum prior complectitur angulum $(q-r)$, posterior vero angulum $(q+r)$; quae duo classes eo magis a se inuicem distingui merentur, quod per sequentes operationes non amplius inter se permiscentur, quam ob rem vtramque classem seorsim tractare licebit, vnde calculo non exiguum subsidium adferetur.

I. Euolutio terminorum prioris classis

§.

Primum igitur ex membris cognitis aequationis no-

	$\sin(q-r)$	$\sin.(2p-q+r)$
$\mathfrak{P}p(-537,703)$ -	+ 266,98594	− 50,87214
$\mathfrak{P}p(-15,442.\cos.2p)$	+ 0,73051	− 3,83385
	− 0,15327	+ 0,00085
	+ 0,57724	− 3,83300
	− 267,56318	− 54,70514
$\mathfrak{P}p(-0,19578.\cos.4p)$	− 0,00009	+ 0,00194
	+ 267,56309	− 54,70320
$Pp(-5,490.\sin.2p)$	− 0,56030	+ 2,78359
	+ 0,09776	+ 0,00184
	− 0,46254	+ 2,78543
	+ 267,10055	− 51,91777
$Pp(-0,1016.\sin.4p)$	− 0,00029	− 0,00181
M=	+ 267,10026	− 51,91958

angulum $q-r$ inuoluentium.

394.

ſtrae, valores litterae M, ſequenti modo colligamus:

ſin. $(2p+q-r)$	ſin. $(4p-q+r)$	ſin. $(4p+q-r)$	
− 10,67342	+ 0,05915	+ 0,01075	
+ 3,83385	− 0,73051	− 0,15327	
+ 0,00015			
+ 3,83400			b
− 6,83942	− 0,67136	− 0,14252	
+ 0,00926	− 0,04861	+ 0,04861	
− 6,83016	− 0,71997	− 0,09391	
+ 2,78359	− 0,56030	− 0,09776	
+ 0,00027			
+ 2,78386			c
− 4,04630	− 1,28027	− 0,19167	
− 0,01037	+ 0,05152	+ 0,05152	
− 4,05667	− 1,22875	− 0,14015	

§.

Antequam hinc respondentem litteram N definire gulis, denominatorem

$\omega =$	$q - r$	$2p - q + r$
$\mu =$	$n - l$	$2m - n + l$
siue $\mu =$	$-0,16659$	$+24,90443$
L. $\mu =$	$-9,2216489$	$+1,3962766$
L. $\mu^2 =$	$+8,4432978$	$+2,7925532$
$\mu^2 =$	$+\ \ 0\ 02775$	$+620,23057$
$-(\lambda + 1) =$	$-180,22893$	$-180,22893$
Den. $=$	$-180,20118$	$+440,00164$
L. M $=$	$+2,4266743$	$-1,7153318$
L. Den. $=$	$-2,2557576$	$+2,6434543$
L. N $=$	$-0,1709167$	$-9,0718775$
N $=$	$-1,48223$	$-0,11800$

395.

liceat, necesse est pro singulis nostris quinque an-
$\mu^2 - \lambda - 1$ euoluere.

$2p+q-r$	$4p-q+r$	$4p+q-r$
$2m+n-l$	$4m-n+l$	$4m+n-l$
+24,57125	+49,64227	+49,30909
+1,3904272	+1,6958517	+1,6929270
+2,7808544	+3,3917034	+3,3858540
+603,74614	+2464,35543	+2431,38667
−180,22893	−180,22893	−180,22893
+423,51721	+2284,12650	+2251,15774
−0,6081697	−0,0894636	−9,1465931
+2,6268712	+3,3587201	+3,3524060
−7,9812985	−6,7307435	−5,7941871
−0,00958	−0,00054	−0,00006

§. 396.

Iam pro parte incognita statuamus:

$$\text{q} = b.\,\text{sin.}\,(q-r) + c.\,\text{sin.}\,(2p-q+r) + d.\,\text{sin.}\,(2p+q-r) + e\,\text{sin.}\,(4p-q+r) + f.\,\text{sin.}\,(4p+q-r)$$

ac primo hanc formam tantum in partem principalem coefficientis α ducamus, formulasque inde natas littera M′ indicemus, particulas autem minimas ipsius α tantisper seponamus, quoad valorem prope verum ipsius q eruerimus. Hinc autem valores litterae M′ sequenti modo deducimus:

	sin. $q-r$	sin. $(2p-q+r)$	sin. $(2p+q-r)$
$\text{q}(+3{,}860645.\cos.2p)$	$-1{,}930322(c-d)$	$-1{,}930322(b-e)$	$+1{,}930322(b+f)$

	sin. $(4p-q+r)$	sin. $(4p+q-r)$
$\text{q}(+3{,}860645.\cos.2p)$	$+1{,}930322.\,c$	$+1{,}930322.\,d$

§. 397.

Hinc iam respondentis litterae N′ valores quaeramus, primam autem columnam in vltimum locum referuemus, donec reliquos coefficientes per primum b definiuerimus. Secundam autem columnam et quartam commode coniunctim euoluere licebit:

$2p-q$

	$2p-q+r$ $(b-e)$	$4p-q+r$ c
Log. M′ =	− 0,2856299	+ 0,2856299
Log. Den. =	+ 2,6434543	+ 3,3587201
Log. N′ =	− 7,6421756	+ 6,9269098
N′ =	− 0,00439 $(b-e)$	+ 0,00084 . c

Quum nunc priori casu N + N′ dare debeat c, posteriore vero e, habebimus has determinationes:

$$c = -0,11800 - 0,00439\,(b-e)$$

$$e = -0,00054 + 0,00084\,c$$

qui posterior in priore substitutus producit:

$$c = -0,11800 - 0,00439\,.\,b$$

hincque $e = -0,00064$.

§. 398.

Eodem modo columnas tertiam et quintam coniunctim expediamus:

	$2p+q-r$ $(b+f)$	$4p+q-r$ d
Log. M′ =	+ 0,2856299	+ 0,2856299
Log. Denom. =	+ 2,6268712	+ 3,3524060
Log. N′ =	+ 7,6587587	+ 6,9332239
N′ =	+ 0,00456 $(b+f)$	+ 0,00086 . d

vnde

vnde colligimus

$$d = -0,00958 + 0,00456.(b+f);$$
$$f = -0,00006 + 0,00086.d \text{ seu}$$
$$d = -0,00958 + 0,00456.b \text{ et } f = -0,00007.$$

§. 399.

Nunc primam columnam adgrediamur, pro qua habebimus $M' = +0,20929 + 0,01728.b$, hinc ergo respondentem litteram N' deriuemus:

	$q-r$	
	1	b
Log. M' =	$+9,3207393$	$+8,2374529$
Log. Denom. =	$-2,2557576$	$-2,2557576$
Log. N' =	$-7,0649817$	$-5,9816953$
N' =	$-0,00116$	$-0,00010.b$

Iam quum hinc fiat $N + N' = b$, erit

$$b = -1,48223 - 0,00116 - 0,00010.b$$
$$b = -1,48339 - 0,00010.b, \text{ ideoque}$$
$$b = -1,48324, \text{ hincque porro}$$
$$c = -0,11149; \; d = -0,01634;$$
$$e = -0,00064 \text{ et } f = -0,00007;$$

ita vt valor quaesitus prope verus iam sit:

$$\phi = -1,48324.\sin.(q-r) - 0,11149.\sin.(2p-q+r) - 0,01634.\sin.(2p+q-r) - 0,00064.\sin.(4p-q+r) - 0,00007\sin.(4p+q-r).$$

§. 400.

§. 400.

Nunc demum etiam particulas minimas ipsius q consideremus, quas sufficiet in valorem prope verum modo inuentum duxisse, vt inde consequamur litteram M'' hoc modo:

	fin. $(q-r)$	fin. $(2p-q+r)$	fin. $(2p+q-r)$
$q(+0,000798)$	$-0,00118$	$-0,00009$	$-0,00001$
$q(+0,038511.\cos 4p)$	$+0,00001$	$+0,00031$	$+0,00215$
ergo M''	$-0,00117$	$+0,00022$	$+0,00214$
L. $M''=$	$-7,0681859$	$+6,3424$	$+7,3304$
L. Den. $=$	$-2,2557576$	$+2,6434$	$+2,6268$
L. $N''=$	$+4,8124283$	$+3,6990$	$+4,7036$
$N''=$	$+0,000007$	$+0,000000$	$+0,000005$

	fi. $(4p-q+r)$	fi. $(4p+q-r)$
$q(+0,000798)$		
$q(+0,038511.\cos 4p)$	$+0,02856$	$-0,02856$
ergo M''	$+0,02856$	$-0,02856$
L. $M''=$	$+8,4557$	$-8,4557$
L. Denom. $=$	$+3,3587$	$+3,3524$
L. $N''=$	$+5,0970$	$-5,1033$
$N''=$	$+0,00001$	$-0,00001$

§. 401.

§. 401.

Hos igitur valores ad ſupra inuentos valores N'' inſuper adiici oportet, hincque praecedentes calculos ſingulos repetamus:

$c = -0,11800 - 0,00439.b;$

$d = -0,00958 + 0,00456.b;$

$e = -0,00063;$

$f = -0,00008.$

ex quibus porro fit pro prima columna, vt ante:

$M' = +0,20929 + 0,01728.b,$

cui ergo reſpondet

$N' = -0,00116 - 0,00010.b$

ex quo colligitur

$b = -1,48338 - 0,00010.b,$

ſicque veri valores ita ſe habebunt:

$b = -1,48323; \quad c = -0,11149;$

$d = -0,01634; \quad e = -0,00063;$

$f = -0,00008.$

§. 402.

§. 402.

Ecce ergo veros valores pro priore parte litterae noſtrae q

$$q = -1,48323 \cdot \sin.(q-r) - 0,11149 \cdot \sin.(2p-q+r)$$
$$-0,01634 \cdot \sin.(2p+q-r)$$
$$-0,00063 \cdot \sin.(4p-q+r)$$
$$-0,00008 \cdot \sin.(4p+q-r)$$

vbi plurimum notaſſe iuuabit, correctiones hic modo inuentas nullius plane eſſe momenti, ita vt iis tuto ſuperſedere potuiſſemus, ex quo etiam intelligitur, minimas particulas a, b, c, etiam ſine errore praetermitti potuiſſe, imprimis quoque manifeſtum eſt, poſtremos terminos angulum $4p$ continentes, tam ipſos eſſe vehementer exiguos, quam nihil inde in praecedentes angulos influere. Interim tamen etiam nunc ſummo rigori inhaereamus, in ſequenti vero capite vbi calculus futurus eſſet immenſus, has obſeruationes ad vſum noſtrum conuertemus:

II. Euolutio terminorum poſterioris

§.

Primum itaque ex membris cognitis poſte-
rum M, ſequenti

	ſin. $(q+r)$	ſin. $(2p-q-r)$
𝔓 p (− 537, 703) -	− 268,95392	+ 40,52134
𝔓 p (15, 442. coſ. $2p$)	− 0,58188	+ 3,86211
	+ 0,00456	− 0,02904
	− 0,57732	+ 3,83307
	− 269,53124	+ 44,35441
𝔓 p (− 0, 195. coſ. $4p$)	+ 0,00037	− 0,00006
	− 269,53087	+ 44,35435
P p (− 5, 490. ſin. $2p$)	+ 0,66668	− 2,76168
	+ 0,00860	+ 0,01974
	+ 0,67528	− 2,74194
	− 268,85559	+ 41,61241
P p (−0, 101. ſin. $4p$)	− 0,00036	− 0,00016
M =	− 268,85595	+ 41,61225

classis angulum $(q+r)$ inuoluentium.

403.

rioribus aequationis nostrae, valores littera-
modo determinemus:

sin. $(2p+q+r)$	sin. $(4p-q-r)$	sin. $(4p+q+r)$
+0,31725	−2,02714	+0,00538
−3,86211 +0,00007	+0,58188	+0,00456
−3,86204		
−3,54479	−1,44526	+0,00994
−0,00738	+0,04896	−0,04896
−3,55217	−1,39630	−0,03902
−2,76168 +0,00006	+0,66668	−0,00860
−2,76162		
−6,31379	−0,72962	−0,04762
+0,01233	−0,05111	−0,05111
−6,30146	−0,78073	−0,09873

§.

Elementa iam numerica ante constitui
litterae N de-

$\omega =$	$q + r$	$2p - q - r$
hinc $\mu =$	$n + l$	$2m - n - l$
seu $\mu =$	26,67867	− 1,94083
Log. $\mu =$	+ 1,4261641	− 0,2879875
Log. $\mu^2 =$	+ 2,8523282	+ 0,5759750
$\mu^2 =$	+ 711,75116	+ 3,76682
$-(\lambda + 1) =$	− 180,22893	− 180,22893
Denom. $=$	+ 531,52223	− 176,46211
Log. M $=$	− 2,4295196	+ 1,6192212
Log. Den. $=$	+ 2,7255214	− 2,2466515
Log. N $=$	− 9,7039982	− 9,3725697
N $=$	− 0,50582	− 0,23581

404.

oportet, quam valores respondentes finire liceat:

$\frac{2p+q+r}{2m+n+l}$	$\frac{4p-q-r}{4m-n-l}$	$\frac{4p+q+r}{4m+n+l}$
+51,41651	+22,79701	+76,15435
+1,7111026	+1,3578779	+1,8816947
+3,4222052	+2,7157558	+3,7633894
+2643,65800	+519,70372	+5799,48474
− 180,22893	−180,22893	− 180,22893
+2463,42907	+339,47479	+5619,25581
−0,7994411	−9,8925009	−8,9944491
+3,3915400	+2,5308075	+3,7496788
−7,4079011	−7,3616934	−5,2447703
−0,00256	−0,00230	−0,00002

§. 405.

§. 405.

Statuamus nunc

$$\mathfrak{q} = b.\ \text{fin.}\,(q + r) + c.\ \text{fin.}\,(2p - q - r)$$
$$+ d.\ \text{fin.}\,(2p + q + r) + e.\ \text{fin.}\,(4p - q - r)$$
$$+ f.\ \text{fin.}\,(4p + q + r)$$

quae ducta in primam partem litterae $\mathfrak{a}$, dabit M′ ex quo deinceps quaeramus N′:

	$q + r$	$2p - q - r$	$2p + q + r$
$\mathfrak{q}(3,86064.\text{cof.}\,2p)$	$-1,930322(c - d)$	$-1,930322(b - e)$	$+1,930322(b + f)$

	$4p - q - r$	$4p + q + r$
$\mathfrak{q}(3,86064.\text{cof.}\,2p)$	$+1,930322.\,c$	$+1,930322.\,d$

Nunc fepofita prima columna expediamus coniunctim fecundam et quartam:

	$2p - q - r$ $(b - e)$	$4p - q - r$ c
Log. M′ =	$-0,2856299$	$+0,2856299$
Log. Denom. =	$-2,2466515$	$+2,5308075$
Log. N′ =	$+8,0389784$	$+7,7548224$
N′ =	$+0,01094(b - e)$	$+0,00569.\,c$

Vnde colligimus has duas determinationes:

$$c = -0,23581 + 0,01094\,b - 0,01094.\,e;$$
$$e = -0,00230 + 0,00569.\,c;$$

proinde

$$c = -0,23577 + 0,01094\,b;$$
$$e = -0,00364 + 0,00006.\,b.$$

§. 406.

§. 406.

Eodem modo tertiam columnam et quintam simul conficiamus:

	$2p+q+r$ $b+f$	$4p+q+r$ d
L. M′ =	$+0{,}2856299$	$+0{,}2856299$
L. Den. =	$+3{,}3915400$	$+3{,}7496788$
L. N′ =	$+6{,}8940899$	$+6{,}5359511$
N′ =	$+0{,}00078.(b+f)$	$+0{,}00034.\,d$

hinc

$$d = -0{,}00256 + 0{,}00078.\,b + 0{,}00078.\,f;$$
$$f = -0{,}00002 + 0{,}00034.\,d \text{ ideoque}$$
$$d = -0{,}00256 + 0{,}00078.\,b;\ f = -0{,}00002.$$

§. 407.

Nunc ob $c - d = -0{,}23321 + 0{,}01016.\,b$, pro prima columna fiet

$M' = -1{,}930322(c-d) = +0{,}45017 - 0{,}01961.\,b$, cui respondens N′ prodibit $= +0{,}00085 - 0{,}00004.\,b$, vnde conficitur

$b = -0{,}50497 - 0{,}00004.\,b$; seu $b = -0{,}50495$, sicque reliquae litterae erunt:

$$c = -0{,}24129;\ d = -0{,}00296;$$
$$e = -0{,}00367;\ f = -0{,}00002.$$

 Valor

Valor igitur prope verus hactenus inuentus ita se habet:

$$\begin{aligned} q = -0{,}50495.\sin.(q+r) &- 0{,}24129.\sin.(2p-q-r) \\ &- 0{,}00296.\sin.(2p+q+r) \\ &- 0{,}00367.\sin.(4p-q-r) \\ &- 0{,}00002.\sin.(4p+q+r) \end{aligned}$$

qui ductus in exiguas particulas ipsius a, dabit M'', indeque reperietur N'' hoc modo:

	$\sin.(q+r)$	$\sin.(2p-q-r)$	$\sin.(2p+q+)$
$q(+0{,}000798)$	$-0{,}00040$	$-0{,}00019$	
$q(+0{,}038511.\cos.4p)$	$+0{,}00007$	$+0{,}00006$	$+0{,}00465$
$M'' =$	$-0{,}00033$	$-0{,}00013$	$+0{,}00465$
$N'' =$	$-0{,}00000$	$+0{,}00000$	$+0{,}00000$

	$\sin.(4p-q-r)$	$\sin.(4+q+r)$
$q(+0{,}000798)$		
$q(+0{,}038511.\cos.2p)$	$+0{,}00972$	$-0{,}00972$
$M'' =$	$+0{,}00972$	$-0{,}0972$
$N'' =$	$+0{,}00003$	$-0{,}0000$

§. 408.

His igitur particulis adiiciendis, operationes praecedentes repetamus

$$c = -0{,}23581 + 0{,}01094.b - 0{,}01094.e, \text{ et}$$
$$e = -0{,}00227 + 0{,}00569.c \text{ proinde}$$
$$c = -0{,}23577 + 0{,}01094.b;$$
$$e = -0{,}00361 + 0{,}00006.b.$$

Simili

Simili ratione

$d = -0,00256 + 0,00078.b$ et $f = -0,00002.$

Hinc $c - d = -0,23321 + 0,01016.b,$

atque pro prima columna

$M'' = +0,45017 - 0,01961.b,$ vnde

$N'' = +0,00085 - 0,00004\,b,$

ideoque

$b = -0,50497 - 0,00004\,b,$ seu $b = -0,50495,$

$c = -0,24129;\ d = -0,00296;$

$e = -0,00364;\ f = -0,00002.$

§. 409.

En ergo verum et completum valorem litterae q:

$$\begin{aligned} q = -1,48323.\sin.(q-r) &- 0,11149.\sin.(2p-q+r) \\ &- 0,01634.\sin.(2p+q-r) \\ &- 0,00063.\sin.(4p-q+r) \\ &- 0,00008.\sin.(4p+q-r) \\ -0,50497.\sin.(q+r) &- 0,24129.\sin.(2p-q-r) \\ &- 0,00296.\sin.(2p+q+r) \\ &- 0,00364.\sin.(4p-q-r) \\ &- 0,00002.\sin.(4p+q+r). \end{aligned}$$

CAPVT III.

EVOLVTIO ORDINIS (III.) $i\,k\,k$ PRO VALORE LITTERAE $\mathfrak{r}$.

§. 410.

Aequatio differentialis, vnde hanc determinationem peti oportet, ita ſupra eſt expoſita:

$$0 = \frac{dd\mathfrak{r}}{dt^2} + (\lambda + 1)\mathfrak{r} + \mathfrak{a}\,\mathfrak{r} + \mathfrak{b}(\mathfrak{P}\,\mathfrak{q} + \mathfrak{Q}\,\mathfrak{p})$$
$$+ \mathfrak{c}(P\,\mathfrak{q} + Q\,\mathfrak{p}) + \mathfrak{d}\,\mathfrak{P}^2\mathfrak{p} + \mathfrak{e}\,\mathfrak{P}P\mathfrak{p} + \mathfrak{f}\,P^2\mathfrak{p}.$$

Si hanc aequationem pari rigore tractare vellemus in calculos maxime taedioſos illaberemur, quam ob rem rationibus ante allegatis innixi, calculum ſequenti modo contrahere conabimur, vt primo omnes terminos, in quos angulus quadruplus $4\,p$ ingreditur praetermittamus; deinde vero in coefficientibus $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$ etc. poſtremas partes vbi litterae $\mathfrak{O}$ et O ad duas dimenſiones aſſurgunt, penitus reiiciamus ex quo iſti coefficientes ſequentes habebunt valores:

$\mathfrak{a} =$

$\mathfrak{a} = + 3,8606454 . \cos. 2p;$

$\mathfrak{b} = - 537,686784 - 15,4425816 . \cos. 2p;$

$\mathfrak{c} = - 5,4906954 . \sin. 2p;$

$\mathfrak{d} = + 1075,373568 + 38,606454 . \cos. 2p;$

$\mathfrak{e} = + 27,4534820 . \sin. 2p;$

$\mathfrak{f} = - 268,843392 - 9,651613 . \cos. 2p.$

hoc compendio eo tutius vti poterimus, quod ipsi termini ex hoc ordine nati ob factorem ikk per se sint valde parui, vt adeo error ad eorum partem sexagesimam assurgens nullius foret momenti; at vero semper longe infra hunc terminum erunt depressi.

§. 411.

Nunc igitur ante omnia valores productorum $\mathfrak{P}\mathfrak{q}$, $\mathfrak{Q}\mathfrak{p}$; $P\mathfrak{q}$ et $Q\mathfrak{p}$ euolui oportet, quem in finem notari conuenit hic fore:

$$\mathfrak{P} = + \cos. q + 0,187695 . \cos.(2p - q) - 0,002703 . \cos.(2p + q)$$

$$P = - 2,012639 . \sin. q - 0,411247 . \sin.(2p-q) - 0,003212 . \sin.(2p+q)$$

$$\mathfrak{Q} = - 0,53896 + 0,21903 . \cos. 2p + 0,50967 . \cos. 2q - 0,20179 . \cos.(2p-2q) + 0,00482 . \cos.(2p + 2q)$$

$$Q = + 0,09800 . \sin. 2p + 0,25209 . \sin 2q + 0,31159 . \sin.(2p - 2q) + 0,00428 . \sin.(2p + 2q)$$

 $\mathfrak{p} =$

$$\mathfrak{p} = \text{sin.}\, r + 0{,}036982.\, \text{sin.}\,(2p - r) \\ + 0{,}001513.\, \text{sin.}\,(2p + r)$$

$$\mathfrak{q} = -1{,}48323\, \text{sin.}(q-r) - 0{,}11149.\, \text{sin.}\,(2p-q+r) \\ - 0{,}01634.\, \text{sin.}(2p+q-r) \\ - 0{,}50497.\, \text{sin.}\,(q+r) - 0{,}24129.\, \text{sin.}\,(2p-q-r) \\ - 0{,}00296.\, \text{sin.}\,(2p+q+r)$$

vnde nostrae binae formulae priores sequenti modo expressae reperiuntur:

$$\mathfrak{P}\mathfrak{q} + \mathfrak{Q}\mathfrak{p} = \Pi = \left\{ \begin{aligned} & -0{,}04156.\text{sin.}\, r - 0{,}39813.\text{sin.}(2p-r) \\ & \qquad +0{,}00219.\text{sin.}(2p+r) \\ & -1{,}00176.\text{sin.}(2q-r) - 0{,}01705.\text{sin.}(2p-2q+r) \\ & \qquad +0{,}00086.\text{sin.}(2p+2q-r) \\ & +0{,}00150.\text{sin.}(2q+r) + 0{,}03706.\text{sin.}(2p-2q-r) \\ & \qquad +0{,}00199.\text{sin.}(2p+2q+r) \end{aligned} \right.$$

$$\mathrm{P}\mathfrak{q} + \mathrm{Q}\mathfrak{p} = \Pi' = \left\{ \begin{aligned} & +2{,}07518.\text{cos.}\, r - 0{,}48153.\text{cos.}(2p-r) \\ & \qquad -0{,}25965.\text{cos.}(2p+r) \\ & -1{,}35725.\text{cos.}(2q-r) + 0{,}26156.\text{cos.}(2p-2q+r) \\ & \qquad -0{,}02134.\text{cos.}(2p+2q-r) \\ & -0{,}63297.\text{cos.}(2q+r) + 0{,}56708.\text{cos.}(2p-2q-r) \\ & \qquad -0{,}00612.\text{cos.}(2p+2q+r). \end{aligned} \right.$$

§. 412.

§. 412.

Deinde quoniam in Partis Primae Capite Tertio iam euoluimus has formulas:

$$\mathfrak{P}^2 = +0,51762 + 0,18490.\cos.2p + 0,49949.\cos.2q + 0,18770.\cos.(2p-2q) - 0,00271.\cos.(2p+2q)$$

$$P^2 = +2,10992 - 0,82093.\cos.2p - 2,02404.\cos.2q + 0,82769.\cos.(2p-2q) - 0,00646.\cos.(2p+2q)$$

$$\mathfrak{P}P = -0,39900.\sin.2p - 1,00713.\sin.2q - 0,01674.\sin.(2p-2q) + 0,00112.\sin.(2p+2q)$$

quae ductae in $\mathfrak{p}$ dabunt producta, quibus hic indigemus:

$$\mathfrak{P}^2\mathfrak{p} = +0,51435.\sin.r - 0,07331.\sin.(2p-r) + 0,09323.\sin.(2p+r) - 0,24632.\sin.(2q-r) + 0,09423.\sin.(2p-2q+r) + 0,01059.\sin.(2p+2q-r) + 0,24992.\sin.(2q+r) - 0,08461.\sin.(2p-2q-r) - 0,00097.\sin.(2p+2q+r)$$

$P^2\mathfrak{p} =$

$$\mathrm{P}^2 \mathfrak{p} = +2,12448.\,\text{fin.}\,r + 0,48849.\,\text{fin.}\,(2p - r) - 0,40727.\,\text{fin.}\,(2p + r)$$
$$+1,02732.\,\text{fin.}(2q-r) + 0,41231.\,\text{fin.}(2p-2q+r) - 0,03419.\,\text{fin.}(2p+2q-r)$$
$$-1,01127.\,\text{fin.}(2q+r) - 0,45126.\,\text{fin.}(2p-2q-r) - 0,00476.\,\text{fin.}(2p+2q+r)$$

$$\mathfrak{P}\,\mathrm{P}\,\mathfrak{p} = -0,00768.\,\text{cof.}\,r - 0,19950.\,\text{cof.}\,(2p - r) + 0,19950.\,\text{cof.}\,(2p + r)$$
$$-0,50387.\,\text{cof.}(2q-r) + 0,00761.\,\text{cof.}(2p-2q+r) + 0,01918.\,\text{cof.}(2p+2q-r)$$
$$+0,50353.\,\text{cof.}(2q+r) - 0,02699.\,\text{cof.}(2p-2q-r) + 0,00020.\,\text{cof.}(2p+2q+r).$$

§. 413.

Singulae istae formulae in terna membra dispescuntur, quorum prima ab angulo q plane sunt immunia, secunda vero inuoluunt angulum $2q - r$, ac tertia angulum $2q + r$, quae membra quum sequentibus operationibus non inter se permisceantur, singula seorsim tractari conueniet, vnde tres sequentes euolutiones nobis erunt expediendae:

I. Euolu-

I. Euolutio terminorum ab angulo q liberorum.

Hic igitur primum formulas modo exhibitas, sigillatim in suos coefficientes $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, $\mathfrak{e}$ et $\mathfrak{f}$ ducamus, indeque valores littera M signatos colligamus:

	sin. r	sin. $(2p-r)$	sin. $(2p+r)$
$\mathfrak{b}\Pi =$	+ 22,34628	+ 214,06930	− 1,17754
	− 3,07408	− 0,32090	+ 0,32090
	− 0,01691		
	+ 19,25529	+ 213,74840	− 0,85664
$\mathfrak{c}\Pi' =$	+ 1,32197	− 5,69709	− 5,69709
	− 0,71283		
	+ 0,60914		
	+ 19,86443	+ 208,05131	− 6,55373
$\mathfrak{d}\mathfrak{P}'p =$	+ 553,11850	− 78,83566	+ 100,25627
	+ 1,41512	− 9,92861	+ 9,92861
	+ 1,79964		
	+ 556,33326	− 88,76427	+ 110,18488
	+ 576,19769	+ 119,28704	+ 103,63115
$\mathfrak{e}\mathfrak{P}Pp =$	− 5,47696	− 0,10542	− 0,10542
	+ 570,72073	+ 119,18162	+ 103,52573
$\mathfrak{f}P^2p =$	− 571,15253	− 131,32736	+ 109,49185
	+ 2,35736	+ 10,25232	− 10,25232
	+ 1,96541		
	− 566,82976	− 121,07504	+ 99,23953
M =	+ 3,89097	− 1,89342	+ 202,76526

§. 414.

Quia prima columna peculiari iudicio eget, pro duabus reliquis quaeramus litteram N per elementa in Cap. I. tradita:

L. M =	$-0,2772468$	$+2,3069947$
L.Den. =	$-1,7176281$	$+3,1058484$
L. N =	$+8,5596187$	$+9,2011463$
N =	$+0,0363$	$+0,1589$

§. 415.

Quod si iam pro parte prima incognita statuamus: $\mathfrak{r} = b.\ \text{sin.}\ r + c.\ \text{sin.}\ (2p - r) + d.\ \text{sin.}\ (2p+r)$; ante omnia notari oportet per hypothesin esse debere $b = 0$, siquidem in ordine primo, terminus sin. r totum coefficientem iam est adeptus, hinc ergo quaeram litteram M′ hoc modo

$$M' = \left|\begin{array}{c|c|c} r & 2p - r & 2p + r \\ = 1,93032.(c-d) & 0 & 0 \end{array}\right.$$

quia ergo pro columnis secunda et tertia prodiit $M' = 0$, erit quoque $N' = 0$, ideoque statim habemus $c = +0,0363$ et $d = +0,1589$.

§. 416.

Pro prima ergo columna habebimus

$M' = -1,93032\,(c-d) = +0,23666.$

vnde fit totus valor $M + M' = +4,12763$, quem valo-

valorem in primum ordinem capitis primi transferri conuenit, quem in finem eum per k^2 multiplicare debemus. Quum igitur ibi esse debuerat $-0,068341 = -0,061970$, nunc iste termino translato prodire deberet $-0,068341 + 4,12763 . k^2 = -0,061970$, quod si minus congruere videtur perpendendum est exiguas illas correctiones, quas hic neglximus in hoc iudicio maximi esse momenti, etiamsi in ipsis Lunae inaequalitatibus sine sensibili errore praetermitti queant, caeterum circa motum nodi, quo haec diiudicatio est referenda nullum dubium superesse aliunde iam nouimus. Quin etiam haec diiudicatio ita est subtilissima, vt si numero l vel leuissima mutatio inducatur, qualem sine dubio actio Planetarum efficit, non obstante hac tanta discrepantia, perfectus consensus obtineri possit. Portio igitur illa ipsius $\mathfrak{r}$ hic eruta ita se habet:

$$\mathfrak{r} = 0, \text{fin.}\, r + 0,0363 . \text{fin.}\,(2p - r)$$
$$+ 0,1589 . \text{fin.}\,(2p + r).$$

Euolutio terminorum angulum $(2q - r)$ inuoluentium.

§. 417.

Hic igitur secundas portiones formularum supra datarum, in suos coefficientes, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, $\mathfrak{e}$, $\mathfrak{f}$ multiplitiplicare debemus, vt obtineamus valores litterae M sequenti modo:

	$\sin(2q-r)$	$\sin(2p-2q+r)$	$\sin(2p+2q-r)$
$b\Pi =$	$+538,63337$	$+9,16756$	$-0,46241$
	$-0,13165$	$-7,73488$	$+7,73488$
	$-0,00664$		
	$+538,49508$	$+1,43268$	$+7,27247$
$c\Pi' =$	$-0,71807$	$+3,72612$	$+3,72612$
	$-0,05859$		
	$-0,77666$		
	$+537,71842$	$+5,15880$	$+10,99859$
$d\mathfrak{P}^2 p =$	$-264,88612$	$+101,33276$	$+11,38821$
	$-1,81895$	$+4,75476$	$-4,75476$
	$+0,20442$		
	$-266,50065$	$+106,08752$	$+6,63345$
$e\mathfrak{P}Pp =$	$+271,21777$	$+111,24632$	$+17,63204$
	$+0,10446$	$-6,91649$	$-6,91649$
	$-0,26328$		
	$-0,15882$		
	$+271,05895$	$+104,32983$	$+10,71555$
$fP^2 p =$	$-276,18823$	$-110,84661$	$+9,19176$
	$+1,98974$	$+4,95765$	$-4,95765$
	$+0,16499$		
	$-274,03350$	$-105,88896$	$+4,23411$
$M =$	$-2,97455$	$-1,55913$	$+14,94966$

§. 418.

§. 418.

Ex his igitur quaeramus litteram N postquam denominatores pro his angulis fuerimus assecuti:

$\omega =$	$2q-r$	$2p-2q+r$	$2p+2q-r$
hinc $\mu =$	$2n-l$	$2m-2n+l$	$2m+2n-l$
siue $\mu =$	$+13,08945$	$+11,64859$	$+37,82729$
Log. $\mu =$	$+1,1169215$	$+1,0662659$	$+1,5778052$
Log. $\mu^2 =$	$+2,2338430$	$+2,1325318$	$+3,1556104$
$\mu^2 =$	$+171,33379$	$+135,68500$	$+1430,90366$
$-(1+\lambda) =$	$-180,22893$	$-180,22393$	$-180,22893$
Denom. $=$	$-8,89514$	$-44,54393$	$+1250,67473$
Log. M $=$	$-0,4734213$	$-0,1928768$	$+1,1746305$
Log. Den. $=$	$-0,9491515$	$-1,6487885$	$+3,0971443$
Log. N $=$	$+9,5242698$	$+8,5440883$	$+8,0774862$
N $=$	$+0,3344$	$+0,0350$	$+0,0120$

§. 419.

Statuamus iam pro hac secunda portione

$$1 = b.\sin.(2q-r) + c.\sin.(2p-2q+r) + d.\sin.(2p+2q-r),$$

hincque definiamus litteram M′

$$M' = \left|\begin{array}{c} 2q-r \\ -1,93032.(c-d) \end{array}\right|\begin{array}{c} 2p-2q+r \\ -1,93032.b \end{array}\left|\begin{array}{c} 2p+2q-r \\ +1,93032.b. \end{array}\right.$$

Hinc autem pro ſecunda et tertia columna, reperiemus litteram N′ vt ſequitur:

	$\frac{2p-2q+r}{b}$	$\frac{2p+2q-r}{b}$
Log. M′ =	−0,2856299	+0,2856299
Log. Den. =	−1,6487885	+3,0971443
Log. N′ =	+8,6368414	+7,1884656
N′ =	+0,0433. b	+0,0015. b

§. 420.

Quocirca litterae c et d ſequenti modo exprimentur:

$$c = +0,0350 + 0,0433 . b$$

$$d = +0,0120 + 0,0015 . b \text{ hinc}$$

$$c - d = +0,0230 + 0,0418 . b$$

ex quo pro prima columna fiet

$$M' = -0,0444 - 0,0807 . b,$$

vnde inuenitur valor reſpondens

$$N' = +0,0050 + 0,0091 . b.$$

§. 421.

Hinc ergo conſequimur hanc determinationem:

$$b = +0,3394 + 0,0091 . b;$$

vnde concluditur $b = +0,3425$, porroque

$$c = +0,0498 \text{ et } d = +0,0125,$$

ita vt ſecunda portio litterae $\mathfrak{r}$ ſit

$$\mathfrak{r} = +0,3425 . \sin.(2q-r) + 0,0498 . \sin.(2p-2q+r) + 0,0125 . \sin.(2p+2q-r).$$

§. 422.

§. 422.

III. Euolutio terminorum angulum $2q+r$ inuoluentium.

Hic iam tertiam portionem formularum supra allatarum in coefficientes $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, $\mathfrak{e}$, $\mathfrak{f}$ multiplicare debemus, vt obtineantur valores litterae M:

	$2q+r$	$2p-2q-r$	$2p+2q+r$
$\mathfrak{b}\Pi =$	− 0,80653	− 19,92668	− 1,07000
	+ 0,28615	+ 0,01158	− 0,01158
	− 0,01536		
	− 0,53574	− 19,91510	− 1,08158
$\mathfrak{c}\Pi' =$	− 1,39211	+ 1,73772	+ 1,73772
	− 0,01680		
	− 1,40891		
	− 1,94465	− 18,17738	+ 0,65614
$\mathfrak{d}\mathfrak{P}^2 p =$	+ 268,75752	− 90,98738	− 1,04311
	+ 1,63314	− 4,82426	+ 4,82426
	− 0,01872		
	+ 270,37194	− 95,81164	+ 3,78115
	+ 268,42729	− 113,98902	+ 4,43729
$\mathfrak{e}\mathfrak{P}Pp =$	− 0,37049	+ 6,91182	+ 6,91182
	− 0,00549		
	− 0,37598		
	+ 268,05131	− 107,07720	+ 11,34911
$\mathfrak{f}P^2 p =$	+ 271,87331	+ 121,31830	+ 1,27969
	− 2,17769	− 4,88019	+ 4,88019
	+ 0,02297		
	+ 269,71859	+ 116,43811	+ 6,15988
M =	+ 537,76990	+ 9,36091	+ 17,50899

§. 423.

§. 423.

Ex his igitur quaeramus litteram N poslquam denominatores debitos fuerimus consecuti:

$\omega =$	$2q+r$	$2p-2q-r$	$2p+2q+r$
$\mu =$	$2n+l$	$2m-2n-l$	$2m+2n+l$
seu $\mu =$	$+39,93471$	$-15,19687$	$+64,67255$
Log. $\mu =$	$+1,6013506$	$-1,1817541$	$+1,8107200$
Log. $\mu^2 =$	$+3,2027012$	$+2,3635082$	$+3,6214400$
$\mu^2 =$	$+1594,78111$	$+230,94480$	$+4182,53958$
$-(1+\lambda) =$	$-180,22893$	$-180,22893$	$-180,22893$
Denom. $=$	$+1414,55218$	$+50,71587$	$+4002,31065$
Log. M $=$	$+2,7305966$	$+0,9713180$	$+1,2432611$
Log. Den. $=$	$+3,1506189$	$+1,7051438$	$+3,6023107$
Log. N $=$	$+9,5799777$	$+9,2661742$	$+7,6409504$
N $=$	$+0,3802$	$+0,1846$	$+0,0044$

§. 424.

Ponamus iam pro tertia portione ipsius r,

$r = b.\,\text{fin.}(2q+r) + c.\,\text{fin.}(2p-2q-r) + d.\,\text{fin.}(2p+2q+r)$

hincque definiamus litteram M':

$$M' = \left| \begin{array}{c} 2q+r \\ -1,93032(c-d) \end{array} \right| \begin{array}{c} 2p-2q-r \\ -1,93032.b \end{array} \left| \begin{array}{c} 2p+2q+r \\ +1,93032.b \end{array} \right.$$

hinc

hinc pro ſecunda et tertia columna reperietur littera N′, vt ſequitur:

	$2p-2q-r$	$2p+2q+r$
Log. M′ =	−0,2856299	+0,2856299
Log. Den. =	+1,7051438	+3,6023107
Log. N′ =	−8,5804861	+6,6833192
N′ =	−0,0381. b	+0,0004. b

§. 425.

Quam ob rem litterae c et d ſequenti modo exprimuntur:

$$c = +0,1846 - 0,0381.b$$

$$d = +0,0044 + 0,0004.b$$

vnde fit

$$c-d = +0,1802 - 0,0385.b,$$

hinc pro prima columna obtinemus

$M' = -0,3478 + 0,0743.b$, vnde $N' = -0,0003$.

§. 426.

Hinc ergo conſequimur has determinationes: $b = +0,3799$, porro $c = +0,1701$ et $d = +0,0045$ ita vt ſit haec tertia portio:

$$r = +0,3799.\text{ſin.}(2q+r) + 0,1701.\text{ſin.}(2p-2q-r) + 0,0045.\text{ſin.}(2p+2q+r).$$

 §. 427.

§. 427.

En ergo totum valorem ex his tribus partibus compositum:

$$\begin{aligned}\mathfrak{r} = 0, \text{ſin.}\, r &+ 0,0363.\,\text{ſin.}\,(2p - r) \\ &+ 0,1589.\,\text{ſin.}\,(2p + r) \\ &+ 0,3425.\,\text{ſin.}\,(2q - r) + 0,0498.\,\text{ſin.}\,(2p - 2q + r) \\ &+ 0,0125.\,\text{ſin.}\,(2p + 2q - r) \\ &+ 0,3799.\,\text{ſin.}\,(2q + r) + 0,1701.\,\text{ſin.}\,(2p - 2q - r) \\ &+ 0,0045.\,\text{ſin.}\,(2p + 2q + r).\end{aligned}$$

CAPVT IV.

CAPVT IV.

EVOLVTIO ORDINIS (IV.) CHARACTERIS $i\varkappa$ PRO LITTERA $\mathfrak{s}$.

§. 428.

Aequatio ſpecialis pro hoc caſu ita ſe habet:

$$\mathfrak{c} = \frac{dd\mathfrak{s}}{dt^2} + (\lambda + 1)\mathfrak{s} + \mathfrak{s}\,\mathfrak{a} + \mathfrak{U}\,p\,\mathfrak{b} + \mathrm{U}\,p\,\mathfrak{c} - 3\,p\,\mathrm{cof.}\,t \;;$$

in cuius reſolutione iisdem compendiis vtemur, quae prius ſunt expoſita, ita vt ſit:

$$\mathfrak{a} = +3,8606454.\ \mathrm{cof.}\ 2\,p;$$
$$\mathfrak{b} = -537,686784 - 15,4425816.\ \mathrm{cof.}\ 2\,p;$$
$$\mathfrak{c} = -5,4906954.\ \mathrm{fin.}\ 2\,p.$$

Tum vero in ſuperioribus iam inuenimus:

$$\mathfrak{U} = -0,006906.\ \mathrm{cof.}\ t + 0,029369.\ \mathrm{cof.}\ (2\,p - t) - 0,003433.\ \mathrm{cof.}\ (2\,p + t)$$

$$U = +0,175997.\sin. t - 0,043196.\sin.(2p - t) + 0,005443.\sin.(2p + t)$$

ex quibus formulae $\mathfrak{U}\mathfrak{p}$ et $U\mathfrak{p}$ colligantur, quae ita reperiuntur expressae:

$$\mathfrak{U}\mathfrak{p} = -0,004000.\sin.(r-t) + 0,001588.\sin.(2p-r+t) + 0,014678.\sin.(2p+r-t) - 0,003368.\sin.(r+t) - 0,014812.\sin.(2p-r-t) - 0,001722.\sin.(2p+r+t)$$

$$U\mathfrak{p} = +0,087203\cos.(r-t) - 0,000533\cos.(2p-r+t) + 0,021731.\cos.(2p+r-t) - 0,087930.\cos.(r+t) - 0,018344.\cos.(2p-r-t) - 0,002854.\cos.(2p+r+t)$$

tum vero

$$+3\mathfrak{p}\cos.t = +1,500000.\sin.(r-t) + 0,055473.\sin.(2p-r+t) + 0,002269.\sin.(2p+r-t) + 1,500000.\sin.(r+t) + 0,055473.\sin.(2p\; r-t) + 0,002269.\sin.(2p+r+t)$$

quae producta sponte se diuidunt in bina membra seorsim euoluenda.

I. Euolu-

I. Euolutio terminorum angulum $(r-t)$ inuoluentium.

§. 429.

Hic ergo istorum productorum tantum membra priora consideramus, vnde numeros littera M insignitos colligamus:

	ſin $(r-t)$	ſin. $(2p-r+t)$	ſin $(2p+r-t)$
Upb =	+ 2,150748	−0,853853	−7,892160
	+ 0,012261	−0,030885	+0,030885
	− 0,113333		
	+ 2,049676	−0,884738	−7,861275
Upc =	+0,001463	−0,239403	− 0,239403
	+0,059659		
	+0,061122		
	+2,110798	−1,124141	−8,100678
-3pcſ.t =	− 1,500000	−0,055473	−0,002269
M =	+0,610798	− 1,179614	−8,102947

§. 430.

§. 430.

Hinc ergo deducamus valorem respondentem N, constitutis ante elementis numericis:

$\omega =$	$r - t$	$2p - r + t$	$2p + r - t$
$\mu =$	$l - 1$	$2p - l + 1$	$2p + l - 1$
seu $\mu =$	$+12,42263$	$+12,31521$	$+37,16047$
Log. $\mu =$	$+1,0942135$	$+1,0904417$	$+1,5700812$
Log. $\mu^2 =$	$+2,1884270$	$+2,1808834$	$+3,1401624$
$\mu^2 =$	$+154,32172$	$+151,66424$	$+1380,90064$
$-(1+\lambda) =$	$-180,22893$	$-180,22893$	$-180,22893$
Denom. $=$	$-25,90721$	$-28,56469$	$+1200,67171$
Log. M $=$	$+9,7858976$	$-0,0717400$	$-0,9086430$
Log. Den. $=$	$-1,4134204$	$-1,4558295$	$+3,0794242$
Log. N $=$	$-8,3724768$	$+8,6159105$	$-7,8292188$
N $=$	$-0,02358$	$+0,04130$	$-0,00675$

§. 431.

Iam pro numeris M′ et N′ ex parte incognita ϑ deducendis ponamus:

$$\vartheta = b.\ \text{sin}\ (r - t) + c.\ \text{sin.}\ (2p - r + t) + d.\ \text{sin.}\ (2p + r - t)$$

et numerus M′ ita colligetur

M′ $=$	sin. $(r - t)$	sin. $(2p - r + t)$	sin. $(2p + r - t)$
	$-1,930322.(c - d)$	$-1,930322.b$	$+1,930322.b.$

vnde

vnde primum pro secunda et tertia columna numerum N′ quaeramus:

	$\sin.(2p-r+t)$	$\sin.(2p+r-t)$
Log. $M'=$	$-0,2856299$	$+0,2856299$
Log. Den. $=$	$-1,4558295$	$+3,0794242$
Log. $N'=$	$+8,8298004$	$+7,2062057$
$N'=$	$+0,06758.b$	$+0,00161.b$

§. 432.

Hinc ergo statim nanciscimur has determinationes:

$$c=+0,04130+0,06758.b$$

$$d=-0,00675+0,00161.b \text{ ergo}$$

$$c-d=+0,04805+0,06597.b$$

vnde pro prima columna adipiscimur

$$M'=-0,09275-0,12729.b,$$

cui respondens numerus N′ inuenitur:

$$=+0,00358+0,00491.b.$$

§. 433.

Sic igitur tandem peruenimus ad hanc aequationem:

$$b=-0,02000+0,00491.b,$$

vnde

vnde fit

$b = -0,02010$, hincque porro

$c = +0,03994$; $d = -0,00686$,

ita vt haec euolutio nobis praebeat:

$$\delta = -0,02010.\,\text{sin.}(r-t) + 0,03994.\,\text{sin.}(2p-r+t) - 0,00686.\,\text{sin.}(2p+r-t).$$

II. Euolutio terminorum angulum $(r+t)$ inuoluentium.

Hic tantum consideramus membra posteriora formularum supra exhibitarum, vnde statim colligamus numeros M:

	sin. $(r+t)$	sin. $(2p-r-t)$	sin. $(2p+r+t)$
$\mathfrak{U}\,p\,\mathfrak{b} =$	+1,810929	+7,964219	+0,925897
	−0,114368	−0,026005	+0,026005
	+0,013296		
	+1,709857	+7,938214	+0,951902
$\mathfrak{U}\,p\,c =$	+0,050361	+0,241398	+0,241398
	−0,007835		
	+0,042526		
	+1,752383	+8,179612	+1,193300
$-3\,p\,c\,\text{f.}\,t =$	−1,500000	−0,055473	−0,002269
M =	+0,252383	+8,124139	+1,191031

§. 434.

§. 434.

Hinc deducamus valorem respondentem N, quaesitis ante elementis numericis:

$\omega =$	$r+t$	$2p-r-t$	$2p+r+t$
$\mu =$	$l+1$	$2m-l-1$	$2m+l+1$
siue $\mu =$	$+14,42263$	$+10,31521$	$+39,16047$
L. $\mu =$	$+1,1590445$	$+1,0134780$	$+1,5928479$
L. $\mu^2 =$	$+2,3180890$	$+2,0269560$	$+3,1856958$
$\mu^2 =$	$+208,01229$	$+106,40354$	$+1533,54211$
$-(1+\lambda) =$	$-180,22893$	$-180,22893$	$-180,22893$
Den. $=$	$+27,78336$	$-73,82539$	$+1353,31318$
L. M $=$	$+9,4020601$	$+0,9097774$	$+0,0759242$
L. Den. $=$	$+1,4437847$	$-1,8682058$	$+3,1313983$
L. N $=$	$+7,9582754$	$-9,0415716$	$+6,9445259$
N $=$	$+0,00908$	$-0,11004$	$+0,00088$

§. 435.

Iam pro numeris M′ et N′ ex parte incognita s deducendis, ponamus

$$s = b.\sin.(r+t) + c.\sin.(2p-r-t) + d.\sin.(2p+r+t)$$

et erit numerus

M′ $=$	$\sin.(r+t)$	$\sin.(2p-r-t)$	$\sin.(2p+r+t)$
	$-1,93032.(c-d)$	$-1,93032.b$	$+1,93032.b$

vnde primum pro ſecunda et tertia columna numerum N′ quaeramus:

	$2p-r-t$	$2p+r+t$
Log. M′ =	$-0,2856299$	$+0,2856299$
Log. Denom. =	$-1,8682058$	$+3,1313983$
Log. N′ =	$+8,4174241$	$+7,1542316$
N′ =	$+0,02614.\,b$	$+0,00143.\,b$

§. 436.

Hinc igitur ſtatim deducuntur hae determinationes:

$$c = -0,11004 + 0,02614.\,b$$

$$d = +0,00088 + 0,00143.\,b \text{ ergo}$$

$$c - d = -0,11092 + 0,02471.\,b$$

ex quo pro prima columna fiet:

$$M' = +0,21411 - 0,04770.\,b,$$

cui reſpondens numerus N′ habetur

$$= +0,00771 - 0,00172.\,b.$$

Sic igitur denique ad hanc peruenimus aequationem:

$$b = +0,01679 - 0,00172.\,b,$$

vnde fit

$$b = +0,01676;\ c = -0,10960;$$
$$d = +0,00090,$$

ita

ita vt haec euolutio nobis praebeat:

$$\vartheta = +0,01676 . \sin.(r+t) - 0,10960 . \sin.(2p-r-t) + 0,00090 . \sin.(2p+r+t).$$

§. 437.

Ecce ergo completum valorem litterae ϑ, quem hoc capite sumus adepti:

$$\vartheta = -0,02010 . \sin.(r-t) + 0,03994 . \sin.(2p-r+t) - 0,00686 . \sin.(2p+r-t) + 0,01676 . \sin.(r+t) - 0,10960 . \sin.(2p-r-t) + 0,00090 . \sin.(2p+r+t).$$

qui termini per $i\varkappa$ multiplicati, ad tertiam nostram coordinatam accedunt.

CAPVT V.

EVOLVTIO ORDINIS (V.) CHARACTERIS i^3 PRO LITTERA $\mathfrak{t}$.

§. 438.

Aequatio ſpecialis huc pertinens ita ex ſuperioribus deducitur:

$$0 = \frac{ddt}{dt^2} + (\lambda + 1.)\mathfrak{t} + \mathfrak{a}\mathfrak{t} + \mathfrak{b}\mathfrak{X}\mathfrak{p} + \mathfrak{c}X\mathfrak{p}$$
$$+ \mathfrak{d}\mathfrak{p}(-\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{Q})$$

vbi vt hactenus habemus:

$$\mathfrak{a} = +\,3,8606454.\ \text{coſ.}\ 2p;$$
$$\mathfrak{b} = -\,537,686784 - 15,4425816.\ \text{coſ.}\ 2p;$$
$$\mathfrak{c} = -\,5,4906954.\ \text{ſin.}\ 2p,$$

tum vero

$$-\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{Q} = -268,843392 - 9,651613.\ \text{coſ.}\ 2p.$$

§. 439.

§. 439.

Hic ergo occurrunt litterae $\mathfrak{X}$ et X, quorum valores in praecedentibus nondum sunt inuestigati siquidem ipsam litterae z determinationem requirunt, interim tamen quia tantum ab eius parte prima p pendent, ne filum nostrae inuestigationis interrumpamus, ex sequentibus hos valores $\mathfrak{X}$ et X depromere licebit, qui ita sunt comparati:

$$\begin{aligned}\mathfrak{X} = -0,25019 &+ 0,01928 . \cos 2p \\ &+ 0,00002 . \cos 4p \\ + 0,24728 . \cos 2r &- 0,01242 . \cos(2p-2r) \\ &+ 0,00038 . \cos(2p+2r) \\ &+ 0,00025 . \cos(4p-2r) \\ &+ 0,00000 . \cos(4p+2r)\end{aligned}$$

$$\begin{aligned}X = -0,02145 . \sin 2p &- 0,00003 . \sin 4p \\ - 0,24645 . \sin 2r &+ 0,03407 . \sin(2p-2r) \\ &- 0,00037 . \sin(2p+2r) \\ &- 0,00014 . \sin(4p-2r) \\ &+ 0,00000 . \sin(4p+2r)\end{aligned}$$

Praeterea quia breuitatis gratia posuimus

$\mathfrak{p} = \mathrm{p}^2$, facto calculo habebimus

$$\mathfrak{p} = +0,500686 - 0,035467.\cos.2p - 0,000097.\cos.4p$$

$$-0,499944.\cos.2r + 0,036982.\cos.(2p-2r) - 0,001513.\cos.(2p+2r) - 0,000637.\cos.(4p-2r) - 0,000008.\cos.(4p+2r)$$

§. 440.

Hinc ergo producta quae in nostra aequatione occurrunt euoluantur:

$$\mathfrak{X}\mathrm{p} = -0,37443.\sin.r - 0,02491.\sin.(2p-r) + 0,01364.\sin.(2p+r) + 0,00045.\sin.(4p-r) + 0,00003.\sin.(4p+r)$$

$$+0,12362.\sin.3r + 0,01078.\sin.(2p-3r) + 0,00038.\sin(2p+3r) - 0,00035.\sin.(4p-3r) + 0,00000.\sin(4p+3r)$$

$\mathrm{X}\,\mathrm{p} =$

$$
\begin{aligned}
Xp = & -0{,}12301.\cos r - 0{,}02794.\cos(2p-r) \\
& \qquad + 0{,}01510.\cos(2p+r) \\
& \qquad + 0{,}00033.\cos(4p-r) \\
& \qquad + 0{,}00006.\cos(4p+r) \\
& + 0{,}12323.\cos 3r + 0{,}01247.\cos(2p-3r) \\
& \qquad + 0{,}00037.\cos(2p+3r) \\
& \qquad - 0{,}00057.\cos(4p-3r) \\
& \qquad + 0{,}00000.\cos(4p+3r)
\end{aligned}
$$

$$
\begin{aligned}
Yp = & +0{,}75196.\sin r + 0{,}05436.\sin(2p-r) \\
& \qquad - 0{,}02545\ \sin(2p+r) \\
& \qquad - 0{,}00087.\sin(4p-r) \\
& \qquad - 0{,}00011.\sin(4p+r) \\
& - 0{,}24991.\sin 3r - 0{,}02773.\sin(2p-3r) \\
& \qquad - 0{,}00114.\sin(2p+3r) \\
& \qquad + 0{,}00099.\sin(4p-3r) \\
& \qquad + 0{,}00000.\sin(4p+3r)
\end{aligned}
$$

quae formulae ſponte in duas partes diſtinguuntur, ex quo tractatio noſtra bipartita eſt conſtituenda.

I. Euo-

I. Euolutio terminorum angu-

Ex partibus igitur cognitis

	fin. r	fin. $(2p-r)$
$\mathfrak{X}\mathfrak{p}(-537,68)=$	+ 201,32141	+ 13,39316
$\mathfrak{X}\mathfrak{p}(-15,44.\ \text{cof.}\ 2p)=$	− 0,19234	− 2,89103
	− 0,10532	− 0,00347
	+ 201,02375	+ 10,49851
$X\mathfrak{p}(-5,48.\ \text{fin.}\ 2p)=$	+ 0,07670	+ 0,33770
	+ 0,04145	+ 0,00091
	+ 0,11815	+ 0,33861
	+ 201,14190	+ 10,83712
$\mathfrak{b}\mathfrak{p}(-268,84)=$	− 202,15958	− 14,61433
$\mathfrak{b}\mathfrak{p}(-9,65.\ \text{cof.}\ 2p)=$	+ 0,26419	+ 3,65397
	+ 0,12367	+ 0,00423
	− 201,77172	− 10,95611
$M=$	− 0,62982	− 0,11899

lum fimplicem r continentium.

Colligamus valores litterae M:

fin. $(2p+r)$	fin. $(4p-r)$	fin $(4p+r)$
−7,33405	−0,24195	−0,01613
+2,89108	+0,19234	−0,10532
−0,00023		
−4,44320	−0,04961	−0,12145
+0,33770	+0,07670	−0,04145
+0,00016		
+0,33786	+0,07670	−0,04145
−4,10534	+0,02709	−0,16290
+6,84206	+0,23389	+0,02957
−3,65397	−0,26419	+0,12367
+0,00053		
+3,18862	−0,03030	+0,15324
−0,91672	−0,00321	−0,00966

 §. 441.

§. 441.

Nunc igitur ſepoſita prima columna ſeu ipſo angulo r, prorſus vti ſupra capite tertio fecimus, pro reliquis angulis definiamus litteram N, denominatoribus indidem ſumtis:

	$2p-r$	$2p+r$	$4p-r$	$4p+r$
L. M =	$-9{,}0755105$	$-9{,}9622367$	$-7{,}5065$	$-7{,}9850$
L.Den. =	$-1{,}7176281$	$+3{,}1058484$	$+3{,}0490$	$+3{,}5770$
L. N =	$+7{,}3578824$	$-6{,}8563883$	$-4{,}4575$	$-4{,}4080$
N =	$+0{,}0023$	$-0{,}0007$	$-0{,}000003$	$-0{,}000003$

§. 442.

Pro partibus autem incognitis ſtatuamus itidem vt in Cap. III. prima euolutione

$$t = 0.\,\text{ſin.}\,r + c.\,\text{ſin.}\,(2p-r) + d.\,\text{ſin.}\,(2p+r) + e.\,\text{ſin.}\,(4p-r) + f.\,\text{ſin.}\,(4p+r)$$

vnde non ſolum iidem valores pro littera M′ ſed etiam pro N′ reperiuntur, quamobrem nanciſcimur ſequentes determinationes

$$c = +0{,}0023 - 0{,}0370.\,e;$$
$$d = -0{,}0007 + 0{,}0015.\,f;$$
$$e = +0{,}0000 + 0{,}0017.\,c;$$
$$f = -0{,}0000 + 0{,}0005.\,d.$$

hi

hi valores in superioribus substituti praebent

$$c = +0,0023;\ d = -0,0007;$$
$$e = +0,0000;\ f = -0,0000.$$

§. 443.

Pro angulo autem r, nunc habebimus

$$M' = -1,930\,(c - d) = -0,0058,$$

omnino igitur $M + M' = -0,6356$, qui valor ad primum ordinem est transferendus, ita vt iam habeamus:

$$-0,061970 = -0,068341 + 4,12763.k^2 - 0,6356.ii,$$

vbi euidens est leuissimam mutationem in numero l factam, sufficere consensui perfecto restituendo.

§. 444.

En ergo priorem partem litterae nostrae t, quam nobis haec euolutio suppeditauit, et quae vlteriori correctione non indiget:

$$\begin{aligned} t = 0.\ \text{fin.}\ r &+ 0,0023.\ \text{fin.}\ (2p - r) \\ &- 0,0007.\ \text{fin.}\ (2p + r) \\ &+ 0,0000.\ \text{fin.}\ (4p - r) \\ &+ 0,0000.\ \text{fin.}\ (4p + r). \end{aligned}$$

II. Euolutio terminorum an-

§.

Primum igitur hic ex partibus posterioribus no-

	sin. $3r$	sin. $(2p-3r)$
$\mathfrak{X}\,p\,(-537,68)$	$-66,46884$	$-5,79626$
$\mathfrak{X}\,p\,(-15,44.\cos.\,2p)$	$+0,08324$	$+0,95450$
	$-0,00292$	$+0,00270$
	$-66,38852$	$-4,83906$
$X\,p\,(-5,48.\sin.\,2p$	$-0,03423$	$-0,33830$
	$+0,00101$	
	$-0,03322$	
	$-66,42174$	$-5,17736$
$\mathfrak{Y}\,p\,(-268,84.)=$	$+67,18665$	$+7,45503$
	$-0,13475$	$-1,21438$
$\mathfrak{Y}\,p\,(-9,65.\cos.\,2p)=$	$+0,00554$	$-0,00481$
	$+67,05744$	$+6,23584$
$M=$	$+0,63570$	$+1,05848$

gulum $3r$ continentium.

445. [illegible]
strarum formularum valores litterae M colligamus:

sin. $(2p+3r)$	sin. $(4p-3r)$	sin. $(4p+3r)$
−0,20432	+0,18819	
−0,95450	−0,08324	
		−0,00292
−1,15882	+0,10495	−0,00292
−0,33830	−0,03423	
+0,00156	[illegible]	
−0,33674		−0,00101
−1,49556	−0,07072	−0,00393
+0,30648	−0,26615	
+1,21438	+0,13475	
		+0,00554
+1,52086	−0,13140	+0,00554
+0,02530	−0,20212	+0,00161

§.

His igitur respondentes litteras N inuestige-
his angulis

$\omega =$	$3r$	$2p - 3r$
$\mu =$	$3l$	$2m - 3l$
seu $\mu =$	$+ 40,26789$	$- 15,53005$
Log. $\mu =$	$+ 1,6049589$	$- 1,1911729$
Log. $\mu^2 =$	$+ 3,2099178$	$+ 2,3823458$
$\mu^2 =$	$+ 1621,5030$	$+ 241,1824$
$-(1 + \lambda) =$	$- 180,2289$	$- 180,2289$
Denom. $=$	$+ 1441,2741$	$+ 60,9535$
Log. M $=$	$+ 9,8032522$	$+ 0,0246827$
Log. Den. $=$	$+ 3,1587466$	$+ 1,7849986$
Log. N $=$	$+ 6,6445056$	$+ 8,2396841$
N $=$	$+ 0,0004$	$+ 0,0174$

446.

mus poſtquam debitos denominatores pro definiuerimus:

$\frac{2p+3r}{2m+3l}$	$\frac{4p-3r}{4m-3l}$	$\frac{4p+3r}{4m+3l}$
+ 65,00573	+ 9,20779	+ 89,74357
+ 1,8129518	+ 0,9641554	+ 1,9530033
+ 3,6259036	+ 1,9283108	+ 3,9060066
+ 4225,7478	+ 84,7834	+ 8053,9074
− 180,2289	− 180,2289	− 180,2289
+ 4045,5189	− 95,4455	+ 7873,6785
+ 8,4031205	− 9,3056093	+ 7,2068259
+ 3,6069753	− 1,9797554	+ 3,8961776
+ 4,7961452	+ 7,3258539	+ 3,3106483
+ 0,0000	+ 0,0021	+ 0,0000

§. 447.

§. 447.

Pro parte autem incognita statuamus

$$t = b.\ \text{sin.}\ 3r + c.\ \text{sin.}(2p - 3r) + d.\ \text{sin.}(2p + 3r)$$
$$+ e.\ \text{sin.}(4p - 3r) + f.\ \text{sin.}(4p + 3r)$$

hincque definiamus M′

	sin. $3r$	sin. $(2p - 3r)$	sin. $(2p + 3r)$
$t(3{,}860.\text{cos}.2p)$	$-1{,}930.(c-d)$	$-1{,}930.(b-e)$	$+1{,}930.(b+f)$

	sin. $(4p - 3r)$	sin. $(4p + 3r)$
$t(3{,}860.\text{cos}.2p)$	$+1{,}930.c$	$+1{,}930.d$

§. 448.

Seposita prima columna, secundam cum quarta coniunctim expediamus:

	$2p - 3r$ $(b - e)$	$4p - 3r$ c
Log. M′ =	$-0{,}2856299$	$+0{,}2856299$
Log. Den. =	$+1{,}7849986$	$-1{,}9797554$
Log. N′ =	$-8{,}5006313$	$-8{,}3058745$
N′ =	$-0{,}0316.(b-e)$	$-0{,}0202.\ c$

ergo $c = +0{,}0174 - 0{,}0316.\ b + 0{,}0316.\ e$

$e = +0{,}0021 - 0{,}0202.\ c.$

qui valor in illa substitutus dat

$c = +0{,}0175 - 0{,}0316.\ b - 0{,}0006.\ c,$

siue $c = +0{,}0175 - 0{,}0316.\ b.$

§. 449.

§. 449.

Simili modo tertia columna cum quarta tractetur:

	$2p + 3r$ $b + f$	$4p + 3r$ d
Log. M′ =	+ 0,2856	+ 0,2856
Log. Den. =	+ 3,6069	+ 3,8961
Log. N′ =	+ 6,6787	+ 6,3895
N′ =	+ 0,0005	+ 0,0002

ergo

$d = + 0,0000 + 0,0005.b + 0,0005.f$

$f = + 0,0000 + 0,0002.d$

ideoque $d = + 0,0005.b$.

§. 450.

Pro angulo $3r$ ob

$M' = - 1,930(c-d) = - 0,0338 + 0,0620.b$,

calculus ita se habebit

	1	b
Log. M′ =	− 2,5286	+ 8,7921
Log. Den. =	+ 3,1587	+ 3,1587
Log. N′ =	− 5,3699	+ 5,6334
N′ =	− 0,0000	+ 0,0000

quare habemus $b = + 0,0004$, ideoque reliquae litterae

$$c = + 0,0175 \,;\; d = + 0,0000 \,;$$
$$e = + 0,0018 \,;\; f = + 0,0000.$$

§. 451.

Quoniam hic correctio manifesto esset superflua completus valor nostrae quantitatis t ita se habebit:

$$t = 0, \text{fin.}\, r + 0,0023.\, \text{fin.}(2p - r) + 0,0000.\, \text{fin.}(4p - r) - 0,0007.\, \text{fin.}(2p + r) + 0,0000.\, \text{fin.}(4p + r)$$
$$+ 0,0004.\, \text{fin.}\, 3r + 0,0175.\, \text{fin.}(2p - 3r) + 0,0000.\, \text{fin.}(2p + 3r) + 0,0018.\, \text{fin.}(4p - 3r) + 0,0000.\, \text{fin.}(4p + 3r).$$

CAPVT VI.

CAPVT VI.

EVOLVTIO ORDINIS CHARACTERE $(i\,a)$ CONTENTI CVI CONVENIT LITTERA $\mathfrak{u}$.

§. 452.

Etsi hunc ordinem supra non commemorauimus, tamen facile intelligere licet eum sequenti aequatione expressum iri:

$$0 = \frac{ddu}{dt^2} + (\lambda + 1)\,u + \mathfrak{u}\,a + \mathfrak{S}\,p\,\mathfrak{b} + S\,p\,c$$
$$+ 3\,p\,\text{cos.}\,p\,(1 + \mathfrak{O}) - 3\,p\,O\,\text{sin.}\,p$$

pro his postremis membris scribemus breuitatis gratia $\mathfrak{P}'(1 + \mathfrak{O}) + P'\,O$, ita vt sit $\mathfrak{P}' = 3\,p\,\text{cos.}\,p$ et $P' = -3p\,\text{sin.}\,p$.

§. 453.

Euoluamus igitur haec producta, quibus membra cognita continentur ac reperiemus:

$$\mathfrak{T}p = -0,05498.\sin.(p-r)+0,05763.\sin.(p+r) \\ +0,00355.\sin.(3p-r) \\ -0,00135.\sin.(3p+r)$$

$$\mathfrak{S}p = -0,12461.\cos.(p-r)+0,12004.\cos.(p+r) \\ +0,00586.\cos.(3p-r) \\ -0,00160.\cos.(3p+r)$$

$$\mathfrak{P}' = -1,44453.\sin.(p-r)+1,50227.\sin.(p+r) \\ +0,05553.\sin.(3p-r) \\ +0,00228.\sin.(3p+r)$$

$$\mathfrak{E}' = -1,55547.\cos.(p+r)+1,49773.\cos.(p+r) \\ +0,05541.\cos.(3p-r) \\ +0,00226.\cos.(3p+r).$$

§. 454.

§. 454.

His iam formulis constitutis colligamus valores litterae nostrae M:

	sin. $(p-r)$	sin. $(p+r)$	sin. $(3p-r)$	sin. $(3p+r)$
𝔖p(− 537. 70.)	+ 29,56202	− 30,98689	− 1,90879	+ 0,72588
	+ 0,44498	− 0,42452	+ 0,42452	− 0,44498
𝔖p(− 15,4. cos. $2p$)	− 0,02741	+ 0,01042		
𝔖p(− ,195. cos $2p$)	− 0,00013	+ 0,00035	+ 0,00564	− 0,00538
	+ 29,97946	− 31,40064	− 1,47863	+ 0,27552
Sp(− 5,49. sin. $2p$)	− 0,32954	+ 0,34209	+ 0,34209	− 0,32954
	+ 0,01609	− 0,00439		
Sp(− 0,101. sin. $4p$)	+ 0,00008	− 0,00030	− 0,00610	+ 0,00633
	− 0,31337	+ 0,33740	+ 0,33599	− 0,32321
	+ 29,66609	− 31,06324	− 1,14264	− 0,04769
𝔓′ =	− 1,44453	+ 1,50227	+ 0,05553	+ 0,00228
𝔓′ 𝔔 =	+ 0,00539	− 0,00518	+ 0,00518	− 0,00539
	− 0,00004	− 0,00020		
		+ 0,00004		
	− 1,43918	+ 1,49693	+ 0,06071	− 0,00311
	+ 28,22691	− 29,56631	− 1,08193	− 0,05080
P′ O =	+ 0,00765	− 0,00794	− 0,00794	+ 0,00765
	− 0,00028	− 0,00001		
	+ 0,00737	− 0,00795		
M =	+ 28,23428	− 29,57426	− 1,08987	− 0,04315

§. 455.

Hinc igitur eliciamus litteram M sequenti modo:

$\omega =$	$p - r$	$p + r$	$3p - r$	$2p + r$
$\mu =$	$m - l$	$m + l$	$3m - l$	$3m + l$
seu $\mu =$	$-1,05371$	$+25,79155$	$+23,68413$	$+50,52939$
Log. $\mu =$	$-0,0227211$	$+1,4114775$	$+1,3744575$	$+1,7035441$
Log. $\mu^2 =$	$+0,0454422$	$+2,8229550$	$+2,7489150$	$+3,4070882$
$\mu^2 =$	$+1,11030$	$+665,20430$	$+560,93822$	$+2553,22000$
$-(1 + \lambda) =$	$-180,22893$	$-180,22893$	$-180,22893$	$-180,22893$
Denom. $=$	$-179,11862$	$+484,97537$	$+380,70929$	$+2372,99107$
Log. M $=$	$+1,4507767$	$-1,4709139$	$-0,0373747$	$-8,6349808$
Log. Den. $=$	$-2,2531408$	$+2,6857198$	$+2,5805935$	$+3,3752963$
Log. N $=$	$-9,1976359$	$-8,7851941$	$-7,4567812$	$-5,2596845$
N $=$	$-0,1576$	$-0,0610$	$-0,0029$	$-0,00002$

§. 456.

§. 456.

Pro prima parte incognita statuamus

$$u = b.\,\text{fin.}(p-r) + c.\,\text{fin.}(p+r)$$
$$+ d.\,\text{fin.}(3p-r) + e.\,\text{fin.}(3p+r)$$

quae formula in partem primam ipsius *a* ducta praebebit nobis M′ hoc modo:

	$p-r$	$p+r$	$3p-r$	$3p+r$
$u(3{,}860.\cos 2p)$	$-1{,}930.(c-d)$	$-1{,}930.(b-e)$	$+1{,}930.b$	$+1{,}930.c$

quibus sequenti modo respondentes valores N′ repeeriuntur:

	$p-r$ $(c-d)$	$p+r$ $(b-e)$	$3p-r$ b	$3p+r$ c
Log. M′ =	$-0{,}28562$	$-0{,}2856$	$+0{,}2856$	$+0{,}2856$
Log. Den. =	$-2{,}25314$	$+2{,}6857$	$+2{,}5805$	$+3{,}3753$
Log N′ =	$+8{,}03248$	$-7{,}5999$	$+7{,}7051$	$+6{,}9103$
N′ =	$+0{,}0108(c-d)$	$-0{,}0040(b-e)$	$+0{,}0051.b$	$+0{,}0008$

§. 457.

Hinc ergo adipiscimur sequentes quatuor determinationes

$$b = -0{,}1576 + 0{,}0108\,c - 0{,}0108.\,d;$$
$$c = -0{,}0610 - 0{,}0040.\,b + 0{,}0040.\,e;$$
$$d = -0{,}0029 + 0{,}0051.\,b;$$
$$e = -0{,}0000 + 0{,}0008.\,c;$$

binis

binis autem posterioribus valoribus, in prioribus substitutis prodit

$$b = -0,1576 + 0,0108.\,c \text{ et}$$
$$c = -0,0610 - 0,0040.\,b$$

hic vero valor ibi inductus producit

$$b = -0,1583 \text{ indeque } c = -0,0604;$$
$$d = -0,0037;\ e = -0,0000.$$

Quoniam facile praeuidere possumus correctiones nullius futuras esse momenti, valor quaesitus litterae u erit ille:

$$u = -0,1583.\,\text{fin.}\,(p-r) - 0,0604.\,\text{fin.}\,(p+r) - 0,0037.\,\text{fin.}\,(3p-r) - 0,0000.\,\text{fin.}\,(3p+r).$$

NOVAE

NOVAE THEORIAE
MOTVVM LVNAE
LIBER PRIMVS
CONTINENS IPSAM LVNAE THEORIAM.

CONTINVATIO
PARTIS SECVNDAE.

EVOLVTIO NVMERICA AEQVATIONVM PRO IIS MEMBRIS COORDINATARVM x ET y, QVORVM CHARACTERES INVOLVVNT ii.

CAPVT I.

EVOLVTIO AEQVATIONVM ORDINIS XI. CHARACTERE $;;$ PRO LITTERIS $\mathfrak{X}$ ET X.

§. 458.

Partes annexae aequationum specialium, quas hic resolui oporter, ita supra sunt exhibitae:

I. $0 = \ldots \mathfrak{X}\mathfrak{A} + X\mathfrak{B} + \mathfrak{b}(-\frac{3}{2}\lambda + 6\lambda\mathfrak{O} - 15\lambda\mathfrak{O}^2 + \frac{15}{4}\lambda O^2)$

II. $0 = \ldots \mathfrak{X}A + XB - \frac{3}{2}\mathfrak{b}\lambda O + \frac{15}{2}\mathfrak{b}\lambda\mathfrak{O}O.$

vbi quia hos valores exactissime definire oportet, postremos coefficientes plenius exhibuimus, quorum valores in numeris ita se habebunt:

$$-\tfrac{3}{4}\lambda + 6\lambda\mathfrak{O} - 15\lambda\mathfrak{O}^2 + \tfrac{15}{4}\lambda O^2 = -268,8518391$$
$$-7,7212908.\,\mathrm{cof.}\,2p - 0,0988907.\,\mathrm{cof.}\,4p$$
$$-\tfrac{3}{2}\text{♄}\lambda O + \tfrac{15}{2}\text{♄}\lambda\mathfrak{O}O = -2,7453482.\,\mathrm{fin.}\,2p$$
$$-0,0508109.\,\mathrm{fin.}\,4p.$$

merito enim hic rationem habemus minimorum terminorum, quoniam determinatio quantitatum $\mathfrak{X}$ et X, etiam fequentes ordines plurimum afficit, vnde etiam pro litteris $\mathfrak{A}$, A, $\mathfrak{B}$, B plenos valores accipi oportebit:

$$\mathfrak{A} = -9,2212908.\,\mathrm{cof.}\,2p$$
$$+0,0264388 - 0,1050568.\,\mathrm{cof.}\,4p.$$
$$\mathfrak{B} = -3,9906964.\,\mathrm{fin.}\,2p - 0,0819104.\,\mathrm{fin.}\,4p.$$
$$A = -3,9906964.\,\mathrm{fin.}\,2p - 0,0819104.\,\mathrm{fin.}\,4p$$
$$B = +5,3606454.\,\mathrm{cof.}\,2p$$
$$-0,0272367 + 0,0665457.\,\mathrm{cof.}\,4p.$$

Denique quum fit $\text{♄} = p^2$ ex valore ipfius p fupra inuento colligimus

$$\text{♄} = +0,500686 - 0,035467\,\mathrm{cof.}\,2p$$
$$-0,000097.\,\mathrm{cof.}\,4p$$
$$-0,499944.\,\mathrm{cof.}\,2r + 0,036982\,\mathrm{cof.}(2p-2r)$$
$$-0,001513.\,\mathrm{cof.}(2p+2r)$$
$$-0,000637.\,\mathrm{cof.}(4p-2r)$$
$$-0,000008.\,\mathrm{cof.}(4p+2r).$$

Quae

Quae expressio sponte se diuidit in duas partes, quas seorsim euolui conuenit:

I. Euolutio terminorum, angulum r non continentium.

§. 459.

Ex partibus cognitis primum litteras $\mathfrak{M}$ et M colligamus:

	absol.	cos. $2p$	cos. $4p$
♄$(-268,85)=$	$-134,610372$	$+9,535369$	$+0,026078$
♄$(-7,7212.\cos.2p)=$	$+0,136927$	$-3,865942$	$+0,136927$
		$+0,000374$	
		$-3,865568$	
	$-134,473445$	$+5,669801$	$+0,163005$
♄$(-0,09889.\cos.4p)=$	$+0,000005$	$+0,001753$	$-0,049514$
$\mathfrak{M}=$	$-134,473440$	$+5,671554$	$+0,113491$

	sin. $2p$	sin. $4p$
♄$(-2,7453.\sin.2p)$	$-1,374559$	$+0,048684$
	$-0,000133$	
	$-1,374692$	
♄$(-0,0508.\sin.4p)$	$+0,000901$	$-0,025440$
M $=$	$-1,373791$	$+0,023244$

pro numero absoluto statim habemus $\mathfrak{N} = \frac{\mathfrak{M}}{\lambda} = -0,25009$ pro angulis autem per elementa numerica in primo capite iam data reperiemus:

	$2p$	$4p$
L. M	$-0,1379207$	$+8,3663109$
L. $\frac{2(m+1)}{\mu}$	$+0,0337640$	$+9,7327340$
L. $\frac{2(m+1)M}{\mu}$	$-0,1716847$	$+8,0990449$
$\frac{2(m+1)M}{\mu}$	$-1,484854$	$+0,012562$
$-\mathfrak{M}$	$-5,671554$	$-0,113491$
Numer.	$-7,156408$	$-0,100929$
L. Num.	$-0,8546951$	$-9,0039160$
L. den.	$-2,6382215$	$-3,3561432$
Log. $\mathfrak{N}$	$+8,2164736$	$+5,6477728$
L. $\frac{2(m+1)}{\mu}$	$+0,0337640$	$+9,7327340$
L. P. II.	$+8,2502376$	$+5,3805068$
Log. M	$-0,1379207$	$+8,3663109$
Log. μ^2	$+2,7867236$	$+3,3887836$
L. P. I.	$-7,3511971$	$+4,9775273$
P. I.	$-0,00224$	$+0,00001$
$-$P. II.	$-0,01779$	$-0,00002$
N	$-0,02003$	$-0,00001$
$\mathfrak{N}$	$+0,01646$	$+0,00004$

§. 460.

§. 460.

At quum ex partibus incognitis principalibus $\mathfrak{M}'$, M′ et $\mathfrak{N}'$ N′ deriuari conuenit statuamus:

$$\mathfrak{X} = \alpha + \beta \cos. 2p + \gamma \cos. 4p \text{ et}$$

$$X = b. \sin. 2p + c. \sin. 4p.$$

et calculus ita se habebit

Pro $\mathfrak{M}'$.

		$\cos. 2p$	$\cos. 4p$
$\mathfrak{X}\mathfrak{A} =$	$-4,6106454.\beta$	$-9,2212908.\alpha$	$-4,6106454.\beta$
		$-4,6106454.\gamma$	
$X\mathfrak{B} =$	$-1,9953482.b$	$-1,9953482.c$	$+1,9953482.b$

Pro M′.

		$\sin. 2p$	$\sin. 4p$
$\mathfrak{X}A =$		$-3,9906964.\alpha$	$-1,9953482.\beta$
		$+1,9953482.\gamma$	
$XB =$		$+2,6803227.c$	$+2,6803227.b$

§. 461.

§. 461.

Primam columnam ipsius $\mathfrak{M}'$ ad extremum reſeruemus, pro binis autem angulis $2p$ et $4p$ calculum ita instituamus:

$2p$

	α	γ	ϵ
Log. M'	$-0{,}6010487$	$+0{,}3000187$	$+0{,}4281870$
L. $\frac{2(m+1)}{\mu}$	$+0{,}0337640$	$+0{,}0337640$	$+0{,}0337640$
l. $\frac{2(m+1)M'}{\mu}$	$-0{,}6348127$	$+0{,}3337827$	$+0{,}4619510$
$\frac{2(m+1)M'}{\mu}$	$-4{,}31333$	$+2{,}15668$	$+2{,}89702$
$-\mathfrak{M}'$	$+9{,}22129$	$+4{,}61064$	$+1{,}99535$
Num.	$+4{,}90796$	$+6{,}76732$	$+4{,}89237$
L. Num.	$+0{,}6909010$	$+0{,}8304167$	$+0{,}6895193$
L. Den.	$-2{,}6382215$	$-2{,}6382215$	$-2{,}6382215$
Log. $\mathfrak{N}'$	$-8{,}0526795$	$-8{,}1921952$	$-8{,}0512978$
L. $\frac{2(m+1)}{\mu}$	$+0{,}0337640$	$+0{,}0337640$	$+0{,}0337640$
L. P. II.	$-8{,}0864435$	$-8{,}2259592$	$-8{,}0850618$
Log. M'	$-0{,}6010487$	$+0{,}3000187$	$+0{,}4281870$
Log. μ^2	$+2{,}7867236$	$+2{,}7867236$	$+2{,}7867236$
Log. P. I.	$-7{,}8143251$	$+7{,}5132951$	$+7{,}6414634$
P. I.	$-0{,}00652$	$+0{,}00326$	$+0{,}00438$
$-$P. II.	$+0{,}01220$	$+0{,}01682$	$+0{,}01216$
N'	$+0{,}00568$	$+0{,}02008$	$+0{,}01654$
$\mathfrak{N}'$	$-0{,}01129$	$-0{,}01556$	$-0{,}01125$

Simili

Simili modo:

$4p$

	β	b
L. M′	−0,3000187	+0,4281870
L. $\frac{2(m+1)}{\mu}$	+9,7327340	+9,7327340
L. $\frac{2(m+1)M'}{\mu}$	−0,0327527	+0,1609210
$\frac{2(m+1)M}{\mu}$	−1,07833	+1,44851
−𝔐′	+4,61064	−1,99535
Numer.	+3,53231	−0,54684
L. Num.	+0,5480588	−9,7378603
L. Den.	−3,3561432	−3,3561432
L. 𝔑′	−7,1919156	+6,3817171
L. $\frac{2(m+1)}{\mu}$	+9,7327340	+9,7327340
L. P. II.	−6,9246496	+6,1144511
Log. M′	−0,3000187	+0,4281870
Log μ^2	+3,3887836	+3,3887836
L. P. I.	−6,9112351	+7,0394034
Pars I.	−0,00082	+0,00110
−P. II.	+0,00084	−0,00013
N′	+0,00002	+0,00097
𝔑′	−0,00155	+0,00024

§. 462.

Hinc igitur iſtas aſſequimur determinationes:

$$\beta = +0,01646 - 0,01129.\alpha - 0,01556.\gamma - 0,01125.c$$

$$b = -0,02003 + 0,00568.\alpha + 0,02008.\gamma + 0,01654.c$$

$$\gamma = +0,00004 - 0,00155.\beta + 0,00024.b$$

$$c = -0,00001 + 0,00002.\beta + 0,00097.b$$

qui bini valores poſteriores in prioribus ſubſtituti praebent:

$$\beta = +0,01646 - 0,01129.\alpha + 0,00002.\beta - 0,00002.b$$

$$b = -0,02003 + 0,00568.\alpha - 0,00003.\beta + 0,00002.b$$

vnde manifeſto fit

$$\beta = +0,01646 - 0,01129.\alpha \text{ et}$$

$$b = -0,02003 + 0,00568.\alpha.$$

§. 463.

§. 463.

Hi iam valores subſtituantur in valore $\mathfrak{M}'$ primae columnae, ac reperietur

$$\mathfrak{M}' = -0,03593 + 0,03872.\alpha,$$

hincque

$$\mathfrak{N}' = \frac{\mathfrak{M}'}{3\lambda} = -0,00007 + 0,00007.\alpha,$$

conſequenter ob $\mathfrak{N} + \mathfrak{N}' = \alpha$, fiet

$$\alpha = -0,25016 + 0,00007.\alpha, \text{ siue}$$

$$\alpha = -0,25018,$$

conſequenter reliquae litterae ita definiuntur

$$\beta = +0,01928;\ b = -0,02145;$$

$$\gamma = +0,00001;\ c = +0,00003;$$

ſicque iſta euolutio nobis ſuppeditat hos valores prope veros:

$$\mathfrak{X} = -0,25018 + 0,01928.\operatorname{cof.} 2p + 0,00001.\operatorname{cof.} 4p$$

$$X = -0,02145.\operatorname{fin.} 2p - 0,00003.\operatorname{fin.} 4p.$$

§. 464.

Valores prope veri, in partes minores litterarum $\mathfrak{A}$, A, $\mathfrak{B}$, B ducantur, vt correctiones pro litteris $\mathfrak{M}''$ et M'' obtineamus:

Pro $\mathfrak{M}''$.

	abſol.	coſ. $2p$	coſ. $4p$
$\mathfrak{X}(+0,0264388)$	$-0,006614$	$+0,000510$	
$\mathfrak{X}(-0,1050568.\text{coſ}.4p)$		$-0,001012$	$+0,026270$
$X(-0,0819.\text{ſin}.4p)$		$+0,000879$	
$\mathfrak{M}''$	$-0,006614$	$+0,000377$	$+0,026270$

Pro M''.

		ſin. $2p$	ſin $4p$
$\mathfrak{X}(-0,0819.\text{ſin}.4p)$		$-0,000789$	$+0,020492$
$X(-0,0272367)$		$+0,000585$	$+0,000001$
$X(+0,0665457.\text{coſ}.4p)$		$+0,000714$	
M''		$+0,000510$	$+0,020493$

§. 465.

Pro prima columna seu numero constante habemus $\mathfrak{N}'' = \frac{\mathfrak{M}''}{3\lambda} = -0,00001$, pro binis autem angulis calculus ita se habebit:

	$2p$	$4p$
$L.\,M''$	$+6,70757$	$+8,3116055$
$L.\frac{2(m+1)}{\mu}$	$+0,03376$	$+9,7327340$
$l.\frac{2(m+1)M''}{\mu}$	$+6,74133$	$+8,0443395$
$\frac{2(m+1)M''}{\mu}$	$+0,000551$	$+0,011075$
$-\mathfrak{M}''$	$-0,000377$	$-0,026270$
Num.	$+0,000174$	$-0,015195$
L. Num.	$+6,24055$	$-8,1817007$
L. Den.	$-2,63822$	$-3,3561432$
$L.\,\mathfrak{N}'$	$-3,60233$	$+4,8255575$
$L.\frac{2(m+1)}{\mu}$	$+0,03376$	$+9,7327340$
L. P. II.	$-3,63609$	$+4,5582915$
$L.\,M''$	$+6,70757$	$+8,3116055$
$L.\,\mu^2$	$+2,78672$	$+3,3887836$
L. P. I.	$+3,92085$	$+4,9228219$
P. I.	$+0,0000008$	$+0,0000084$
$-$ P. II.	$+0,0000004$	$-0,0000036$
N''	$+0,0000012$	$+0,0000048$
$\mathfrak{N}''$	$-0,0000004$	$+0,0000067$

§.

Quum igitur hi valores ex correctione deducti
bendi, dummodo ob $\mathfrak{N}''$ pro prima columna $-0{,}00001$
ex hac euolutione oriundi sunt:

$$\mathfrak{X} = -0{,}25019 + 0{,}01928.\text{cos}.\,2p + 0{,}00002.\text{cos}.\,4p.$$

II. Euolutio terminorum

§.

Hic etiam primo ex partibus cogni-

	cos. $2r$	cos. $(2p-2r)$
♄$(-268{,}85.)$	$+134{,}410868$	$-9{,}942680$
♄$(-7{,}72.\text{cos}.\,2p)$	$-0{,}142774$	$+1{,}930107$
	$+0{,}005841$	$+0{,}002459$
	$-0{,}136933$	$+1{,}932566$
	$-134{,}273935$	$-8{,}010114$
♄$(-0{,}09889.\text{cos}.\,4p)$	$+0{,}000031$	$+0{,}000075$
$\mathfrak{M} =$	$134{,}273966$	$-8{,}010039$

	fin. $2r$	fin. $(2p-2r)$
♄$(-2{,}745.\text{fin}.\,2p)$	$-0{,}050764$	$+0{,}686260$
	$-0{,}002077$	$-0{,}000874$
	$-0{,}052841$	$+0{,}685386$
♄$(-0{,}0508.\text{fin}.\,4p)$	$+0{,}000016$	$+0{,}000038$
$M =$	$-0{,}052825$	$+0{,}685424$

466.

euanescant, valores supra inuenti pro veris sunt ha-
valor litterae $\alpha\ \frac{1}{103603}$ minuatur, ideoque valores veri

$$X = \qquad -0{,}02145.\sin.2p - 0{,}00003.\sin.4p.$$

angulum $2r$ inuoluentium.

467.

tis litteras 𝔐 et M deriuemus:

$\cos.(2p+2r)$	$\cos.(4p-2r)$	$\cos.(4p+2r)$
+ 0,406772	+ 0,171259	+ 0,002151
+ 1,930107	− 0,142774	+ 0,005841
+ 0,000031		
+ 1,930138		
+ 2,336910	+ 0,028485	+ 0,007992
− 0,001829	+ 0,024720	+ 0,024720
+ 2,335081	+ 0,053205	+ 0,032712

$\sin.(2p+2r)$	$\sin.(4p-2r)$	$\sin.(4p+2r)$
+ 0,686260	− 0,050764	+ 0,002077
− 0,000011		
+ 0,686249	− 0,050764	+ 0,002077
− 0,000939	+ 0,012701	+ 0,012701
+ 0,685310	− 0,038063	+ 0,014778

§. 468.

§.

Antequam vlterius progredi liceat
angulis constitui

$\omega =$	$2r$	$2p - 2r$
$\mu =$	$2l$	$2m - 2l$
seu $\mu =$	$-26,84526$	$-2,10742$
Log. $2(m+1) =$	$+1,4271258$	$+1,4271258$
Log. $\mu =$	$+1,4288676$	$-0,3237511$
Log. $\frac{2(m+1)}{\mu} =$	$-9,9982582$	$-1,1033747$
Log. $\mu^2 =$	$+2,8577352$	$+0,6475022$
$\lambda - 2 =$	$177,22893$	$+177,22893$
$-\mu^2 =$	$-720,66800$	$-4,44122$
Denom. $=$	$-543,43907$	$+172,78771$
Log. Den. $=$	$-2,7351509$	$+2,2375129$

468.

elementa numerica pro his quinque
necesse est:

$2p+2r$	$4p-2r$	$4p+2r$
$2m+2l$	$4m-2l$	$4m+2l$
+51,58310	+22,63042	+76,32094
+1,4271258	+1,4271258	+1,4271258
+1,7125075	+1,3546927	+1,8826437
+9,7146183	+0,0724331	+9,5444821
+3,4250150	+2,7093854	+3,7652874
177,22893	177,22893	177,22893
−2660,81743	−512,13511	−5824,88600
−2483,58850	−334,90618	−5647,65707
−3,3950798	−2,5249231	−3,7518683

§. 469.

Hic etiam commode primam columnam ad finem remittimus, vnde calculum numerorum $\mathfrak{N}$ et N, tantum pro quatuor reliquis columnis expediamus:

	$2p-2r$	$2p+2r$	$4p-2r$	$4p+2r$
L. M	+ 9,8359594	+ 9,8358871	− 8,5805030	+ 8,1696157
L. $\frac{2(m+1)}{\mu}$	− 1,1033747	+ 9,7146183	+ 0,0724331	+ 9,5444821
L. $\frac{2(m+1)M}{\mu}$	− 0,9393341	+ 9,5505054	− 8,6529361	+ 7,7140978
$\frac{2(m+1)M}{\mu}$	− 8,696292	+ 0,355226	− 0,044971	+ 0,005177
− $\mathfrak{M}$	+ 8,010039	− 2,335081	− 0,053205	− 0,032712
Numer	− 0,686253	− 1,979855	− 0,098176	− 0,027535
L. Num.	− 9,8364843	− 0,2966334	− 8,9920053	− 8,4398851
L. Den.	+ 2,2375129	− 3,3950798	− 2,5249231	− 3,7518683
L. $\mathfrak{N}$	− 7,5989714	+ 6,9015536	+ 6,4670822	+ 4,6880168
L. $\frac{2(m+1)}{\mu}$	− 1,1033747	+ 9,7146183	+ 0,0724331	+ 9,5444821
L. P. II.	+ 8,7023461	+ 6,6161719	+ 6,5395153	+ 4,2324989
L. M	+ 9,8359594	+ 9,8358871	− 8,5805030	+ 8,1696157
L. μ^2	+ 0,6475022	+ 3,4250150	+ 2,7093854	+ 3,7652874
L. P. I.	+ 9,1884572	+ 6,4108721	− 5,8711176	+ 4,4043283
P. I.	+ 0,15433	+ 0,00026	− 0,00007	+ 0,000003
− P. II.	− 0,05039	− 0,00041	− 0,00034	− 0,000002
N	+ 0,10394	− 0,00015	− 0,00041	+ 0,000001
at $\mathfrak{N}$	− 0,00397	+ 0,00080	+ 0,00029	+ 0,000005

§. 470.

Iam pro ipsis partibus incognitis, quatenus in partes praecipuas litterarum $\mathfrak{A}$, A et $\mathfrak{B}$, B ducuntur, quibus respondeant litterae $\mathfrak{M}'$, M' et $\mathfrak{N}'$, N' ponamus:

$$\mathfrak{X} = \beta.\text{cos}.\,2r + \gamma.\text{cos}.(2p-2r) + \delta.\text{cos}.(2p+2r) + \varepsilon.\text{cos}.(4p-2r) + \zeta.\text{cos}.(4p+2r)$$

$$X = b.\text{fin}.\,2r + c.\text{fin}.(2p-2r) + d.\text{fin}.(2p+2r) + e.\text{fin}.(4p-2r) + f.\text{fin}.(4p+2r)$$

vnde primum litteras $\mathfrak{M}'$ et M' formemus:

Pro $\mathfrak{M}'$.

	$2r$	$2p-2r$	$2p+2r$
$\mathfrak{X}\mathfrak{A} =$	$-4{,}6106454.(\gamma+\delta)$	$-4{,}6106454.(\beta+\varepsilon)$	$-4{,}6106454.(\beta+\zeta)$
$X\mathfrak{B} =$	$-1{,}9953482.(c+d)$	$-1{,}9953482.(b+e)$	$+1{,}9953482.(b-f)$

	$4p-2r$	$4p+2r$
$\mathfrak{X}\mathfrak{A} =$	$-4{,}61064.\gamma$	$-4{,}61064.\delta$
$X\mathfrak{B} =$	$+1{,}99534.c$	$+1{,}99534.d$

Pro M'.

	$2r$	$2p-2r$	$2p+2r$
$\mathfrak{X}A =$	$-1{,}9953482.(\gamma-\delta)$	$-1{,}9953482.(\beta-\varepsilon)$	$-1{,}9953482.(\beta-\zeta)$
$XB =$	$-2{,}6803227.(c-d)$	$-2{,}6803227.(b-e)$	$+2{,}6803227.(b+f)$

	$4p-2r$	$4p+2r$
$\mathfrak{X}A =$	$-1{,}99534.\gamma$	$-1{,}99534.\delta$
$XB =$	$+2{,}68032.c$	$+2{,}68032.d$

§. 471.

Quia columna secunda et quarta singulari nexu inter se cohaerent coniunctim ad eas sequentem calculum accomodemus:

	$2p - 2r$			
	β	ε	b	e
Log. M'	−0,3000187		−0,4281870	
L. $\frac{2(m+1)}{\mu}$	−1,1033747		−1,1033747	
L. $\frac{2(m+1)M'}{\mu}$	+1,4033934	−1,4033934	+1,5315617	−1,5315617
$\frac{2(m+1)M'}{\mu}$	+25,31590	−25,31590	+34,00649	−34,00649
− $\mathfrak{M}'$	+ 4,61064	+ 4,61064	+ 1,99534	+ 1,99534
Numer.	+29,92654	−20,70526	+36,00183	−32,01115
L.Num.	+1,4760566	−1,3160807	+1,5563245	−1,5053013
L. Den	+2,2375129	+2,2375129	+2,2375129	+2,2375129
L. $\mathfrak{N}'$	+9,2385437	−9,0785678	+9,3188116	−9,2677884
L. $\frac{2(m+1)}{\mu}$	−1,1033747	−1,1033747	−1,1033747	−1,1033747
L. P. II.	−0,3419184	+0,1819425	−0,4221863	+0,3711631
L. M'	−0,3000187		−0,4281870	
L. μ^2	+0,6475022		+0,6475022	
L. P. I.	−9,6525165	+9,6525165	−9,7806848	+9,7806848
P. I.	−0,44928	+0,44928	−0,60351	+0,60351
− P. II.	+2,19745	−1,52035	+2,64354	−2,35052
N'	+1,74817	−1,07107	+2,04003	−1,74701
$\mathfrak{N}'$	+0,17320	−0,11983	+0,20836	−0,18526

Simili

Simili modo:

$4p - 2r$

	γ	c
L. M′	−0,3000187	+0,4281870
L. $\frac{2(m+1)}{\mu}$	+0,0724331	+0,0724331
l. $\frac{2(m+1)M'}{\mu}$	−0,3724518	+0,5006201
$\frac{2(m+1)M'}{\mu}$	−2,35750	+3,16680
− 𝔐′	+4,61064	−1,99534
Numer.	+2,25314	+1,17146
Log.	+0,3527882	+0,0687274
L. Den.	−2,5249231	−2,5249231
Log. 𝔑′	−7,8278651	−7,5438043
L. $\frac{2(m+1)}{\mu}$	+0,0724331	+0,0724331
L. P II.	−7,9002982	−7,6162374
Log. M′	−0,3000187	+0,4281870
Log. μ^2	+2,7093854	+2,7093854
L. P. I.	−7,5906333	+7,7188016
P. I.	−0,00390	+0,00523
− P. II.	+0,00795	+0,00413
N′	+0,00405	+0,00936
𝔑′	−0,00673	−0,00349

vnde hos quatuor valores adipiſcimur:

$$\gamma = -0,00397 + 0,17320.\beta - 0,11983.\varepsilon$$
$$+ 0,20836.b - 0,18526.e$$

$$c = +0,10394 + 1,74817.\beta - 1,07107.\varepsilon$$
$$+ 2,04003.b - 1,74701.e$$

$$\varepsilon = +0,00029 - 0,00673.\gamma - 0,00349.c$$

$$e = -0,00041 + 0,00405.\gamma + 0,00936.c.$$

§. 472.

Nunc bini valores poſteriores in binis prioribus ſubſtituantur, vnde habebitur:

$$\gamma = -0,00392 + 0,17320.\beta + 0,20836.b$$
$$+ 0,00006.\gamma - 0,00131.c$$

$$c = +0,10435 + 1,74817.\beta + 2,04003.b$$
$$+ 0,00013.\gamma - 0,01261.c$$

ex illa fit

$$\gamma = -0,00392 + 0,17321.\beta + 0,20837.b$$
$$- 0,00131.c$$

qui valor in hac ſubſtitutus praebet

$$c = +0,10435 + 1,74819.\beta + 2,04006.b$$
$$- 0,01261.c$$

vnde

$$c = +0,10305 + 1,72632.\beta + 2,01466.b$$

ac proinde

$$\gamma = -0,00406 + 0,17095.\beta + 0,20573.b.$$

§. 473.

§. 473.

Simili modo tertiam columnam cum quinta iunctim expediamus:

$2p + 2r$

	β	ζ	b	f
L. M′	− 0,3000187		+ 0,4281870	
L. $\frac{2(m+1)}{\mu}$	+ 9,7146183		+ 9,7146183	
L. $\frac{2(m+1)M}{\mu}$	− 0,0146370	+ 0,0146370	+ 0,1428053	
$\frac{2(m+1)M'}{\mu}$	− 1,03428	+ 1,03428	+ 1,38933	+ 1,38933
− $\mathfrak{M}'$	+ 4,61064	+ 4,61064	− 1,99534	+ 1,99534
Numer.	+ 3,57636	+ 5,64492	− 0,60601	+ 3,38467
L.Num.	+ 0,5534401	+ 0,7516578	− 9,7824798	+ 0,5295176
L. Den.	− 3,3950798	− 3,3950798	− 3,3950798	− 3,3950798
Log. $\mathfrak{N}'$	− 7,1583603	− 7,3565780	+ 6,3874000	− 7,1344378
L. $\frac{2(m+1)}{\mu}$	+ 9,7146183	+ 9,7146183	+ 9,7146183	+ 9,7146183
L. P. II.	− 6,8729786	− 7,0711963	+ 6,1020183	− 6,8490561
Log. M′	− 0,3000187		+ 0,4281870	
Log. μ^2	+ 3,4250150		+ 3,4250150	
L P. I.	− 6,8750037	+ 6,8750037	+ 7,0031720	+ 7,0031720
P. I.	− 0,00075	+ 0,00075	+ 0,00101	+ 0,00101
− P. II.	+ 0,00075	+ 0,00118	− 0,00013	+ 0,00071
N′	− 0,00000	+ 0,00193	+ 0,00088	+ 0,00172
$\mathfrak{N}'$	− 0,00144	− 0,00227	+ 0,00024	− 0,00136

Simili

Simili modo:

$4p + 2r$

	δ	d
L. M′	− 0,3000187	+ 0,4281870
$L.\frac{2(m+1)}{\mu}$	+ 9,5444821	+ 9,5444821
$L.\frac{2(m+1)M'}{\mu}$	− 9,8445008	+ 9,9726691
$\frac{2(m+1)M'}{\mu}$	− 0,69903	+ 0,93901
− 𝔐′	+ 4,61064	− 1,99534
Numer.	+ 3,91161	− 1,05633
L.Num.	+ 0,5923555	− 0,0237996
L. Den.	− 3,7518683	− 3,7518683
Log. 𝔑′	− 6,8404872	+ 6,2719313
$L.\frac{2(m+1)}{\mu}$	+ 9,5444821	+ 9,5444821
L. P. II.	− 6,3849693	+ 5,8164134
Log. M′	− 0,3000187	+ 0,4281870
Log. μ^2	+ 3,7652874	+ 3,7652874
L. P. I.	− 6,5347313	+ 6,6628996
P. I.	− 0,00034	+ 0,00046
− P. II.	+ 0,00024	− 0,00007
N′	− 0,00010	+ 0,00039
𝔑′	− 0,00069	+ 0,00019

vnde

vnde hos quatuor valores deducimus

$$\delta = +0,00080 - 0,00144.\beta - 0,00227.\zeta$$
$$+0,00024.b - 0,00136.f$$
$$d = -0,00015 + 0,00000.\beta + 0,00193.\zeta$$
$$+0,00088.b + 0,00172.f$$
$$\zeta = +0,00000 - 0,00069.\delta + 0,00019.d$$
$$f = +0,00000 - 0,00010.\delta + 0,00039.d.$$

vnde ſtatim ſequitur

$$\delta = +0,00080 - 0,00144.\beta + 0,00024.b$$
$$d = -0,00015 + 0,00000.\beta + 0,00088.b$$
$$\zeta = 0;\ f = 0.$$

§. 474.

Nunc demum primam columnam anguli 2 r adgrediamur, ac primo litteras 𝔐′ et M′ per β et b exprimamus et quum ſit

$$\gamma = -0,00406 + 0,17095.\beta + 0,20573.b$$
$$\delta = +0,00080 - 0,00144.\beta + 0,00024.b$$

erit

$$\gamma + \delta = -0,00326 + 0,16951.\beta + 0,20597.b$$
$$\gamma - \delta = -0,00486 + 0,17239.\beta + 0,20549.b$$

similiter ob

$$c = +0,10305 + 1,72632.\beta + 2,01466.b$$
$$d = -0,00015 + 0,00000.\beta + 0,00088.b$$
$$c+d = +0,10290 + 1,72632.\beta + 2,01554.b$$
$$c-d = +0,10320 + 1,72632.\beta + 2,01378.b$$

vnde colligimus

$$\begin{aligned}\mathfrak{M}' = &+0,015031 - 0,78155.\beta - 0,94965.b\\ &-0,205321 - 3,44461 \quad - 4,02170.\\ \hline &-0,190290 - 4,22616.\beta - 4,97135.b\end{aligned}$$

$$\begin{aligned}M' = &+0,009697 - 0,34398\,\beta - 0,41002.b\\ &-0,276609 - 4,62709 \quad - 5,39758.\\ \hline &-0,266912 - 4,97107.\beta - 5,80760.b\end{aligned}$$

ad hos valores addantur litterae $\mathfrak{M}$ et M, et habebimus

$$\mathfrak{M} + \mathfrak{M}' = 134,083676 - 4,22616.\beta - 4,97135.b$$
$$M + M' = -0,319737 - 4,97107.\beta - 5,80760.b$$

quos numeros simpliciter litteris $\mathfrak{M}$ et M indicemus.

§. 475.

§. 475.

His ergo quaeramus numeros analogos $\mathfrak{N}$ et N pro angulo $2r$:

	1	β	b
L. M	− 9,5047929	− 0,6964499	− 0,7639967
L. $\frac{2(m+1)}{\mu}$	+ 9,9982582	+ 9,9982582	+ 9,9982582
L. $\frac{2(m+1)M}{\mu}$	− 9,5030511	− 0,6947081	− 0,7622549
$\frac{2(m+1)M}{\mu}$	− 0,318457	− 4,95117	− 5,78435
− $\mathfrak{M}$	− 134,083676	+ 4,22616	+ 4,97135
Numer.	− 134,402133	− 0,72501	− 0,81306
L. Num.	− 2,1284062	− 9,8603440	− 9,9100905
L. den.	− 2,7351509	− 2,7351509	− 2,7351509
Log. $\mathfrak{N}$	+ 9,3932553	+ 7,1251931	+ 7,1749396
L. $\frac{2(m+1)}{\mu}$	+ 9,9982582	+ 9,9982582	+ 9,9982582
L. P. II.	+ 9,3915135	+ 7,1234513	+ 7,1731978
Log. M	− 9,5047929	− 0,6964499	− 0,7639967
Log. μ^2	+ 2,8577352	+ 2,8577352	+ 2,8577352
L. P. I.	− 6,6470577	− 7,8387147	− 7,9062615
P. I.	− 0,00044	− 0,00690	− 0,00806
− P. II.	− 0,24633	− 0,00133	− 0,00149
N	− 0,24677	− 0,00823	− 0,00955
$\mathfrak{N}$	+ 0,24732	+ 0,00134	+ 0,00150

§.

Quum ergo fieri debeat

$$\beta = +0,24732 + 0,00134.\beta + 0,00150.b$$

ex priori statim habetur $\beta = +0,24765 + 0,00150.b$, $-0,00956.b$, vnde concluditur $b = -0,24645$ et

$$\gamma = -0,01249;\ c = +0,03341;$$
$$\varepsilon = +0,00025;\ e = -0,00015;$$

§.

Valores igitur prope veri

$$\mathfrak{X} = +0,24728.\text{cos.}\,2r - 0,01249.\text{cos.}\,(2p-2r) + 0,00038.\text{cos.}\,(2p+2r)$$

$$X = -0,24645.\text{sin.}\,2r + 0,03341.\text{sin.}\,(2p-2r) - 0,00037.\text{sin.}\,(2p+2r)$$

hos igitur valores prope veros ducamus in partes nostras litteras

Pro

	$2r$	$2p-2r$
$\mathfrak{X}\,(+0,02643)$ -	$+0,006538$	$-0,000330$
$\mathfrak{X}\,(-0,10505.\text{cos.}\,4p)$	$-0,000013$	$-0,000020$
$X\,(-0,08191.\text{sin.}\,4p)$	$+0,000006$	$+0,000015$
$\mathfrak{M}'' =$	$+0,006531$	$-0,000335$
$\mathfrak{X}\,(-0,08191.\text{sin.}\,4p)$	$-0,000010$	$-0,000016$
$X\,(-0,02723)$ -	$+0,006721$	$-0,000911$
$X\,(+0,0665.\text{cos.}\,4p)$	$+0,000005$	$+0,000012$
$M'' =$	$+0,006716$	$-0,000915$

476.
$\mathfrak{N} = \beta$ et. $N = b$, erit

$$b = -0,24677 - 0,00823.\beta - 0,00955.b$$

qui valor in posteriori substitutus dat $b = -0,24881$
$\beta = +0,24728$, atque hinc porro
$\delta = +0,00038$; $d = -0,00037$;
$\zeta = 0$; $f = 0$.

477.
hactenus inuenti sunt:
$+ 0,00025.\cos.(4p - 2r)$
$+ 0,00000.\cos.(4p + 2r)$
$- 0,00015.\sin.(4p - 2r)$
$+ 0,00000.\sin.(4p + 2r)$
minimas litterarum $\mathfrak{A}$, A, $\mathfrak{B}$, B, vt inde eliciamus
$\mathfrak{M}''$ et M'':

$\mathfrak{M}''$:

$2p + 2r$	$4p - 2r$	$4p + 2r$
+ 0,000010	+ 0,000007	
+ 0,000656	− 0,012983	− 0,012983
− 0,001368	+ 0,010093	− 0,010093
− 0,000702	− 0,002883	− 0,023076
+ 0,000511	− 0,010127	− 0,010127
+ 0,000010	+ 0,000004	
− 0,001111	+ 0,008200	− 0,008200
− 0,000590	− 0,001923	− 0,018327

§. 478.

Seposita iterum prima columna pro reliquis calculus erit:

	$2p-2r$	$2p+2r$	$4p-2r$	$4p+2r$
Log. M''	$-6,9614211$	$-6,77085$	$-7,28398$	$-8,26309$
L. $\frac{2(m+1)}{\mu}$	$-1,1033747$	$+9,71462$	$+0,07243$	$+9,54448$
l. $\frac{2(m+1)M'}{\mu}$	$+8,0647958$	$-6,48547$	$-7,35641$	$-7,80757$
$\frac{2(m+1)M''}{\mu}$	$+0,011609$	$-0,000306$	$-0,002272$	$-0,006420$
$-\mathfrak{M}''$	$+0,000335$	$+0,000702$	$+0,002823$	$+0,023076$
Num.	$+0,011944$	$+0,000396$	$+0,000551$	$+0,016656$
L. Num.	$+8,0771498$	$+6,59769$	$+6,74115$	$+8,22157$
L. Den.	$+2,2375129$	$-3,39508$	$-2,52492$	$-3,75187$
Log. $\mathfrak{N}''$	$+5,8396369$	$-3,20261$	$-4,21623$	$-4,46970$
L. $\frac{2(m+1)}{\mu}$	$-1,1033747$	$+9,71462$	$+0,07243$	$+9,54448$
L. P. II.	$-6,9430116$	$-2,91723$	$-4,28866$	$-4,01418$
Log. M''	$-6,9614211$	$-6,77085$	$-7,28398$	$-8,26309$
Log. μ^2	$+0,6475022$	$+3,42501$	$+2,70938$	$+3,76528$
Log. P. I.	$-6,3139189$	$-3,34584$	$-4,57460$	$-4,49781$
P. I.	$-0,00021$	$-0,00000$	$-0,000004$	$-0,000003$
$-$P. II.	$+0,00088$	$+0,00000$	$+0,000002$	$+0,000001$
N''	$+0,00067$	$+0$	-0	-0
$\mathfrak{N}''$	$+0,00007$	-0	-0	-0

§. 479.

§. 479.

Hos igitur valores, qui praeterquam pro angulo $2p - 2r$ sunt euanescentes, ad superiores litteras $\mathfrak{N}$ et N adiungi oportet, vnde istas habebimus aequationes:

$$\gamma = -0,00390 + 0,17320.\beta - 0,11983.\varepsilon$$
$$+ 0,20836.b - 0,18526.e$$
$$c = +0,10461 + 1,74817.\beta - 1,07107.\varepsilon$$
$$+ 2,04003.b - 1,74701.e$$
$$\varepsilon = +0,00029 - 0,00673.\gamma - 0,00349.c$$
$$e = -0,00041 + 0,00405.\gamma + 0,00936.c$$

quodsi iam posteriores in prioribus substituamus, habebimus

$$\gamma = -0,00385 + 0,17320.\beta + 0,20836.b$$
$$+ 0,00006.\gamma - 0,00131.c$$
$$c = +0,10502 + 1,74817.\beta + 2,04003.b$$
$$+ 0,00013.\gamma - 0,01261.c$$

ex quibus fit

$$\gamma = -0,00385 + 0,17321.\beta + 0,20837.b - 0,00131.c$$
$$c = +0,10502 + 1,74819.\beta + 2,04006.b - 0,01261.c$$

vnde $c = +0,10371 + 1,72632.\beta + 2,01466.b$

ac $\gamma = -0,00399 + 0,17095.\beta + 0,20573.b$

δ vero et d manent vt ante.

§. 480.

§. 480.

Vt iam hinc correctionem pro primo angulo $2r$ inueniamus, notamus esse:

$$\gamma+\delta=-0,00319+0,16951.\beta+0,20597.b$$

$$\gamma-\delta=-0,00479+0,17239.\beta+0,20549.b$$

$$c+d=+0,10356+1,72632.\beta+2,01554.b$$

$$c-d=+0,10386+1,72632.\beta+2,01378.b$$

vnde colligimus

$$\mathfrak{M}'=-0,191936-4,22616.\beta-4,97135.b$$

$$M'=-0,268820-4,97107.\beta-5,80760.b$$

huc igitur addamus $\mathfrak{M}+\mathfrak{M}''$ et $M+M''$, quatenus scilicet conueniunt angulo $2r$, vt obtineamus valores completos

$$\mathfrak{M}=134,088567-4,22616.\beta-4,97135.b$$

$$M=-0,314929-4,97107.\beta-5,80760.b$$

§. 481.

His ergo numeris quaeramus respondentes completos $\mathfrak{N}$ et N, quem calculum sufficit pro numeris absolutis instituisse vt sequitur:

Log. M

Log. M =	$-9{,}4982126$
L. $\frac{2(m+1)}{\mu}$ =	$+9{,}9982582$
L. $\frac{2(m+1)M}{\mu}$ =	$-9{,}4964708$
$\frac{2(m+1)M}{\mu}$ =	$-0{,}313668$
$-\mathfrak{M}$ =	$-134{,}088567$
Num. =	$-134{,}402235$
P. I. =	$-0{,}00044$
$-$P. II. =	$-0{,}24633$
N =	$-0{,}24677$

L. Num. =	$-2{,}1284064$
L. Den. =	$-2{,}7351509$
Log. $\mathfrak{N}$ =	$+9{,}3932555$
L. $\frac{2(m+1)}{\mu}$ =	$+9{,}9982582$
L. P. II. =	$+9{,}3915137$
Log. M =	$-9{,}4982126$
Log. μ^2 =	$+2{,}8577352$
L. P. I. =	$-6{,}6404774$
$\mathfrak{N}$ =	$+0{,}24732$

vnde impetramus

$$\beta = +0{,}24732 + 0{,}00134 . \beta + 0{,}00150 . b$$
$$b = -0{,}24677 - 0{,}00823 . \beta - 0{,}00955 . b$$

atque hinc tandem nanciſcimur:

$$\beta = +0{,}24728;\ b = -0{,}24645,$$

ex quibus valoribus facile ſequentes elicimus

$$\gamma = -0{,}01242;\ \delta = +0{,}00038;$$
$$\varepsilon = +0{,}00025;\ \zeta = 0{,}00000;$$
$$c = +0{,}03407;\ d = -0{,}00037;$$
$$e = -0{,}00014;\ f = 0{,}00000.$$

§. 482.

Valores igitur veri & completi litterarum $\mathfrak{X}$ et X, ad quos hae binae euolutiones nos deduxerunt ita exprimuntur:

$$\begin{aligned}
\mathfrak{X} = & -0,25019 + 0,01928.\,\text{coſ.}\,2p \\
& \qquad\qquad\quad + 0,00002.\,\text{coſ.}\,4p \\
& + 0,24728.\,\text{coſ.}\,2r - 0,01242.\,\text{coſ}(2p-2r) \\
& \qquad\qquad\qquad\quad + 0,00038.\,\text{coſ.}(2p+2r) \\
& \qquad\qquad\qquad\quad + 0,00025.\,\text{coſ}(4p-2r) \\
& \qquad\qquad\qquad\quad + 0,00000.\,\text{coſ.}(4p+2r)
\end{aligned}$$

$$\begin{aligned}
X = & \qquad\qquad\quad - 0,02145.\,\text{ſin.}\,2p \\
& \qquad\qquad\quad - 0,00003.\,\text{ſin.}\,4p \\
& - 0,24645.\,\text{ſin.}\,2r + 0,03407.\,\text{ſin}\,(2p-2r) \\
& \qquad\qquad\qquad\quad - 0,00037.\,\text{ſin.}(2p+2r) \\
& \qquad\qquad\qquad\quad - 0,00014\,\text{ſin.}(4p-2r) \\
& \qquad\qquad\qquad\quad + 0,00000.\,\text{ſin.}(4p+2r)
\end{aligned}$$

CAPVT II.

CAPUT II.

EVOLUTIO AEQUATIONUM ORDINIS XII, CHARACTERE *iik* PRO LITTERIS $\mathfrak{Y}$ ET Y.

§. 483.

Partes annexae aequationum hic consideratarum, ita se habent:

I. $0 = \ldots \mathfrak{Y}\mathfrak{A} + Y\mathfrak{B} + 2\mathfrak{P}\mathfrak{X}\mathfrak{C} + (\mathfrak{P}X + P\mathfrak{X})\mathfrak{D} + 2PX\mathfrak{E}$
$+ \text{♄}\mathfrak{P}(6\lambda - 30.\lambda\mathfrak{O} + 90.\lambda\mathfrak{O}^2 - \tfrac{45}{4}\lambda O^2) + \text{♄}P(\tfrac{15}{2}\lambda O - 45\lambda\mathfrak{O}O)$
$+ \text{♃}(-\tfrac{3}{2}\lambda + 6\lambda\mathfrak{O} - 15\lambda\mathfrak{O}^2 + \tfrac{15}{4}\lambda O^2)$

II. $0 = \ldots \mathfrak{Y}A + YB + 2\mathfrak{P}\mathfrak{X}C + (\mathfrak{P}X + P\mathfrak{X})D + 2PXE$
$+ \text{♄}P(\tfrac{15}{2}\lambda O - 45\lambda\mathfrak{O}O) + \text{♄}P(-\tfrac{3}{2}\lambda + \tfrac{15}{2}\lambda\mathfrak{O} - \tfrac{45}{2}\lambda\mathfrak{O}^2 + \tfrac{45}{4}\lambda O^2)$
$+ \text{♃}(-\tfrac{3}{2}\lambda O + \tfrac{15}{2}\lambda\mathfrak{O}O)$.

vbi posteriorum membrorum coefficientes ex natura aequationum compleuimus, quamquam in calculo sequente minima membra tuto negligere licet.

§. 484.

Ante omnia igitur singula producta, quae hic occurrunt, euolui debent, calculo autem subducto reperimus:

$$2\mathfrak{P}\mathfrak{X} = -0,49681.\cos.q - 0,07464.\cos.(2p-q) + 0,02063.\cos.(2p+q)$$
$$+0,24495.\cos.(q-2r) + 0,04679.\cos.(2p-q+2r) - 0,01309.\cos.(2p+q-2r)$$
$$+0,24738.\cos.(q+2r) + 0,03399.\cos.(2p-q-2r) - 0,00029.\cos.(2p+q+2r)$$

eius multiplicatores

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} +537,635924 + 15,4425816.\cos.2p \\ +10,9813978.\sin.2p. \end{matrix} \right.$$

$$\Pi = \mathfrak{P}X + P\mathfrak{X} = +0,50543.\sin.q + 0,11157.\sin.(2p-q) - 0,02932.\sin.(2p+q)$$
$$-0,12497.\sin.(q-2r) - 0,07378.\sin.(2p-q+2r) + 0,02880.\sin.(2p+q-2r)$$
$$-0,37195.\sin.(q+2r) - 0,02319.\sin.(2p-q-2r) - 0,00063.\sin.(2p+q+2r)$$

eius multiplicatores

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} +21,9627856.\sin.2p \\ -537,5635056 - 15,442580.\cos.2p \end{matrix} \right.$$

2PX =

$$2PX = +0{,}00889.\cos q + 0{,}04315.\cos(2p-q)$$
$$-0{,}04315.\cos(2p+q)$$
$$+0{,}48203.\cos(q-2r) - 0{,}10063\cos(2p-q+2r)$$
$$+0{,}06934.\cos(2p+q-2r)$$
$$-0{,}49600.\cos(q+2r) + 0{,}03280\cos(2p-q-2r)$$
$$-0{,}00151.\cos(2p+q+2r)$$

eius multiplicatores

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} -268{,}7817528 - 7{,}721290\cos 2p \\ -8{,}2360446.\sin. 2p. \end{matrix} \right.$$

§. 485.

Valorem autem ipsius $\mathfrak{p}$, qui est p' iam ante euoluimus et inuenimus

$$\mathfrak{p} = +0{,}500686 - 0{,}035467.\cos 2p - 0{,}000097.\cos 4p$$
$$-0{,}499944.\cos 2r + 0{,}036982.\cos(2p-2r)$$
$$-0{,}001513\cos(2p+2r)$$
$$-0{,}000637.\cos(4p-2r)$$
$$-0{,}000008.\cos(4p+2r)$$

quem ergo in $\mathfrak{P}$ et P duci oportet, quae producta reperiuntur

$$\mathfrak{p}\mathfrak{P} = +0{,}49741.\cos q + 0{,}07624.\cos(2p-q)$$
$$-0{,}01909.\cos(2p+q)$$
$$-0{,}24650.\cos(q-2r) - 0{,}04767.\cos(2p-q+2r)$$
$$+0{,}01911.\cos(2p+q-2r)$$
$$-0{,}25016.\cos(q+2r) - 0{,}02843.\cos(2p-q-2r)$$
$$-0{,}00008.\cos(2p+q+2r)$$

 eius

eius multiplicatores

$$\begin{array}{l} \mathrm{I.} \\ \mathrm{II.} \end{array}\left\{ \begin{array}{l} 1075,55190 + 38,606454 . \cos . 2p \\ + 13,726741 . \sin . 2p . \end{array}\right.$$

$$\begin{aligned} ♄P = & -1,01493 . \sin . q - 0,24160 . \sin . (2p-q) \\ & + 0,03408 . \sin . (2p+q) \\ & + 0,51071 . \sin (q-2r) + 0,10128 . \sin (2p-q+2r) \\ & - 0,03643 . \sin (2p+q-2r) \\ & + 0,50273 . \sin . (q+2r) + 0,14001 . \sin . (2p-q-2r) \\ & + 0,00232 . \sin . (2p+q+2r) \end{aligned}$$

cuius multiplicatores

$$\begin{array}{l} \mathrm{I.} \\ \mathrm{II.} \end{array}\left\{ \begin{array}{l} + 13,726741 . \sin . 2p \\ - 268,809950 - 9,651614 . \cos . 2p . \end{array}\right.$$

Quum denique sit $♃ = 2pq$ colligitur

$$\begin{aligned} ♃ = & + 0,96857 . \cos . q + 0,07419 . \cos . (2p-q) \\ & + 0,03430 . \cos . (2p+q) \\ & - 1,48738 . \cos . (q-2r) + 0,10924 . \cos . (2p-q+2r) \\ & + 0,03851 . \cos . (2p+q-2r) \\ & + 0,50449 . \cos . (q+2r) - 0,25996 . \cos . (2p-q-2r) \\ & + 0,00372 . \cos (2p+q+2r) \end{aligned}$$

cuius multiplicatores sunt:

$$\begin{array}{l} \mathrm{I.} \\ \mathrm{II.} \end{array}\left\{ \begin{array}{l} - 268,8518391 - 7,7212908 . \cos 2p \\ - 2,7453482 . \sin . 2p . \end{array}\right.$$

Singula haec producta sponte se diuidunt in terna membra, quae seorsim tractasse iuuabit.

I. Euolu-

I. Euolutio terminorum ab angulo r liberorum.

§. 486.

Pro his igitur terminis, quaeramus ex priori aequatione valores litterae 𝔐:

	cos. q	cos. $(2p-q)$	cos. $(2p+q)$
2𝔓𝔛 (537, 63) -	− 267,10286	− 40,12014	+ 11,09143
	− 0,57632	− 3,83602	− 3,83602
2𝔓𝔛 (+ 15,442. cos 2p)	+ 0,15929		
	− 0,41703		
	− 267,51989	− 43,96516	+ 7,25541
	+ 1,22519	+ 5,55032	− 5,55032
Π (+ 21,962. sin. 2p)	− 0,32197		
	+ 0,90322		
	− 266,61667	− 38,41484	+ 1,70509
2PX (− 268, 78) -	− 2,38947	− 11,59793	+ 11,59793
2PX (− 7,721. cos. 2p)		− 0,03434	− 0,03434
		− 11,63227	+ 11,56359
	− 269,00614	− 50,04711	+ 13,26868
♄𝔓 (+ 1075, 55) -	+ 534,99025	+ 82,00008	− 20,53228
♄𝔓 (+ 38,606. cos. 2p)	+ 1,47168	+ 9,60161	+ 9,60161
	− 0,36850		
	+ 536,09343	+ 91,60169	− 10,93067
	+ 267,08729	+ 41,55458	+ 2,33801
♄P (+ 13,726. sin. 2p)	− 1,65819	− 6,96584	+ 6,96584
	+ 0,23300		
	− 1,42429		
	+ 265,66300	+ 34,58874	+ 9,30385
♃ (− 268, 85) -	− 260,40442	− 19,94611	− 9,22183
	− 0,28662	− 3,74193	− 3,74193
♃ (− 7, 721. cos. 2p)	− 0,13251		
	− 260,82355	− 23,688 4	− 12,96376
𝔐 =	+ 4,83945	+ 10,90070	− 3,65991

§. 487.

§. 487.

Simili modo pro M habemus:

	ſin. q	ſin. $(2p-q)$	ſin $(2p+q)$
$2\mathfrak{P}\mathfrak{X}\,(+10,98.\,\text{ſin}.\,2p)$	− 0,40983	− 2,72783	− 2,72783
	− 0,11327		
	− 0,52310		
$\Pi\,(-537,563)$ -	− 271,70069	− 59,97694	+ 15,76136
	+ 0,86146	+ 3,90257	− 3,90257
$\Pi\,(-15,442.\,\text{coſ}.\,2p)$	+ 0,22639		
	− 270,61284	− 56,07437	+ 11,85879
	− 271,13594	− 58,80220	+ 9,13096
$2PX\,(-8,2360\ \text{ſin}.\,2p)$	− 0,35538	− 0,03661	− 0,03661
	− 271,49132	− 58,83881	+ 9,09435
♄$\mathfrak{P}\,(+13,726.\,\text{ſin}.\,2p)$	+ 0,52326	+ 3,41391	+ 3,41391
	+ 0,13102		
	+ 0,65428		
	− 270,83704	− 55,42490	+ 12,50826
♄$P\,(-268,809)$ -	+ 272,82319	+ 64,94445	− 9,16104
♄$P\,(-9,651.\,\text{coſ}.\,2p)$	− 1,16591	− 4,89786	+ 4,89786
	− 0,16446		
	+ 271,49282	+ 60,04659	− 4,26318
	+ 0,65578	+ 4,62169	+ 8,24508
♃ $(-2,745.\,\text{ſin}.\,2p)$	− 0,10183	− 1,32954	− 1,32954
	+ 0,04708		
	− 0,05475		
M =	+ 0,60103	+ 3,29215	+ 6,91554

§. 488.

§. 488.

Seposita iterum prima columna pro binis reliquis quaeramus consueto more litteras 𝔑 et N:

	$2p-q$	$2p+q$
L. M	+ 0,5174795	+ 0,8398261
L. $\frac{2(m+1)}{\mu}$	+ 0,3671158	+ 9,8474122
L $\frac{2(m+1)M}{\mu}$	+ 0,8845953	+ 0,6872383
$\frac{2(m+1)M}{\mu}$	+ 7,66647	+ 4,86674
− 𝔐	− 10,90070	+ 3,65991
Numer.	− 3,23423	+ 8,52665
L. Num.	− 0,5097709	+ 0,9307784
L. Den.	+ 1,6570290	− 3,1025375
Log. 𝔑	− 8,8527419	− 7,8282409
L. $\frac{2(m+1)}{\mu}$	+ 0,3671158	+ 9,8474122
L. P. II	− 9,2198577	− 7,6756531
Log. M	+ 0,5174795	+ 0,8398261
Log. μ^2	+ 2,1200200	+ 3,1594272
L. P. I.	+ 8,3974595	+ 7,6803989
P. I	+ 0,0250	+ 0,0048
− P. II	+ 0,1659	+ 0,0047
N	+ 0,1909	+ 0,0095
𝔑	− 0,0712	− 0,0067

§. 489.

§. 489.

Pro partibus incognitis ſtatuamus, ſecundum praecepta ſupra data

$$\mathfrak{Y} = 0.\ \text{coſ.}\ q + \gamma.\ \text{coſ.}\ (2p - q) + \delta.\ \text{coſ.}\ (2p + q)$$

et

$$Y = b.\ \text{ſin.}\ q + c.\ \text{ſin.}\ (2p - q) + d.\ \text{ſin.}\ (2p + q)$$

quas formulas nunc ducamus in factores, $\mathfrak{A}$, $\mathfrak{B}$ et A, B, excluſis tantum terminis angulum $4p$ continentibus, vnde impetrabimus litteras $\mathfrak{M}'$ et M':

Pro $\mathfrak{M}'$.

	coſ. q	coſ. $(2p-q)$	coſ. $(2p+q)$
$\mathfrak{Y}\,(-9,221.\ \text{coſ.}\ 2p)$	$-4,610645.(\gamma+\delta)$		
$\mathfrak{Y}\,(+0,026438.)$		$+0,0264388.\ \gamma$	$+0,0264388.\ \delta$
$Y\,(-3,990696.\text{ſin.}\,2p)$	$-1,995348.(c+d)$	$-1,995348.\ b$	$+1,9953488.\ b$

Pro M'.

	ſin. q	ſin $(2p-q)$	ſin. $(2p+q)$
$\mathfrak{Y}\,(-3,990696.\text{ſin.}\,2p)$	$-1,995348.(\gamma-\delta)$		
$Y\,(+5,36064.\text{coſ.}\,2p)$	$-2,6803227.(c-d)$	$-2,6803227.\ b$	$+2,6803223.\ b.$
$Y\,(-0,0272367)$	$-0,0272367.\ b$	$-0,0272367.\ c$	$-0,0272367.\ d$

§. 490.

§. 490.

Seposita iterum prima columna pro duabus reliquis iunctim inuestigemus litteras $\mathfrak{M}'$ et M':

$2p - q$

	b	γ	c
Log. M'	− 0,4281870		− 8,4351545
L. $\frac{2(m+1)}{\mu}$	+ 0,3671158		+ 0,3671158
L. $\frac{2(m+1)M'}{\mu}$	− 0,7953028		− 8,8022703
$\frac{2(m+1)M'}{\mu}$	− 6,2417		− 0,0634
− $\mathfrak{M}'$	+ 1,9953	− 0,0264	
Numer.	− 4,2464	− 0,0264	− 0,0634
L. Num	− 0,6280209	− 8,4216039	− 8,8022703
L. Den.	+ 1,6570290	+ 1,6570290	+ 1,6570290
L. $\mathfrak{M}'$	− 8,9709910	− 6,7645749	− 7,1452413
L. $\frac{2(m+1)}{\mu}$	+ 0,3671158	+ 0,3671158	+ 0,3671158
L. P. II	− 9,3381068	− 7,1316907	− 7,5123571
L. M'	− 0,4281870		− 8,4351545
L. μ^2	+ 2,1200200		+ 2,1200200
L. P. I.	− 8,3081670		− 6,3151345
P. I.	− 0,0203		− 0,0002
− P. II.	+ 0,2178	+ 0,0014	+ 0,0033
N'	+ 0,1975	+ 0,0014	+ 0,0031
$\mathfrak{M}'$	− 0,0935	− 0,0006	− 0,0014

Simili modo:

$2p+q$

	b	δ	d
L. M′	+ 0,4281870		− 8,4351545
L. $\frac{2(m+1)}{\mu}$	+ 9,8474122		+ 9,8474122
l. $\frac{2(m+1)M'}{\mu}$	+ 0,2755992		− 8,2825667
$\frac{2(m+1)M'}{\mu}$	+ 1,8863		− 0,0192
− 𝔐′	− 1,9953	− 0,0264	
Numer.	− 0,1090	− 0,0264	− 0,0192
L. Num.	− 9,0374265	− 8,4216039	− 8,2825667
L. Den.	− 3,1025375	− 3,1025375	− 3,1025375
Log. 𝔑′	+ 5,9348890	+ 5,3190664	+ 5,1800292
L. $\frac{2(m+1)}{\mu}$	+ 9,8474122	+ 9,8474122	+ 9,8474122
L. P II.	+ 5,7823012	+ 5,1664786	+ 5,0274414
Log. M′	+ 0,4281870		− 8,4351545
Log. μ^2	+ 3,1594272		+ 3,1594272
L. P. I.	+ 7,2687598		− 5,2757273
P. I.	+ 0,0019		− 0,00002
− P. II.	− 0,00006	− 0,00001	− 0,00001
N′	+ 0,0018	− 0,0000	− 0,0000
𝔑′	+ 0,0001	− 0,0000	− 0,0000

§. 491.

§. 491.

Hinc ergo obtinemus has quatuor determinationes:

$$\gamma = -0,0712 - 0,0935 . b - 0,0006 . \gamma - 0,0014 . c$$
$$c = +0,1909 + 0,1975 . b + 0,0014 . \gamma + 0,0031 . c$$
$$\delta = -0,0067 + 0,0001 . b$$
$$d = +0,0095 + 0,0018 . b,$$

inde autem deducimus

$$\gamma = -0,0715 - 0,0938 . b$$
$$c = +0,1914 + 0,1980 . b.$$

His vero inuentis erit pro angulo q propter

$$\gamma + \delta = -0,0782 - 0,0937 . b,$$
$$\gamma - \delta = -0,0648 - 0,0939 . b, \text{ et}$$
$$c + d = +0,2009 + 0,1998 . b,$$
$$c - d = +0,1819 + 0,1962 . b$$
$$\mathfrak{M}' = -0,04032 + 0,03335 . b$$
$$M' = -0,35825 - 0,36576 . b$$

his addamus ipſas litteras $\mathfrak{M}$ et M, vt obtineamus valores completos

$$\mathfrak{M} = +4,79913 + 0,03335 . b,$$
$$M = +0,24278 - 0,36576 . b.$$

§. 492.

His igitur reſpondeant litterae $\mathfrak{N}$ et N et quum per hypotheſin hic ſit $\mathfrak{N} = 0$ et $N = b$, erit $b = \frac{M}{nn}$

fiue $nnb = +0,24278 - 0,36576.b$ hincque concluditur $b = +0,0014$, ideoque $\mathfrak{M} = +4,79918$ et $M = +0,24225$.

§. 493.

Ex his igitur calculo confueto, quaeramus numeratorem, quafi vellemus litteram $\mathfrak{N}$ definire

$L.\ M = +9,3842638$	$\frac{2(m+1)}{\mu} M = +0,48862$
$L.\ \frac{2(m+1)}{\mu} = +0,3047120$	$-\mathfrak{M} = -4,79918$
$L.\ \frac{2(m+1)}{\mu} M = +9,6889758$	$Num. = -4,31056$

quem numerum per ii multiplicatum in ordinem fecundum transponi oportet, ita vt aequatio in §. 178 data, nunc ita fe habebit:

$$1,50640 = 1,55301 - 11,1273.k^2 - 4,31056.ii.$$

§. 494.

Inuento autem valore $b = +0,0014$, reliquae ita determinantur

$$\gamma = -0,0716;\ c = +0,1917;$$
$$\delta = -0,0067;\ d = +0,0095;$$

ex quo prima haec euolutio nobis fuppeditat

$$\mathfrak{Y} = 0.\ cof.\ q - 0,0716.\ cof.\ (2p - q) - 0,0067.\ cof.\ (2p + q)$$

$$Y = +0,0014.\ fin.\ q + 0,1917.\ fin.\ (2p - q) + 0,0095.\ fin.\ (2p + q).$$

II. Euolu-

II. Euolutio terminorum angulum $(q-2r)$ inuoluentium.

§. 495.

Hic igitur vt ante primum valorem litterae 𝔐 euoluamus:

	$(q-2r)$	$(2p-q+2r)$	$(2p+q-2r)$
2 𝔓 𝔛 (537, 63) -	+ 131,69388	+ 25,15598	− 7,03765
2 𝔓 𝔛 (+ 15,442. cos. 2 p)	+ 0,36128	+ 1,89133	+ 1,89133
	− 0,09877		
	+ 131,95639	+ 27,04731	− 5,14632
Π (+ 21, 962. sin. 2 p)	− 0,81021	− 1,37234	+ 1,37234
	+ 0,31626		
	− 0,49395		
	+ 131,46244	+ 25,67497	− 3,77398
2 P X (− 268, 78) -	− 129,56087	+ 27,04751	− 18,63732
	+ 0,38877	− 1,86223	− 1,86223
2 P X (− 7,721. cos. 2 p)	− 0,26788		
	− 129,43998	+ 25,18528	− 20,49955
	+ 2,02246	+ 50,86025	− 24,27353
♄ 𝔓 (1075, 55) -	− 265,12354	− 51,27157	+ 20,55376
	− 0,92018	− 4,75824	− 4,75824
♄ 𝔓 (+ 38,606. cos. 2 p)	+ 0,36889		
	− 265,67483	− 56,02981	+ 15,79552
	− 263,65237	− 5,16956	− 8,47801
♄ P (+ 13, 726. sin. 2 p)	+ 0,69512	+ 3,50519	− 3,50519
	− 0,25003		
	+ 0,44509		
	− 263,20728	− 1,66437	− 11,98320
♃ (− 268, 85) - -	+ 399,88473	− 29,36937	− 10,35350
	− 0,42203	+ 5,74622	+ 5,74622
♃ (− 7, 721. cos. 2 p)	− 0,14878		
	+ 399,31392	− 23,62315	− 4,60728
𝔐 =	+ 136,10664	− 25,28752	− 16,59048

§. 496.

§. 496.

Simili modo procedamus pro littera M:

	$(q-2r)$	$(2p-q+2r)$	$(2p+q-2r)$
2𝔓𝔛(+10,98. sin 2p)	+ 0,25691	+ 1,34495	+ 1,34495
	+ 0,07187		
	+ 0,32878		
Π(− 537, 563.) -	+ 67,17930	+ 39,66142	− 15,48183
Π(− 15, 442. cos. 2p)	− 0,56968	− 0,96493	+ 0,96493
	− 0,22237		
	+ 66,38725	+ 38,69649	− 14,51690
	+ 66,71603	+ 40,04144	− 13,17195
2PX(− 8,236. sin. 2p)	+ 0,41440	− 1,98501	− 1,98501
	+ 0,28554		
	+ 0,69994		
	+ 67,41597	+ 38,05643	− 15,15696
♄𝔓(+ 13,72. sin. 2p)	− 0,32718	− 1,69182	− 1,69182
	− 0,13116		
	− 0,45834		
	+ 66,95763	+ 36,36461	− 16,84878
♄P(− 268, 809.) -	− 137,28390	− 27,22513	+ 9,79274
♄P(− 9, 651. cos. 2p)	+ 0,48876	+ 2,46459	− 2,46459
	+ 0,17580		
	− 136,61934	− 24,76054	+ 7,32815
	− 69,66171	+ 11,60407	− 9,52063
♃(− 2, 745. sin. 2p)	− 0,14995	+ 2,04169	+ 2,04169
	+ 0,05286		
	− 0,09709		
M =	− 69,75880	+ 13,64576	− 7,47894

§. 497

§. 497.

Sequuntur ergo elementa numerica:

$\omega =$	$q - 2r$	$2p - q + 2r$	$2p + q - 2r$
$\mu =$	$n - 2l$	$2m - n + 2l$	$2m + n - 2l$
seu $\mu =$	$-13,58922$	$+38,32706$	$+11,14862$
Log. $\mu =$	$-1,1331945$	$+1,5835055$	$+1,0472212$
L. $2(m+1) =$	$+1,4271258$	$+1,4271258$	$+1,4271258$
L. $\frac{2(m+1)}{\mu} =$	$-0,2939313$	$+9,8436203$	$+0,3799046$
Log. $\mu^2 =$	$+2,2663890$	$+3,1670110$	$+2,0944424$
$\lambda - 2 =$	$+177,22893$	$+177,22893$	$+177,22893$
$-\mu^2 =$	$-184,66689$	$-1468,96337$	$-124,29177$
Denom. $=$	$-7,43796$	$-1291,73444$	$+52,93716$
Log. Den. $=$	$-0,8714538$	$-3,1111732$	$+1,7237606$

§. 498.

Omissa columna prima calculum nostrum pro binis reliquis instituamus:

	$2p-q+2r$	$2p+q-2r$
L. M	+ 1,1349977	− 0,8738400
L. $\frac{2(m+1)}{\mu}$	+ 9,8436203	+ 0,3799046
L. $\frac{2(m+1)M}{\mu}$	+ 0,9786180	− 1,2537446
$\frac{2(m+1)M}{\mu}$	+ 9,51958	− 17,93678
− $\mathfrak{M}$	+ 25,28752	+ 16,59048
Numer.	+ 34,80710	− 1,34630
L. Num.	+ 1,5416679	− 0,1291418
L. Den.	− 3,1111732	+ 1,7237606
L. $\mathfrak{N}$	− 8,4304947	− 8,4053812
L. $\frac{2(m+1)}{\mu}$	+ 9,8436203	+ 0,3799046
L. P. II.	− 8,2741150	− 8,7852858
L. M	+ 1,1349977	− 0,8738400
L. μ^2	+ 3,1670110	+ 2,0944424
L. P. I.	+ 7,9679867	− 8,7793976
P. I.	+ 0,0093	− 0,0602
− P. II.	+ 0,0188	+ 0,0610
N	+ 0,0281	+ 0,0008
at $\mathfrak{N}$	− 0,0269	− 0,0254

§. 499.

§. 499.

Pro partibus incognitis ponamus

$$\mathfrak{Y}=\beta.\cos.(q-2r)+\gamma.\cos.(2p-q+2r)$$
$$+\delta.\cos.(2p+q-2r)$$

$$Y=b.\sin.(q-2r)+c.\sin.(2p-q+2r)$$
$$+d\sin.(2p+q-2r)$$

quibus respondeant litterae $\mathfrak{M}'$ et M' ita definiendae:

Pro $\mathfrak{M}'$.

	$q-2r$	$2p-q+2r$	$2p+q-2r$
$\mathfrak{Y}(-9,221.\cos.2p)=$	$-4,61064.(\gamma+\delta)$	$-4,61064.\beta$	$-4,61064.\beta$
$\mathfrak{Y}(+0,02644)=$	$+0,02644.\beta$	$+0,02644\,\gamma$	$+0,02644.\delta$
$Y(-3,990.\sin.2p)=$	$-1,99534.(c+d)$	$-1,99534.b$	$-1,99534.b$

Pro M'.

	$q-2r$	$2p-q+2r$	$2p+q-2r$
$\mathfrak{Y}(-3,990\cos.2p)=$	$-1,9953.(\gamma-\delta)$	$-1,9953.\beta$	$-1,9953.\beta$
$Y(+5,360.\cos.2p)=$	$-2,6803.(c-d)$	$-2,6803.b$	$+2,6803.b$
$Y(-0,02723.)=$	$-0,02723.b$	$-0,02723.c$	$-0,02723.d$

§. 500.

Omissa iterum prima columna, pro secunda quaeramus $\mathfrak{N}'$ et N':

Pro angulo $(2p - q + 2r)$

	β	b	γ	c
L. M'	− 0,3000187	− 0,4281868		− 8,4351545
L. $\frac{2(m+1)}{\mu}$	+ 9,8436203	+ 9,8436203		+ 9,8436203
L $\frac{2(m+1)M}{\mu}$	− 0,1436390	− 0,2718071		− 8,2787748
$\frac{2(m+1)M'}{\mu}$	− 1,3920	− 1,8699		− 0,0190
− $\mathfrak{M}'$	+ 4,6106	+ 1,9953	− 0,0264	
Numer.	+ 3,2186	+ 0,1254	− 0,0264	− 0,0190
L. Num.	+ 0,5076670	+ 9,0982975	− 8,4216039	− 8,2787748
L. Den.	− 3,1111732	− 3,1111732	− 3,1111732	− 3,1111732
Log. $\mathfrak{N}'$	− 7,3964938	− 5,9871243	+ 5,3104307	+ 5,1676016
L. $\frac{2(m+1)}{\mu}$	+ 9,8436203	+ 9,8436203	+ 9,8436203	+ 9,8436203
L. P. II.	− 7,2401141	− 5,8307446	+ 5,1540510	+ 5,0112219
Log. M'	− 0,3000187	− 0,4281868		− 8,4351545
Log. μ^2	+ 3,1670110	+ 3,1670110		+ 3,1670110
L. P. I.	− 7,1330077	− 7,2611758		− 5,2681435
P. I.	− 0,00136	− 0,00182		− 0,00002
− P. II.	+ 0,00174	+ 0,00007	− 0,00001	− 0,00001
N'	+ 0,0004	− 0,0017	− 0,	− 0,
$\mathfrak{N}'$	− 0,0025	− 0,0001	+ 0,	+ 0,

hinc

hinc habentur hae determinationes

$$\gamma = -0{,}0269 - 0{,}0025.\beta - 0{,}0001.b$$
$$c = +0{,}0281 + 0{,}0004.\beta - 0{,}0017.b.$$

§. 501.

Similem instituamus calculum pro angulo $(2p+q-hr)$

	β	b	δ	d
L. M′	− 0,3000187	+ 0,4281868		− 8,4351545
L. $\frac{2(m+1)}{\mu}$	+ 0,3799046	+ 0,3799046		+ 0,3799046
L. $\frac{2(m+1)M'}{\mu}$	− 0,6799233	+ 0,8080914		− 8,8150591
$\frac{2(m+1)M}{\mu}$	− 4,7854	+ 6,4282		− 0,0653
− $\mathfrak{M}'$	+ 4,6106	+ 1,9953	− 0,0264	
Numer.	− 0,1748	+ 8,4235	− 0,0264	− 0,0653
L.Num.	− 9,2425414	+ 0,9254926	− 8,4216039	− 8,8150591
L. Den.	+ 1,7237606	+ 1,7237606	+ 1,7237606	+ 1,7237606
L. $\mathfrak{N}'$	− 7,5187808	+ 9,2017320	− 6,6978433	− 7,0912985
L. $\frac{2(m+1)}{\mu}$	+ 0,3799046	+ 0,3799046	+ 0,3799046	+ 0,3799046
L. P. II.	− 7,8986854	+ 9,5816366	− 7,0777479	− 7,4712031
Log. M′	− 0,3000187	+ 0,4281868		− 8,4351545
Log μ^2	+ 2,0944424	+ 2,0944424		+ 2,0944424
L. P. I.	− 8,2055763	+ 8,3337444		− 6,3407121
Pars I.	− 0,01605	+ 0,02156		− 0,00022
−P. II.	+ 0,00792	− 0,38162	+ 0,0012	+ 0,00296
N′	− 0,0081	− 0,3601	+ 0,0012	+ 0,0027
$\mathfrak{N}'$	− 0,0033	+ 0,1591	− 0,0005	− 0,0012

 hinc

hinc fiet

$$\delta = -0,0254 - 0,0033.\beta + 0,1591.b$$
$$- 0,0005.\delta - 0,0012.d$$

et

$$d = +0,0608 - 0,0081.\beta - 0,3601.b$$
$$+ 0,0012.\delta + 0,0027.d$$

vnde concluditur

$$\delta = -0,0254 - 0,0033.\beta + 0,1594.b$$
$$d = +0,0008 - 0,0081.\beta - 0,3609.b.$$

§. 502.

His inuentis pro primo angulo $q - 2r$ propter

$$\gamma + \delta = -0,0523 - 0,0057.\beta + 0,1593.b$$
$$\gamma - \delta = -0,0015 + 0,0009.\beta - 0,1595.b$$
$$c + d = +0,0289 - 0,0077.\beta - 0,3626.b$$
$$c - d = +0,0273 + 0,0085.\beta + 0,3592.b$$

colligimus

$$\mathfrak{M} + \mathfrak{M}' = \mathfrak{M} = 136,29012 + 0,06808.\beta + 0,01096.b$$

et

$$M + M' = M = -69,82898 - 0,02458.\beta - 0,67175.b.$$

§. 503.

§. 503.

His igitur litteris $\mathfrak{M}$ et M, quaeramus respondentes $\mathfrak{N}$ et N:

Pro angulo $(q - 2r)$

	1.	β	b
L. M	− 1,8440357	− 8,3905819	− 9,8272077
L. $\frac{2(m+1)}{\mu}$	− 0,2939313	− 0,2939313	− 0,2939313
L. $\frac{2(m+1)M}{\mu}$	+ 2,1379670	+ 8,6845132	+ 0,1211390
$\frac{2(m+1)M}{\mu}$	+ 137,39376	+ 0,04836	+ 1,32172
− $\mathfrak{M}$	− 136,29012	− 0,06808	− 0,01096
Numer.	+ 1,10364	− 0,01972	+ 1,31076
L. Num.	+ 0,0428275	+ 8,2949069	+ 0,1175232
L. den.	− 0,8714538	− 0,8714538	− 0,8714538
Log. $\mathfrak{N}$	− 9,1713737	+ 7,4234531	− 9,2460694
L. $\frac{2(m+1)}{\mu}$	− 0,2939313	− 0,2939313	− 0,2939313
L. P. II.	+ 9,4653050	− 7,7173844	+ 9,5400007
Log. M	− 1,8440357	− 8,3905819	− 9,8272077
Log. μ^2	+ 2,2663890	+ 2,2663890	+ 2,2663890
L. P. I.	− 9,5776467	− 6,1241929	− 7,5608187
P. I.	− 0,3781	− 0,0001	− 0,0036
− P. II.	− 0,2920	+ 0,0052	− 0,3467
N	− 0,6701	+ 0,0051	− 0,3503
$\mathfrak{N}$	− 0,1483	+ 0,0027	− 0,1762

§. 504.

Quare quum fieri debeat $\mathfrak{N} = \beta$ et $N = b$ erit

$$\beta = -0,1483 + 0,0027.\beta + 0,1762.b$$

$$b = -0,6701 + 0,0051.\beta - 0,3503.b$$

ex posteriori concludimus

$$b = -0,49626 + 0,0038.\beta,$$

qui valor in priore substitutus producit

$$\beta = -0,0609 + 0,0020\beta \text{ vnde}$$

$$\beta = -0,0610 \text{ et } b = -0,4965.$$

§. 505.

Retrogrediendo igitur nanciscemur

$$\gamma = -0,0268;\ c = +0,0289;$$

$$\delta = -0,1044;\ d = +0,1805;$$

ita vt haec euolutio nobis largiatur

$$\mathfrak{Y} = -0,0610.\text{cos.}(q-2r) - 0,0268.\text{cos.}(2p-q+2r) - 0,1044.\text{cos.}(2p+q-2r)$$

$$Y = -0,4965.\text{sin.}(q-2r) + 0,0289.\text{sin.}(2p-q+2r) + 0,1805.\text{sin.}(2p+q-2r).$$

III. Euolu-

III. Euolutio terminorum angulum $q+2r$ inuoluentium.

§. 506.

Primum valorem litterae $\mathfrak{M}$ euoluamus:

	$q+2r$	$2p-q-2r$	$2p+q+2r$
$2\mathfrak{P}\mathfrak{X}(537,63)$ -	$+133,00036$	$+18,27424$	$-0,15591$
	$+0,26245$	$+1,91009$	$+1,91009$
$2\mathfrak{P}\mathfrak{X}(+15,442.\cos 2p)$	$-0,00224$		
	$+133,26057$	$+20,18433$	$+1,75418$
	$-0,25466$	$-4,08453$	$+4,08453$
$\Pi(+21,962.\sin.2p)$	$-0,00692$		
	$-0,26158$		
	$+132,99899$	$+16,09980$	$+5,83871$
$2PX(-268,78)$ -	$+133,31574$	$-8,81604$	$+0,40586$
	$-0,12672$	$+1,91620$	$+1,91620$
$2PX(-7,721.\cos.2p)$	$+0,00583$		
	$+133,19485$	$-7,89984$	$+2,32206$
	$+266,19384$	$+8,19996$	$+8,16077$
♄$\mathfrak{P}(+1075,55)$ -	$-269,06012$	$-30,57794$	$-0,08604$
♄$\mathfrak{P}(+38,606.\cos.2p)$	$-0,54879$	$-4,82889$	$-4,82889$
	$-0,01544$		
	$-269,62435$	$-35,40683$	$-4,91493$
	$-3,43051$	$-27,20687$	$+3,24584$
♄$P(+13,726.\sin.2p)$	$+0,96094$	$+3,45042$	$-3,45042$
	$+0,01592$		
	$+0,97686$		
	$-2,45365$	$-23,75645$	$-0,20458$
♃$(-268,85)$ -	$-135,63303$	$+69,89072$	$-1,00013$
	$+1,00431$	$-1,94900$	$-1,94900$
♃$(-7,721.\cos.2p)$	$-0,01437$		
	$-134,64309$	$+67,94172$	$-2,94913$
$\mathfrak{M} =$	$-137,09674$	$+44,18527$	$-3,15371$

§. 507.

Simili modo euoluamus valores litterae M:

	$q+2r$	$2p-q-2r$	$2p+q+2r$
2𝔓𝔛$(+10,98.\,\text{fin.}\,2p)$	+ 0,18663	+ 1,35829	+ 1,35829
	+ 0,00159		
	+ 0,18822		
Π$(-537,56)$ -	+ 199,94678	+ 12,46610	+ 0,33867
	− 0,17947	− 2,87194	+ 2,87194
Π$(-15,44\,\text{cof.}\,2p)$	+ 0,00487		
	+ 199,77218	+ 9,59416	+ 3,21061
	+ 199,96040	+ 10,95245	+ 4,56890
$2PX(-8,236.\,\text{fin.}\,2p)$	− 0,13507	+ 2,04254	+ 2,04254
	− 0,00622		
	− 0,14129		
	+ 199,81911	+ 12,99499	+ 6,61144
♄𝔓$(+13,726.\,\text{fin.}\,2p)$	− 0,19512	− 1,71694	− 1,71694
	+ 0,00055		
	− 0,19457		
	+ 199,62454	+ 11,27805	+ 4,89450
♄$P(-268,809)$ -	− 135,13878	− 37,63607	− 0,62364
	+ 0,67566	+ 2,42607	− 2,42607
♄$P(-9,651.\,\text{cof.}\,2p)$	− 0,01120		
	− 134,47432	− 35,21000	− 3,04971
	+ 65,15022	− 23,93195	+ 1,84479
♃$(-2,745.\,\text{fin.}\,2p)$	+ 0,35684	− 0,69250	− 0,69250
	+ 0,00511		
	+ 0,36195		
M =	+ 65,51217	− 24,62445	+ 1,15229

§. 508.

§. 508.

Nunc ergo elementa numerica exhibeamus:

$\omega =$	$q + 2r$	$2p - q - 2r$	$2p + q + 2r$
$\mu =$	$n + 2l$	$2m - n - 2l$	$2m + n + 2l$
seu $\mu =$	+40,10130	−15.36346	+64,83914
Log. $2(m+1) =$	+ 1,4271258	+ 1,4271258	+ 1,4271258
Log. $\mu =$	+ 1,6031585	− 1,1864890	+ 1,8118372
Log. $\frac{2(m+1)}{\mu} =$	+ 9,8239673	−0,2406368	+ 9,6152886
Log. $\mu^2 =$	+ 3,2063170	+2,3729780	+3,6236744
$\lambda - 2 =$	+177,22893	+177,22893	+ 177,22893
$-\mu^2 =$	−1608,11444	− 236,03589	−4204,11388
Denom. $=$	−1430,88551	− 58,80496	−4026,88495
Log. Den. $=$	− 3,1556047	− 1,7694140	− 3,6049692

 §. 509.

§. 509.

Calculum litterarum $\mathfrak{N}$ et N vt hactenus, tantum pro binis columnis posterioribus instituamus:

	$2p-q-2r$	$2p+q+2r$
L. M	$-1,3913665$	$+0,0615618$
$L.\frac{2(m+1)}{\mu}$	$-0,2406368$	$+9,6152886$
$l.\frac{2(m+1)M}{\mu}$	$+1,6320033$	$+9,6768504$
$\frac{2(m+1)M}{\mu}$	$+42,85518$	$+0,47517$
$-\mathfrak{M}$	$-44,18527$	$+3,15371$
Num.	$-1,33009$	$+3,62888$
L.Num.	$-0,1238811$	$+0,5597726$
L. Den.	$-1,7694140$	$-3,6049692$
L. $\mathfrak{N}$	$+8,3544671$	$-6,9548034$
$L.\frac{2(m+1)}{\mu}$	$-0,2406368$	$+9,6152886$
L. P. II.	$-8,5951039$	$-6,5700920$
L. M	$-1,3913665$	$+0,0615618$
L. μ^2	$+2,3729780$	$+3,6236744$
L. P. I.	$-9,0183885$	$+6,4378874$
P. I.	$-0,1043$	$+0,00027$
$-$ P. II.	$+0,0394$	$+0,00037$
N	$-0,0649$	$+0,0006$
$\mathfrak{N}$	$+0,0226$	$-0,0009$

§. 510.

§. 510.

Pro partibus incognitis statuamus:

$$\mathfrak{Y} = \beta . \cos . (q + 2r) + \gamma . \cos . (2p - q - 2r) + \delta . \cos . (2p + q + 2r)$$

$$Y = b . \sin . (q + 2r) + c . \sin . (2p - q - 2r) + d . \sin . (2p + q + 2r)$$

quibus respondent litterae $\mathfrak{M}'$ et M' ita definiendae:

Pro $\mathfrak{M}'$:

	$q+2r$	$2p-q-2r$	$2p+q+2r$
$\mathfrak{Y}(-9,221 . \cos . 2p)$	$-4,6106 . (\gamma+\delta)$	$-4,6106 . \beta$	$-4,6106\ \beta$
$\mathfrak{Y}(+0,0264)$	$+0,0264 . \beta$	$+0,0264\ \gamma$	$+0,0264 . \delta$
$Y(-3,990 . \sin . 2p)$	$-1,9953 . (c+d)$	$-1,9953 . b$	$-1,9953 . b$

Pro M':

$\mathfrak{Y}(-3,990 . \sin . 2p)$	$-1,9953 . (\gamma-\delta)$	$-1,9953 . \beta$	$-1,9953 . \beta$
$Y(-5,3606 . \cos . 2p)$	$-2,6803 . (c-d)$	$-2,6803 . b$	$+2,6803 . b$
$Y(-0,0272)$	$-0,0273 . b$	$-0,0273 . c$	$-0,0273 . d$

§. 511.

Omissa iterum prima columna, pro secunda quaeramus $\mathfrak{N}'$ et N':

Pro angulo $2p - q - 2r$.

	β	b	γ	c
Log. M'	− 0,3000187	− 0,4281868		− 8,4351545
$L.\frac{2(m+1)}{\mu}$	− 0,2406368	− 0,2406368		− 0,2406368
$l.\frac{2(m+1)M'}{\mu}$	+ 0,5406555	+ 0,6688236		+ 8,6757913
$\frac{2(m+1)M'}{\mu}$	+ 3,4726	+ 4,6647		+ 0,0474
$-\mathfrak{N}'$	+ 4,6106	+ 1,9953	− 0,0264	
Num.	+ 8,0832	+ 6,6600	− 0,0264	+ 0,0474
L. Num.	+ 0,9075833	+ 0,8234742	− 8,4216	+ 8,6757
L. Den.	− 1,7694140	− 1,7694140	− 1,7694	− 1,7694
Log. $\mathfrak{N}'$	− 9,1381693	− 9,0540602	+ 6,6522	− 6,9063
$L.\frac{2(m+1)}{\mu}$	− 0,2406368	− 0,2406368	− 0,2406	− 0,2406
L. P. II.	+ 9,3788061	+ 9,2946970	− 6,8928	+ 7,1469
Log. M'	− 0,3000187	− 0,4281868		− 8,4351
Log. μ^2	+ 2,3729780	+ 2,3729780		+ 2,3729
Log. P. I.	− 7,9270407	− 8,0552088		− 6,0622
P. I.	− 0,0084	− 0,0114		− 0,0001
− P. II.	− 0,2392	− 0,1971	+ 0,0008	− 0,0014
N'	− 0,2476	− 0,2085	+ 0,0008	− 0,0015
$\mathfrak{N}'$.	− 0,1375	− 0,1133	+ 0,0005	− 0,0008

hinc prodit $\gamma = +0,0226 - 0,1375.\beta - 0,1133\, b + 0,0005.\gamma - 0,0008.c$

$c = -0,0649 - 0,2476\, \beta - 0,2085.b + 0,0008.\gamma - 0,0015.c$

vnde deducitur $\gamma = +0,0226 - 0,1374\, \beta - 0,1132.b;$

$c = -0,0649 - 0,2473.\beta - 0,2083\, b.$

§. 512.

§. 512.

Pro angulo vero $2p+q+2r$ facile praeuidemus, ex litteris δ et d nihil plane oriri, vnde calculum tantum pro litteris β et b faciamus:

Pro angulo $(2p+q+2r)$

	β	b
L. M′	− 0,30002	+ 0,42818
L. $\frac{2(m+1)}{\mu}$	+ 9,61529	+ 9,61529
l. $\frac{2(m+1)M'}{\mu}$	− 9,91531	+ 0,04347
$\frac{2(m+1)M'}{\mu}$	− 0,8228	+ 1,1053
− 𝔐′	+ 4,6106	+ 1,9953
Numer.	+ 3,7878	+ 3,1006
L. Num.	+ 0,57839	+ 0,49145
L. Den.	− 3,60497	− 3,60497
Log. 𝔑′	− 6,97342	− 6,88648
L. $\frac{2(m+1)}{\mu}$	+ 9,61529	+ 9,61529
L. P II.	− 6,58871	− 6,50177
Log. M′	− 0,30002	+ 0,42818
Log. μ^2	+ 3,62367	+ 3,62367
L. P. I.	− 6,67635	+ 6,80451
P. I.	− 0,0005	+ 0,0007
− P. II.	+ 0,0004	+ 0,0003
N′	− 0,0001	+ 0,0010
𝔑′	− 0,0009	− 0,0008

hinc

hinc

$$\delta = -0,0009 - 0,0009.\beta - 0,0008.b$$
$$d = +0,0006 - 0,0001.\beta + 0,0010.b.$$

§. 513.

Nunc angulum $(q + 2r)$ adgressi, quoniam

$$\gamma + \delta = +0,0217 - 0,1383.\beta - 0,1140.b$$
$$c + d = -0,0643 - 0,2474.\beta - 0,2073.b$$

colligimus

$$\mathfrak{M} + \mathfrak{M}' = -137,06849 + 1,15774.\beta + 0,93925.b = \mathfrak{M}$$

tum vero quia

$$\gamma - \delta = +0,0235 - 0,1365.\beta - 0,1124.b$$
$$c - d = -0,0655 - 0,2472.\beta - 0,2093.b$$

reperitur

$$M + M' = +65,64084 + 0,93494.\beta + 0,75796.b = M.$$

his

his ergo valoribus subnectimus sequentem calculum

Pro angulo $(q + 2r)$

	I.	β	b
L. M	+ 1,8171741	+ 9,9707837	+ 9,8796463
$L.\frac{2(m+1)}{\mu}$	+ 9,8239673	+ 9,8239673	+ 9,8239673
$L.\frac{2(m+1)M}{\mu}$	+ 1,6411414	+ 9,7947510	+ 9,7036136
$\frac{2(m+1)M}{\mu}$	+ 43,76645	+ 0,62338	+ 0,50537
$-\mathfrak{M}$	+ 137,06849	− 1,15774	− 0,93925
Numer.	+ 180,83494	− 0,53436	− 0,43388
L. Num.	+ 2,2572823	− 9,7277527	− 9,6373696
L. Den.	− 3,1556047	− 3,1556047	− 3,1556047
Log. $\mathfrak{N}$	− 9,1016776	+ 6,5721480	+ 6,4817649
$L.\frac{2(m+1)}{\mu}$	+ 9,8239673	+ 9,8239673	+ 9,8239673
L. P. II	− 8,9256449	+ 6,3961153	+ 6,3057322
Log. M	+ 1,8171741	+ 9,9707837	+ 9,8796463
Log. μ^2	+ 3,2063170	+ 3,2063170	+ 3,2063170
L. P. I.	+ 8,6108571	+ 6,7644667	+ 6,6733293
P. I	+ 0,0408	+ 0,0006	+ 0,0005
− P. II.	+ 0,0843	− 0,0003	− 0,0002
N	+ 0,1251	+ 0,0003	+ 0,0003
$\mathfrak{N}$	− 0,1264	+ 0,0004	+ 0,0003

hinc fit

$$\beta = -0,1264 + 0,0004.\beta + 0,0003.b \text{ atque}$$
$$b = +0,1251 + 0,0003.\beta + 0,0003.b$$

perinde

$$\beta = -0,1264 \text{ et } b = +0,1251,$$

reliquae vero litterae prodibunt

$$\gamma = +0,0258;\ c = -0,0597;$$
$$\delta = -0,0009;\ d = +0,0007.$$

Conclusio.

§. 514.

Valores igitur completi, quos hoc capite inuestigauimus, ita se habent:

$$\begin{aligned}
\mathfrak{Y} = &+ 0,\text{cos}.\,q - 0,0716.\text{cos}.(2p-q) \\
&- 0,0067.\text{cos}.(2p+q) \\
&- 0,0610.\text{cos}.(q-2r) - 0,0268.\text{cos}.(2p-q+2r) \\
&- 0,1044.\text{cos}.(2p+q-2r) \\
&- 0,1264.\text{cos}(q+2r) + 0,0258.\text{cos}(2p-q-2r) \\
&- 0,0009.\text{cos}(2p+q+2r)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Y} = &+ 0,0014.\text{sin}.\,q + 0,1917.\text{sin}.(2p-q) \\
&+ 0,0095.\text{sin}.(2p+q) \\
&- 0,4965.\text{sin}.(q-2r) + 0,0289.\text{sin}(2p-q+2r) \\
&+ 0,1805.\text{sin}.(2p+q-2r) \\
&+ 0,1251.\text{sin}.(q+2r) - 0,0597.\text{sin}.(2p-q-2r) \\
&+ 0,0007.\text{sin}.(2p+q+2r).
\end{aligned}$$

CAPVT III.

CAPVT III.

EVOLVTIO ORDINIS *iikk* SEV TERMINORVM $\mathfrak{Z}$, Z.

§. 515.

Partes annexae harum aequationum ita exhibentur:

$$\text{I. } 0 = \ldots \mathfrak{Z}\mathfrak{A} + Z\mathfrak{B} + 2(\mathfrak{P}\mathfrak{Y} + \mathfrak{Q}\mathfrak{X})\mathfrak{C}$$
$$+ (\mathfrak{P}Y + P\mathfrak{Y} + \mathfrak{Q}X + Q\mathfrak{X})\mathfrak{D} + 2(PY + QX)\mathfrak{E}$$
$$+ 3\mathfrak{P}^2\mathfrak{X}\mathfrak{F} + (\mathfrak{P}^2X + 2\mathfrak{P}P\mathfrak{X})\mathfrak{G} + (P^2\mathfrak{X} + 2\mathfrak{P}PX)\mathfrak{H}$$
$$+ 3P^2X\mathfrak{I} + (♄\mathfrak{Q} + ♃\mathfrak{P})(6\lambda - 30\lambda\mathfrak{O} + 90\lambda\mathfrak{O}^2 - \tfrac{45}{4}\lambda O^2)$$
$$+ (♄Q + ♃P)(\tfrac{15}{2}\lambda O - \tfrac{45}{2}\lambda\mathfrak{O}O)$$
$$+ \mathfrak{P}^2♄(-15\lambda + 90\lambda\mathfrak{O} - \tfrac{315}{2}\lambda\mathfrak{O}^2 + \tfrac{315}{4}\lambda O^2)$$
$$+ \mathfrak{P}P♄(-45\lambda O + 315\lambda\mathfrak{O}O)$$
$$+ P^2♄(+\tfrac{15}{4}\lambda - \tfrac{45}{2}\lambda\mathfrak{O} + \tfrac{315}{4}\lambda\mathfrak{O}^2 - \tfrac{315}{8}\lambda O^2)$$
$$+ ♂(-\tfrac{5}{8}\lambda + 6\lambda\mathfrak{O} - 15\lambda\mathfrak{O}^2 + \tfrac{15}{4}\lambda O^2).$$

$$\text{II. } 0 = \ldots \mathfrak{Z} A + Z B + 2(\mathfrak{P}\mathfrak{Y} + \mathfrak{Q}\mathfrak{X}) C$$
$$+ (\mathfrak{P} Y + P \mathfrak{Y} + \mathfrak{Q} X + Q \mathfrak{X}) D + 2(P Y + Q X) E$$
$$+ 3 \mathfrak{P}^2 \mathfrak{X} F + (\mathfrak{P}^2 X + 2 \mathfrak{P} P \mathfrak{X}) G + (P^2 \mathfrak{X} + 2 \mathfrak{P} P X) H$$
$$+ 3 P^2 . X I + (\mathfrak{P} \text{♃} + \text{♄} \mathfrak{Q})(\tfrac{15}{2} \lambda O - \tfrac{45}{2} \lambda \mathfrak{O} O)$$
$$+ (\text{♃} P + \text{♄} Q)(- \tfrac{3}{2} \lambda + \tfrac{15}{2} \lambda \mathfrak{O} - \tfrac{45}{2} \lambda \mathfrak{O}^2 + \tfrac{45}{4} \lambda O^2)$$
$$+ \mathfrak{P}^2 \text{♄} (- \tfrac{45}{2} \lambda O + \tfrac{315}{2} \lambda \mathfrak{O} O)$$
$$+ \mathfrak{P} P \text{♄} (+ \tfrac{15}{2} \lambda - 45 \lambda \mathfrak{O} + \tfrac{315}{2} \lambda \mathfrak{O}^2 - \tfrac{315}{4} \lambda O^2)$$
$$+ P^2 \text{♄} (+ \tfrac{45}{4} \lambda O - \tfrac{315}{4} \lambda \mathfrak{O} O)$$
$$+ \text{♂} (- \tfrac{3}{2} \lambda O + \tfrac{15}{2} \lambda \mathfrak{O} O)$$

vbi ex natura rei coefficientes posteriorum membrorum compleuimus; caeterum meminisse oportet esse $\text{♂} = q^2 + 2 p r$.

§. 516.

Euolutio harum aequationum, tam ob multitudinem terminorum, quam ob ipsam eorum complicationem, sine dubio immensum laborem requireret, quem vix sine vllo calculi errore expedire liceret, minimus autem error in hoc calculo commissus, totum negotium irritum esset redditurus, quas ob caussas, hunc laborem suscipere merito pertimescimus, idque eo magis, quod minimae illae particulae, quas quidem hactenus negleximus ob crebram replicationem, hic insignis momenti fieri possent, ita vt etiamsi

ipsum

ipsum calculum sine vllo errore ad finem perducere liceret, vix tamen vllam fiduciam in conclusionibus inde ortis ponere possemus.

§. 517.

De caetero hic perpendisse iuuabit, characterem huius ordinis $iikk$ vix ad $\frac{1}{40000}$ assurgere, vnde sequitur si euolutis omnibus terminis coefficiens cuiusquam esset vnitas, eius valorem in loco Lunae 5″ superare non posse, quum ergo coefficientes, qui hactenus prodierunt, vix binarium superauerint, si idem in hoc ordine eueniat, eius effectum in Luna non esse 10″ excessurum, id quod sine dubio operae pretium non foret, tam prolixum et taediosum calculum moliri, praecipue quum denique ancipites haerere circa certitudinem conclusionis deberemus.

§. 518.

Praeterea euolutio omnium productorum in his aequationibus occurrentium insignem terminorum numerum suppeditaret, quos ratione angulorum in sequentes quinque classes distribuere licet:

I.	II.	III.	IV.	V.
0	$2q$	$2r$	$2q-2r$	$2q+2r$
$2p$	$2p-2q$	$2p-2r$	$2p-2q+2r$	$2p-2q-2r$
	$2p+2q$	$2p+2r$	$2p+2q-2r$	$2p+2q+2r$

inter quos angulos, hic imprimis $2q-2r$ est notatu dignus, quippe pro quo numerus μ fit $-0,33318$,

 hoc

hoc est $\mu = -\frac{1}{3}$, vnde elementa numerica ita se haberent $\frac{2(m+1)}{\mu} = -80,25036$ et $\frac{1}{\mu^3} = 9$, quare cum his tantis numeris quantitas M sit multiplicanda, euidens est minimos errores in litteris 𝔐 et M, totum nostrum laborem esse subuersuros, quae circumstantia quum potissimum in hoc angulo $2p - 2r$ locum inveniat, si forte hinc inaequalitas sensibilis in motum Lunae redundet, eam multo tutius et certius ex ipsis obseruationibus concludere licebit, si enim omnes reliquae inaequalitates recte se habeant, ex comparatione plurium obseruationum, interuallo sesquianni inter se distantium, facile dignoscere poterimus, quantum ista inaequalitas valere queat.

§. 519.

Praetermisso ergo hoc ordine atque omnibus reliquis, quorum characteres infra $\frac{1}{20000}$ cadunt, huic capiti finem imponamus, eius vero loco ordinem $i\,i\,\varkappa$ supra non commemoratum pertractemus, quandoquidem eius character $i\,i\,\varkappa$ multo est maior et fere ad $\frac{1}{7033}$ assurgit, hocque modo omnes ordines expediverimus quorum characteres $\frac{1}{20000}$ superant, dummodo excipiamus ordinem i^4 ex quo nulla inaequalitas notabilis oriri potest, litterasque 𝔜 et Z huic capiti destinatas, ordini sequenti $i\,i\,\varkappa$ tribuamus.

CAPVT IV.

CAPUT IV.

EVOLUTIO AEQUATIONUM ORDINIS $_{iii}$ PRO LITTERIS $\mathfrak{Z}$ ET Z.

§. 520.

Partes annexae harum aequationum ex aequationibus generalibus facile colliguntur sequentes:

I. $o = \ldots + \mathfrak{Z}\mathfrak{A} + Z\mathfrak{B} + 2\mathfrak{U}\mathfrak{X}\mathfrak{C} + (\mathfrak{U}X + U\mathfrak{X})\mathfrak{D}$
$+ 2UX\mathfrak{E} + \text{♄}\mathfrak{U}(6\lambda - 30\lambda\mathfrak{D} + 90.\lambda\mathfrak{D}^2 - \frac{45}{4}\lambda O^2)$
$+ \text{♄}U(\frac{15}{2}\lambda O - 45.\lambda\mathfrak{D}O)$
$+ \text{♁}(-\frac{3}{2}\lambda + 6\lambda\mathfrak{D} - 15.\lambda\mathfrak{D}^2 + \frac{15}{4}\lambda O^2)$
$+ \frac{3}{4}\mathfrak{X}(2\cos. t + 7.\cos.(2p - t) - \cos.(2p + t))$
$+ \frac{3}{4}X(-7\sin.(2p - t) + \sin.(2p + t))$.

II. $o = \ldots + \mathfrak{Z}A + ZB + 2\mathfrak{U}\mathfrak{X}C + (\mathfrak{U}X + U\mathfrak{X})D$
$+ 2UXE + \text{♄}\mathfrak{U}(\frac{15}{2}\lambda O - 45.\lambda\mathfrak{D}O)$
$+ \text{♄}U(-\frac{3}{2}\lambda + \frac{15}{2}\lambda\mathfrak{D} - \frac{45}{2}\lambda\mathfrak{D}^2 + \frac{45}{4}\lambda O^2)$
$+ \text{♁}(-\frac{3}{2}\lambda O + \frac{15}{2}\lambda\mathfrak{D}O)$
$+ \frac{3}{4}\mathfrak{X}(-7.\sin.(2p - t) + \sin.(2p + t))$
$+ \frac{3}{4}X(2\cos. t - 7.\cos.(2p - t) + \cos.(2p + t))$

vbi $\zeta = 2p\delta$, coefficientes vero hic occurrentes, iam in capite penultimo euoluimus.

§. 521.

Ante omnia igitur singula producta, quae hic occurrunt euolui conueniet, quae quidem nouem angulos in has tres classes distribuendos continebunt:

I.	II.	III.
t	$t-2r$	$t+2r$
$2p-t$	$2p-t+2r$	$2p-t-2r$
$2p+t$	$2p+t-2r$	$2p+t+2r$

His notatis superiores formulas ordine petractabimus:

I. $2\,\mathfrak{U}\,\mathfrak{X}$ dat

$$+0,00392.\cos. t - 0,01484.\cos.(2p-t)$$
$$+0,00160.\cos.(2p+t)$$
$$-0,00206.\cos.(t-2r) + 0,00727.\cos(2p-t+2r)$$
$$-0,00076.\cos.(2p+t-2r)$$
$$-0,00164.\cos.(t+2r) + 0,00736.\cos.(2p-t-2r)$$
$$-0,00085.\cos.(2p+t+2r)$$

eius multiplicator

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} +537,635924 + 15,4425816.\cos. 2p \\ +10,9813978.\sin. 2p. \end{matrix} \right.$$

II.

II. $(\mathfrak{U}X + U\mathfrak{X})$ dat

$-0{,}04756.\sin. t + 0{,}00907.\sin.(2p-t)$
$+0{,}00102.\sin.(2p+t)$
$+0{,}02295.\sin.(t-2r) - 0{,}00902.\sin.(2p-t+2r)$
$+0{,}00104.\sin.(2p+t-2r)$
$+0{,}02443.\sin.(t+2r) - 0{,}00065.\sin.(2p-t-2r)$
$+0{,}00114.\sin.(2p+t+2r)$

eius multiplicator

I. $\left\{ \begin{array}{l} +21{,}9627856.\sin. 2p \\ -537{,}5635056 - 15{,}4452816.\cos. 2p \end{array}\right.$
II.

III. $2\,\mathfrak{U}X$ dat

$+0{,}00081.\cos. t - 0{,}00409.\cos.(2p-t)$
$+0{,}00409.\cos.(2p+t)$
$-0{,}04845.\cos.(t-2r) - 0{,}01074.\cos.(2p-t+2r)$
$-0{,}00785.\cos.(2p+t-2r)$
$+0{,}04718.\cos.(t+2r) + 0{,}01716.\cos.(2p-t-2r)$
$+0{,}00143.\cos.(2p+t+2r)$

eius multiplicator

I $\left\{ \begin{array}{l} -268{,}7817528 - 7{,}721290.\cos. 2p \\ -8{,}2360446.\sin. 2p. \end{array}\right.$
II.

IV. $\mathfrak{b}\,\mathfrak{U}$ dat

$-0{,}00388.\cos. t + 0{,}01484.\cos.(2p-t)$
$-0{,}00161.\cos.(2p+t)$
$+0{,}00225.\cos.(t-2r) - 0{,}00738.\cos.(2p-t+2r)$
$+0{,}00073.\cos.(2p+t-2r)$
$+0{,}00162.\cos.(t+2r) - 0{,}00748.\cos.(2p-t-2r)$
$+0{,}00087.\cos.(2p+t+2r)$

eius multiplicator

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} +1075,55190+38,606454.\cos.2p \\ +13,726741.\sin.2p. \end{matrix} \right.$$

V. ♄ U dat

$$+0,09455.\sin.t-0,01831.\sin.(2p-t)$$
$$-0,00061.\sin.(2p+t)$$
$$-0,04684.\sin.(t-2r)+0,01097.\sin.(2p-t+2r)$$
$$+0,00214.\sin.(2p+t-2r)$$
$$-0,04757.\sin.(t+2r)+0,00731.\sin.(2p-t-2r)$$
$$-0,00152.\sin.(2p+t+2r)$$

eius multiplicator

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} +13,726741.\sin.2p \\ -268,809950-9,651614.\cos.2p. \end{matrix} \right.$$

VI. ♅ dat

$$-0,00592.\cos.t+0,10356.\cos.(2p-t)$$
$$-0,03969.\cos.(2p+t)$$
$$+0,01991.\cos.(t-2r)+0,00678.\cos.(2p-t+2r)$$
$$+0,03920.\cos.(2p+t-2r)$$
$$-0,01590.\cos.(t+2r)-0,10898.\cos.(2p-t-2r)$$
$$-0,00093.\cos.(2p+t+2r)$$

eius

eius multiplicator

$$\begin{matrix} \text{I.} \\ \text{II.} \end{matrix} \left\{ \begin{matrix} -268,8518391 - 7,7212908.\mathrm{cos}.2p \\ -2,7453482.\mathrm{sin}.2p. \end{matrix} \right.$$

Postremae partes aequationis primae dant:

$$\begin{aligned} &-0,28373.\mathrm{cos}.t - 1,29893.\mathrm{cos}(2p-t) \\ &\qquad\qquad + 0,20210.\mathrm{cos}.(2p+t) \\ &+0,06313.\mathrm{cos}.(t-2r) + 0,00256.\mathrm{cos}.(2p-t+2r) \\ &\qquad\qquad -0,19447.\mathrm{cos}.(2p+t-2r) \\ &+0,20288.\mathrm{cos}.(t+2r) + 1,28673.\mathrm{cos}.(2p-t-2r) \\ &\qquad\qquad -0,00002.\mathrm{cos}.(2p+t+2r) \end{aligned}$$

Postremae partes secundae aequationis dant

$$\begin{aligned} &+0,12218.\mathrm{sin}\, t + 1,29739\ \mathrm{sin}.(2p-t) \\ &\qquad\qquad -0,20373.\mathrm{sin}.(2p+t) \\ &+0,06307.\mathrm{sin}.(t-2r) - 0,00217\ \mathrm{sin}.(2p-t+2r) \\ &\qquad\qquad +0,18515\ \mathrm{sin}.(2p+t-2r) \\ &-0,20028.\mathrm{sin}.(t+2r) - 1,29605.\mathrm{sin}.(2p-t-2r) \\ &\qquad\qquad +0,00002.\mathrm{sin}\,(2p+t+2r). \end{aligned}$$

Hae expressiones singulae manifeste in tres portiones distinguuntur, quas seorsim tractari conuenit.

Euolutio I. terminorum angulum t simpliciter continentium.

§. 522.

Ex partibus cognitis primae aequationis eliciatur 𝔐 hoc modo:

	t	$2p-t$	$2p+t$
$2\,\mathfrak{U}\mathfrak{X}\,(+537{,}63)$ -	+2,10753	−7,97852	+0,86022
	−0,11459		
$2\,\mathfrak{U}\mathfrak{X}\,(+15{,}442.\cos.2p)$	+0,01235	+0,03027	+0,03027
	+2,00529	−7,94825	+0,89049
$(\mathfrak{U}X+U\mathfrak{X})\,(+21.\sin.2p)$	+0,09960	−0,52227	+0,52227
	+0,01120		
	+0,11080		
	+2,11609	−8,47052	+1,41276
$2\,UX\,(-268{,}78)$ -	−0,21772	+1,09932	−1,09932
$2\,UX\,(-7{,}721.\cos.2p)$	−0,00000	−0,00313	−0,00313
		+1,09619	−1,10245
	+1,89837	−7,37433	+0,31031
♄ $\mathfrak{U}\,(+1075{,}55)$ -	−4,17314	+15,96119	−1,73164
	+0,28646	−0,07489	−0,07489
♄ $\mathfrak{U}\,(+38{,}606.\cos.2p)$	−0,03108		
	−3,91776	+15,88630	−1,80653
	−2,01939	+8,51197	−1,49622
♄ $U\,(+13{,}726.\sin.2p)$	−0,12567	+0,64893	−0,64893
	−0,00419		
	−0,12986		
	−2,14925	+9,16090	−2,14515
♁ $(-268{,}85)$ - -	+1,59160	−27,84229	+10,67027
♁ $(-7{,}721.\cos.2p)$	−0,40008	+0,02287	+0,02287
	−0,15334		
	+1,03818	−27,81942	+10,69314
	−1,11107	−18,65852	+8,54799
Pars post.	−0,28373	−1,29893	+0,20210
𝔐 =	−1,39480	−19,95745	+8,75009

§. 523.

§. 523.

Eodem modo ex ſecunda aequatione eliciamus M:

	t	$2p-t$	$2p+t$
$2\mathfrak{U}\mathfrak{X}(+10,981.\,\text{ſin}.\,2p)$	− 0,08148	+ 0,02152	+ 0,02152
	− 0,00879		
	− 0,09027		
$(\mathfrak{U}X+U\mathfrak{X})(-537,56.)$	+ 25,56651	− 4,87570	− 0,54831
	+ 0,07003	− 0,36723	+ 0,36723
$(\mathfrak{U}X+U\mathfrak{X})(-15,44.\,\text{coſ}.\,2p)$	− 0,00788		
	+ 25,62866	− 5,24293	− 0,18108
	+ 25,53839	− 5,22141	− 0,15956
$2UX(-8,236.\,\text{ſin}.\,2p)$	+ 0,03368	− 0,00334	− 0,00334
	+ 25,57207	− 5,22475	− 0,16290
$\mathfrak{b}\mathfrak{U}(+13,72.\,\text{ſin}.\,2p)$	+ 0,10185	− 0,02603	− 0,02663
	+ 0,01105		
	+ 0,11290		
	± 25,68497	− 5,25138	− 0,18953
$\mathfrak{b}U(-268,809.)$ −	− 25,41597	+ 4,92191	+ 0,16397
	− 0,08836	+ 0,45628	− 0,45628
$\mathfrak{b}U(-9,65.\,\text{coſ}.\,2p)$	+ 0,00294		
	− 25,50139	+ 5,37819	− 0,29231
	+ 0,18358	+ 0,12681	− 0,48184
$\mathfrak{d}(-2,745.\,\text{ſin}.\,2p)$	− 0,14215	+ 0,00812	+ 0,00812
	− 0,05448		
	− 0,19663		
	− 0,01305	+ 0,13493	− 0,47372
Pars poſter.	+ 0,12218	+ 1,29739	− 0,20373
M =	+ 0,10913	+ 1,43232	− 0,67745

§. 524.

Quoniam elementa numerica iam supra Capite VII. sunt tradita, hinc statim quaeramus litteras $\mathfrak{N}$ et N:

	t	$2p-t$	$2p+t$
L. M	+ 9,0379442	+ 0,1560400	− 9,8308772
L. $\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 0,0516847	+ 0,0165527
L. $\frac{2(m+1)M}{\mu}$	+ 0,4650700	+ 0,2077247	− 9,8474299
$\frac{2(m+1)M}{\mu}$	+ 2,9179	+ 1,6133	− 0,7037
− $\mathfrak{M}$	+ 1,3948	+ 19,9574	− 8,7501
Numer.	+ 4,3127	+ 21,5707	− 9,4538
L. Num.	+ 0,6347492	+ 1,3338642	− 0,9756064
L. den.	+ 2,2460771	− 2,5722026	− 2,6859274
Log. $\mathfrak{N}$	+ 8,3886721	− 8,7616616	+ 8,2896790
L. $\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 0,0516847	+ 0,0165527
L. P. II.	+ 9,8157979	− 8,8133463	+ 8,3062317
Log. M	+ 9,0379442	+ 0,1560400	− 9,8308772
Log. μ^2		+ 2,7408822	+ 2,8211442
L. P. I.	+ 9,0379442	+ 7,4151578	− 7,0097330
P. I.	+ 0,1091	+ 0,0026	− 0,0010
− P. II.	− 0,6543	+ 0,0651	− 0,0202
N	− 0,5452	+ 0,0677	− 0,0212
$\mathfrak{N}$	+ 0,0244	− 0,0577	+ 0,0195

§. 525.

§. 525.

Pro partibus autem incognitis statuamus

$$\mathfrak{Z} = \beta.\cos.t + \gamma.\cos.(2p-t) + \delta.\cos.(2p+t)$$

$$Z = b.\sin.t + c.\sin.(2p-t) + d\,\sin.(2p+t)$$

vnde quaeramus litteras $\mathfrak{M}'$ et M':

Pro $\mathfrak{M}'$.

	$\cos.t$	$\cos.(2p-t)$	$\cos.(2p+t)$
$\mathfrak{Z}(-9,221.\cos 2p)=$	$-4,61064.(\gamma+\delta)$	$-4,61064\,\beta$	$-4,61064.\beta$
$\mathfrak{Z}(+0,02644)=$	$+0,02644.\beta$	$+0,02644\,\gamma$	$+0,02644.\delta$
$Z(-3,990.\sin.2p)=$	$-1,99534.(c+d)$	$-1,99534.b$	$-1,99534.b$

Pro M'.

	$\sin.t$	$\sin.(2p-t)$	$\sin.(2p+t)$
$\mathfrak{Z}(-3,990\cos.2p)=$	$-1,995.(\gamma-\delta)$	$-1,995.\beta$	$-1,995.\beta$
$Z(+5,360.\cos.2p)=$	$-2,680.(c-d)$	$-2,680.b$	$+2,680.b$
$Z(-0,0272)=$	$-0,0272.b$	$-0,0272.c$	$-0,0272.d$

§. 526.

§. 526.

Euoluamus igitur vt hactenus secundum angulum $(2p-t)$:

	β	b	γ	c
L. M′	− 0,3000187	− 0,4281870		− 8,43515
L. $\frac{2(m+1)}{\mu}$	+ 0,0516847	+ 0,0516847		+ 0,05168
L. $\frac{2(m+1)M}{\mu}$	− 0,3517034	− 0,4798717		− 8,48683
$\frac{2(m+1)M'}{\mu}$	− 2,2475	− 3,0191		− 0,0307
− $\mathfrak{M}'$	+ 4,6106	+ 1,9953	− 0,0264	
Numer.	+ 2,3631	− 1,0238	− 0,0264	− 0,0307
L. Num.	+ 0,3734821	− 0,0102151	− 8,42160	− 8,48683
L. Den.	− 2,5722026	− 2,5722026	− 2,57220	− 2,57220
Log. $\mathfrak{N}'$	− 7,8012795	+ 7,4380125	+ 5,84940	+ 5,91463
L. $\frac{2(m+1)}{\mu}$	+ 0,0516847	+ 0,0516847	+ 0,05168	+ 0,05168
L. P. II.	− 7,8529642	+ 7,4896972	+ 5,90108	+ 5,96631
Log. M′	− 0,3000187	− 0,4281870		− 8,43515
Log. μ^2	+ 2,7408822	+ 2,7408822		+ 2,74088
L. P. I.	− 7,5591365	− 7,6873048		− 5,69427
P. I.	− 0,0036	− 0,0049		− 0,00005
− P. II.	+ 0,0071	− 0,0031	− 0,00008	− 0,00009
N′	+ 0,0035	− 0,0080	− 0,00008	− 0,00014
$\mathfrak{N}'$	− 0,0063	+ 0,0027	+ 0,00007	+ 0,00008

hinc $\gamma = -0,0577 - 0,0063\,\beta + 0,0027.b$

$c = +0,0677 + 0,0035\,\beta - 0,0080.b$

impune enim bina postrema membra omittere licet.

§. 527.

§. 527.

In sequente ergo calculo statim, multo magis binas postremas columnas omittere poterimus:

Pro angulo $(2p+i)$

	β	b
L. M′	− 0,3000187	+ 0,4281870
L. $\frac{2(m+1)}{\mu}$	+ 0,0165527	+ 0,0165527
L. $\frac{2(m+1)M'}{\mu}$	− 0,3165714	+ 0,4447397
$\frac{2(m+1)M'}{\mu}$	− 2,0729	+ 2,7844
− 𝔐′	+ 4,6106	+ 1,9953
Numer.	+ 2,5377	+ 4,7797
L. Num.	+ 0,4044493	+ 0,6794097
L. Den.	− 2,6859274	− 2,6859274
L. 𝔑′	− 7,7185129	− 7,9934823
L. $\frac{2(m+1)}{\mu}$	+ 0,0165527	+ 0,0165527
L. P. II.	− 7,7350656	− 8,0100350
Log. M′	− 0,3000187	+ 0,4281870
Log. μ^2	+ 2,8211442	+ 2,8211442
L. P. I.	− 7,4788745	+ 7,6070428
Pars I.	− 0,0030	+ 0,0040
− P. II.	+ 0,0054	+ 0,0102
N′	+ 0,0024	+ 0,0142
𝔑′	− 0,0052	− 0,0099

Bbbb hinc

hinc

$$\delta = + 0,0195 - 0,0052.\beta - 0,0099.b$$

$$d = - 0,0212 + 0,0024.\beta + 0,0142.b$$

§. 528.

Quum iam hinc fit

$$\gamma + \delta = - 0,0382 - 0,0115.\beta - 0,0072.b;$$

$$\gamma - \delta = - 0,0772 - 0,0011.\beta + 0,0126.b;$$

$$c + d = + 0,0465 + 0,0059.\beta + 0,0062.b;$$

$$c - d = + 0,0889 + 0,0011.\beta - 0,0222.b;$$

pro angulo primo t reperimus

$$\mathfrak{M}' = + 0,0833 + 0,0412.\beta + 0,0211.b$$

$$M' = - 0,0842 - 0,0007.\beta + 0,0344.b.$$

§. 529.

§. 529.

His igitur quaeramus respondentes $\mathfrak{N}'$ et N':

Pro angulo ι.

	1.	β	b
Log. M'	− 8,9253121	− 6,8450980	+ 8,5365584
$L.\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 1,4271258	+ 1,4271258
$L.\frac{2(m+1)M'}{\mu}$	− 0,3524379	− 8,2722238	+ 9,9636842
$\frac{2(m+1)M'}{\mu}$	− 2,2513	− 0,0187	+ 0,9197
− $\mathfrak{M}'$	− 0,0833	− 0,0412	− 0,0211
Numer.	− 2,3346	− 0,0599	+ 0,8986
L. Num.	− 0,3682125	− 8,7774268	+ 9,9535664
L. Den.	+ 2,2460771	+ 2,2460771	+ 2,2460771
L. $\mathfrak{N}'$	− 8,1221354	− 6,5313497	+ 7,7074893
$L.\frac{2(m+1)}{\mu}$	+ 1,4271258	+ 1,4271258	+ 1,4271258
L. P. II.	− 9,54926612	− 7,9584755	+ 9,1346151
L. P. I.	− 8,92531	− 6,84509	+ 8,53655
P. I.	− 0,0842	− 0,0007	+ 0,0344
− P. II.	+ 0,3542	+ 0,0091	− 0,1363
N'	+ 0,2700	+ 0,0084	− 0,1019
$\mathfrak{N}'$	− 0,0133	− 0,0003	+ 0,0051

 Hinc

Hinc

$$\beta = +0,0111 - 0,0003.\beta + 0,0051.b$$
$$b = -0,2752 + 0,0084.\beta - 0,1019.b$$

ex priori fit

$$\beta = +0,0111 + 0,0051.b,$$

qui valor in altera substitutus praebet

$$b = -0,2751 - 0,1019.b,$$

vnde colligitur

$$b = -0,2496;\ \text{ideoque}\ \beta = +0,0098;$$
$$\gamma = -0,0584;\ c = +0,0697;$$
$$\delta = +0,0220;\ d = -0,0247;$$

ita vt haec euolutio prima nobis exhibeat:

$$\mathfrak{Z} = +0,0098.\cos t - 0,0584.\cos(2p-t) + 0,0220.\cos(2p+t)$$

$$Z = -0,2496.\sin t + 0,0697.\sin(2p-t) - 0,0247.\sin(2p+t).$$

II. Euolu-

II. Euolutio terminorum angulum $(t-2r)$ inuoluentium.

§. 530.

Hinc igitur superiorum formularum partes secundas sumimus, ac primo quidem ex priori aequatione colligamus litteram 𝔐:

	$t-2r$	$2p-t+2r$	$2p+t-2r$
2 𝔘𝔛 (537, 63) -	− 1,10753	+ 3,90861	− 0,40860
	+ 0,05613	− 0,01591	− 0,01591
2 𝔘𝔛 (+ 15,442. cos. 2p)	− 0,00587		
	− 1,05727	+ 3,89270	− 0,42451
(𝔘X + U𝔛)(21. sin. 2p)	− 0,09905	+ 0,25202	− 0,25202
	− 0,01142		
	− 0,11047		
	− 1,16774	+ 4,14472	− 0,67652
2 U X (− 268, 78) -	+ 13,02247	+ 2,88672	+ 2,10994
	+ 0,04149	+ 0,18718	+ 0,18718
2 U X (− 7, 721. cos. 2p)	+ 0,03033		
	+ 13,09429	+ 3,07390	+ 2,29712
	+ 11,92655	+ 7,21862	+ 1,62060
♄ 𝔘 (+ 1075, 55) -	+ 2,41999	− 7,91606	+ 0,78515
	− 0,14207	+ 0,04343	+ 0,04343
♄ 𝔘 (+ 38,606. cos. 2p)	+ 0,01409		
	+ 2,29201	− 7,87263	+ 0,82858
	+ 14,21856	− 0,65401	+ 2,44918
♄ U (+ 13, 726. sin. 2p)	+ 0,07529	− 0,32148	+ 0,32148
	+ 0,01469		
	+ 0,08998		
	+ 14,30854	− 0,97549	+ 2,77066
♁ (− 268, 85) -	− 5,35284	− 1,82282	− 10,53899
	− 0,02619	− 0,07692	− 0,07692
♁ (− 7, 72. cos. 2p)	− 0,15144		
	− 5,53047	− 1,89974	− 10,61591
	+ 8,77807	− 2,87523	− 7.84525
Part. poster.	+ 0,06313	+ 0,00256	− 0,19447
𝔐 =	+ 8,84120	− 2,87267	− 8,03972

 §. 531.

§. 531.

Simili modo ex secunda aequatione elicimus M:

	$t-2r$	$2p-t+2r$	$2p+t-2r$
$2\mathfrak{U}\mathfrak{X}$ (+10,98. sin. $2p$)	+ 0,03991	− 0,01131	− 0,01131
	+ 0,00417		
	+ 0,04408		
($\mathfrak{U}X+U\mathfrak{X}$)(−537,56)	− 12,33709	+ 4,84882	+ 0,55906
	− 0,06964	+ 0,17721	− 0,17721
($\mathfrak{U}X+U\mathfrak{X}$)(−15,44. cs. $2p$)	+ 0,00803		
	− 12,39870	+ 5,02603	+ 0,38185
	− 12,35462	+ 5,01472	+ 0,37054
$2UX$(− 8,236. sin. $2p$)	+ 0,04423	+ 0,19952	+ 0,19952
	− 0,03233		
	+ 0,01190		
	− 12,34272	+ 5,21424	+ 0,57006
$\mathfrak{V}\mathfrak{U}$ (+13,726. sin. $2p$)	− 0,05051	+ 0,01544	+ 0,01544
	− 0,00501		
	− 0,05552		
	− 12,39824	+ 5,22968	+ 0,58550
$\mathfrak{V}U$ (− 268, 809)	+ 12,59106	− 2,94884	− 0,57525
	+ 0,05294	− 0,22604	+ 0,22604
$\mathfrak{V}U$ (− 9,651. cos. $2p$)	− 0,01032		
	+ 12,63368	− 3,17488	− 0,34921
	+ 0,23544	+ 2,05480	+ 0,23629
$\mathfrak{d}$ (− 2, 745. sin. $2p$)	− 0,01044	− 0,02733	− 0,02733
	+ 0,05381		
	+ 0,04337		
	+ 0,27881	+ 2,02747	+ 0,20896
Part. poster.	+ 0,06307	− 0,00217	+ 0,18515
M =	+ 0,34188	+ 2,02530	+ 0,39411

§. 532.

§. 532.

Euoluantur iam pro his angulis elementa numerica:

$\omega =$	$t - 2r$	$2p - t + 2r$	$2p + t - 2r$
$\mu =$	$1 - 2l$	$2m - 1 + 2l$	$2m + 1 - 2l$
seu $\mu =$	$-25,84526$	$+50,58310$	$-1,10742$
Log. $\mu =$	$-1,4123792$	$+1,7040055$	$-0,0443123$
Log. $2(m+1) =$	$+1,4271258$	$+1,4271258$	$+1,4271258$
Log. $\frac{2(m+1)}{\mu} =$	$-0,0147466$	$+9,7231203$	$-1,3828135$
Log. $\mu^2 =$	$-2,8247584$	$+3,4080110$	$+0,0886246$
$\lambda - 2 =$	$177,22893$	$177,22893$	$177,22893$
$-\mu^2 =$	$-667,97275$	$-2558,65059$	$-1,22638$
Denom. $=$	$-490,74382$	$-2381,42166$	$-176,00255$

§. 533.

Horum elementorum ope determinemus litteras 𝔑 et N pro binis angulis posterioribus:

L. M	+0,3064894	+9,5956175
L. $\frac{2(m+1)}{\mu}$	+9,7231203	−1,3828135
L. $\frac{2(m+1)M}{\mu}$	+0,0296097	−0,9784310
$\frac{2(m+1)M}{\mu}$	+1,07056	−9,51549
−𝔐	+2,87267	+8,03972
Numer.	+3,94323	−1,47577
L. Num.	+0,5958521	−0,1690192
L. Den.	−3,3768361	+2,2455189
L. 𝔑	−7,2190160	−7,9235003
L. $\frac{2(m+1)}{\mu}$	+9,7231203	−1,3828135
L. P. II.	−6,9421363	+9,3063138
L. M	+0,3064894	+9,5956175
L. μ^2	+3,4080110	+0,0886246
L. P. I.	+6,8984784	+9,5069929
P. I.	+0,0008	+0,3214
−P. II.	+0,0009	−0,2025
N	+0,0017	+0,1189
at 𝔑	−0,0017	−0,0084

§. 534.

§. 534.

Pro partibus incognitis ponamus vt hactenus:

$$\mathfrak{Z} = \beta.\cos.(t-2r) + \gamma.\cos.(2p-t+2r) + \delta.\cos.(2p+t-2r)$$

$$Z = b.\sin.(t-2r) + c.\sin.(2p-t+2r) + d.\sin.(2p+t-2r)$$

vnde quaeramus litteras $\mathfrak{M}'$ et M':

Pro $\mathfrak{M}'$:

	$\cos.(t-2r)$	$\cos.(2p-t+2r)$	$\cos.(2p+t-2r)$
$\mathfrak{Z}(-9,221.\cos.2p)$	$-4,6106.(\gamma+\delta)$	$-4,6106.\beta$	$-4,6106.\beta$
$\mathfrak{Z}(+0,0264)$	$+0,0264.\beta$	$+0,0264.\gamma$	$+0,0264.\delta$
$Z(-3,990.\sin.2p)$	$-1,995.(c+d)$	$-1,995.b$	$-1,995.b$

Pro M':

	$\sin.(t-2r)$	$\sin.(2p-t+2r)$	$\sin.(2p+t-2r)$
$\mathfrak{Z}(-3,990.\sin.2p)$	$-1,995.(\gamma-\delta)$	$-1,995.\beta$	$-1,995.\beta$
$Z(+5,360.\cos.2p)$	$-2,6803.(c-d)$	$-2,6803.b$	$+2,6803.b$
$Z(-0,0272)$	$-0,0272.b$	$-0,0272.c$	$-0,0272.d$

§. 535.

Hinc calculum litterarum $\mathfrak{N}'$ et N′ incipiamus ab angulo postremo

$2p + t - 2r.$

	β	b	δ	d
Log. M′	− 0,3000187	+ 0,4281870		− 8,4351545
$L.\frac{2(m+1)}{\mu}$	− 1,3828135	− 1,3828135		− 1,3828135
$L.\frac{2(m+1)M'}{\mu}$	+ 1,6828322	− 1,8110005		+ 9,8179680
$\frac{2(m+1)M'}{\mu}$	+ 48,17616	− 64,71434		+ 0,65761
$-\mathfrak{M}'$	+ 4,61064	+ 1,99534	− 0,02644	
Num.	+ 52,78680	− 62,71900	− 0,02644	+ 0,65761
L.Num.	+ 1,7225253	− 1,7973991	− 8,4216039	+ 9,8179680
L. Den.	+ 2,2455189	+ 2,2455189	+ 2,2455189	+ 2,2455189
Log. $\mathfrak{N}'$	+ 9,4770064	− 9,5518802	− 6,1760850	+ 7,5724491
$L.\frac{2(m+1)}{\mu}$	− 1,3828185	− 1,3828185	− 1,3828135	− 1,3828135
L. P. II.	− 0,8598199	+ 0,9346937	+ 7,5588985	− 8,9552626
Log. M′	− 0,3000187	+ 0,4281870		− 8,4351545
Log. μ^3	+ 0,0886246	+ 0,0886246		+ 0,0886246
Log. P.I.	− 0,2113941	+ 0,3395624		− 8,3465299
P. I.	− 1,62703	+ 2,18556		− 0,02221
P. II.	+ 7,24135	− 8,60387	− 0,00362	+ 0,09021
N′	+ 5,6143	− 6,4183	− 0,0036	+ 0,0680
$\mathfrak{N}'$	+ 0,2999	− 0,3564	− 0,0002	+ 0,0038

hinc

$$\delta = -0{,}0084 + 0{,}2999\,\beta - 0{,}3564.b - 0{,}0002\,\delta + 0{,}0038.d$$

$$d = +0{,}1189 + 5{,}6143.\beta - 6{,}4183.b - 0{,}0036.\delta + 0{,}0680.d$$

vnde

vnde concludimus

$$\delta = -0,0079 + 0,3226.\beta - 0,3825.b$$
$$d = +0,1279 + 6,0225.\beta - 6,8852.b.$$

§. 536.

Eodem modo tractemus secundum angulum $2p - t + 2r$, vbi facile patet binas postremas columnas γ et c praetermitti posse:

	β	b
L. M′	−0,3000187	−0,4281870
$L.\frac{2(m+1)}{\mu}$	+9,7231203	+9,7231203
$h\frac{2(m+1)M'}{\mu}$	−0,0231390	−0,1513073
$\frac{2(m+1)M'}{\mu}$	−1,0547	−1,4168
−𝔐′	+4,6106	+1,9953
Numer.	+3,5559	+0,5785
L.Num.	+0,5509495	+9,7623034
L. Den.	−3,3768361	−3,3768361
Log. 𝔑′	−7,1741134	−6,3854673
$L.\frac{2(m+1)}{\mu}$	+9,7231203	+9,7231203
L. P II.	−6,8972337	−6,1085876
Log. M′	−0,3000187	−0,4281870
Log. μ^2	+3,4080110	+3,4080110
L. P. I.	−6,8920077	−7,0201760
P. I.	−0,0008	−0,0010
−P. II.	+0,0008	+0,0001
N′	−0,0000	−0,0009
𝔑′	−0,0015	−0,0002

 hinc

hinc

$$\gamma = -0{,}0017 - 0{,}0015.\beta - 0{,}0002.b$$
$$c = +0{,}0017 + 0{,}0000.\beta - 0{,}0009.b$$

§. 537.

Quum igitur nunc habeamus:

$$\gamma + \delta = -0{,}0096 + 0{,}3211.\beta - 0{,}3827.b$$
$$\gamma - \delta = +0{,}0062 - 0{,}3241.\beta + 0{,}3823.b$$
$$c + d = +0{,}1296 + 6{,}0225.\beta - 6{,}8861.b$$
$$c - d = -0{,}1262 - 6{,}0225.\beta + 6{,}8843.b$$

reperiemus pro angulo primo $t - 2r$

$$\mathfrak{M} + \mathfrak{M}' = +8{,}62686 - 13{,}47102.\beta + 15{,}50467.b$$

et

$$M + M' = +0{,}66777 + 16{,}78893.\beta - 19{,}24220.b.$$

§. 538.

§. 538.

His ergo valoribus ſequens calculus ſuperſtruatur pro angulo $t - 2r$.

	I.	β	γ
L. M	+ 9,8246334	+ 1,2240233	− 1,2842547
L. $\frac{2(m+1)}{\mu}$	− 0,0147466	− 0,0147466	− 0,0147466
L. $\frac{2(m+1)M}{\mu}$	− 9,8393800	− 1,2387699	+ 1,2990013
$\frac{2(m+1)M}{\mu}$	− 0,69085	− 17,32886	+ 19,90679
− 𝔐	− 8,62686	+ 13,47102	− 15,50467
Num.	− 9,31771	− 3,85784	+ 4,40212
L. Num.	− 0,9693092	− 0,5863443	+ 0,6436619
L. Den.	− 2,6908549	− 2,6908549	− 2,6908549
L. 𝔑	+ 8,2784543	+ 7,8954894	− 7,9528070
L. $\frac{2(m+1)}{\mu}$	− 0,0147466	− 0,0147466	− 0,0147466
L. P. II.	− 8,2932009	− 7,9102360	+ 7,9675536
L. M	+ 9,8246334	+ 1,2240233	− 1,2842547
L. μ^2	+ 2,8247584	+ 2,8247584	+ 2,8247584
L. P. I.	+ 6,9998750	+ 8,3992649	− 8,4594963
P. I.	+ 0,0010	+ 0,0251	− 0,0288
− P. II.	+ 0,0196	+ 0,0082	− 0,0073
N	+ 0,0206	+ 0,0333	− 0,0381
𝔑	+ 0,0190	+ 0,0079	− 0,0090

hinc

$$\beta = +0,0190 + 0,0079.\beta - 0,0090.b \text{ et}$$
$$b = +0,0206 + 0,0333.\beta - 0,0381.b$$

ex posteriori deducimus

$$b = +0,0198 + 0,0321.\beta,$$

qui valor in prima substitutus praebet

$$\beta = +0,0188 + 0,0076.\beta,$$

vnde

$$\beta = +0,0189 \text{ et } b = +0,0204,$$

vnde sequentes deducuntur valores:

$$\gamma = -0,0017;\ c = +0,0017;$$
$$\delta = -0,0096;\ d = +0,1016.$$

ita vt haec secunda euolutio nobis suppeditet:

$$\mathfrak{Z} = +0,0189.\cos.(t-2r) - 0,0017.\cos.(2p-t+2r) - 0,0096.\cos.(2p+t-2r)$$

$$Z = +0,0204.\sin.(t-2r) + 0,0017.\sin.(2p-t+2r) + 0,1016.\sin.(2p+t-2r).$$

III. Euo-

III. Euolutio terminorum angulum $t+2r$ inuoluentium.

§. 539.

Calculus pro litteris 𝔐 ita se habet:

	$t+2r$	$2p-t-2r$	$2p+t+2r$
2 𝔘𝔛 (+537,63) -	− 0,88172	+ 3,95700	−0,45699
	+ 0,05683	− 0,01266	−0,01266
2 𝔘𝔛 (+15,442.cos. 2 p)	− 0,00656		
	− 0,83145	+ 3,94434	−0,46965
(𝔘X+U𝔛)(+21.sin.2 p)	− 0,00714	+ 0,26828	− 0,26828
	+ 0,01252		
	+ 0,00538		
	− 0,82607	+ 4,21262	−0,73793
2 UX (−268,78) -	− 12,68110	− 4,61230	−0,38436
	− 0,06629	− 0,18227	−0,18227
2 UX (−7,721.cos. 2 p)	− 0,00553		
	− 12,75292	− 4,79457	−0,56663
	− 13,57899	− 0,58195	− 1,30456
♄𝔘 (+1075,55) -	+ 1,74239	− 8,04513	+0,93573
	− 0,14439	+ 0,03127	+0,03127
♄𝔘 (+38,606.cos. 2 p)	+ 0,01679		
	+ 1,61479	− 8,01386	+ 0,96700
	− 11,96420	− 8,59581	− 0,33756
♄U (+13,726.sin. 2 p)	+ 0,05017	− 0,32649	+0,32649
	− 0,01043		
	+ 0,03974		
	− 11,92446	− 8,62230	−0,01107
♃ (−268,85) - -	+ 4,54359	+29,29945	+0,25003
♃ (−7,721.cos. 2 p)	+ 0,42102	+ 0,06529	+0,06529
	+ 0,00359		
	+ 4,96820	+29,36474	+0,31532
	− 6,95626	+20,74244	+0,30425
Pars post.	+ 0,20288	+ 1,28673	− 0,00002
𝔐 =	− 6,75338	+22,02917	+0,30423

§. 540.

§. 540.

Simili modo ſequens calculus ſe habebit pro M:

	$t+2r$	$2p-t-2r$	$2p+t+2r$
2𝔘𝔛(+10,981. ſin 2p)	+0,04041	−0,00900	−0,00900
	+0,00467		
	+0,04508		
(𝔘X+U𝔛)(−537,56.)	−13,13268	+0,34942	−0,61282
	−0,00502	+0,18863	−0,18863
(𝔘X+U𝔛)(−15,44 coſ. 2p)	−0,00880		
	−13,14650	+0,53805	−0,80145
	−13,10142	+0,52905	−0,81045
2UX(−8,236. ſin. 2p)	−0,07066	−0,19429	−0,19429
	+0,00589		
	−0,06477		
	−13,16619	+0,33476	−1,00474
♄𝔘(+13,72. ſin. 2p)	−0,05134	+0,01112	+0,01112
	−0,00597		
	−0,05731		
	−13,22350	+0,34588	−0,99362
♄U(−268,809.)	+12,78729	−1,96500	+0,40859
	+0,03528	−0,22957	+0,22957
♄U(−9,65. coſ. 2p)	+0,00734		
	+12,82991	−2,19457	+0,53816
	−0,39359	−1,84869	−0,45546
♄(−2,745. ſin. 2p)	+0,14959	+0,02320	+0,02320
	−0,00128		
	+0,14831		
	−0,24528	−1,82549	−0,43226
Pars poſter.	−0,20028	−1,29605	+0,00002
M =	−0,44556	−3,12154	−0,43224

§. 541.

§. 541.

Elementa numerica ex praecedentibus facile formantur:

$\omega =$	$t + 2r$	$2p - t - 2r$	$2p + t + 2r$
$\mu =$	$1 + 2l$	$2m - 1 - 2l$	$2m + 1 + 2l$
seu $\mu =$	$+ 27,84526$	$- 3,10742$	$+ 52,58310$
$L. 2(m+1) =$	$+ 1,4271258$	$+ 1,4271258$	$+ 1,4271258$
$Log. \mu =$	$+ 1,4447512$	$- 0,4924000$	$+ 1,7208462$
$L. \frac{2(m+1)}{\mu} =$	$+ 9,9823746$	$- 0,9347258$	$+ 9,7062796$
$Log. \mu^2 =$	$+ 2,8895024$	$+ 0,9848000$	$+ 3,4416924$
$\lambda - 2 =$	$+ 177,22893$	$+ 177,22893$	$+ 177,22893$
$- \mu^2 =$	$- 775,35830$	$- 9,65606$	$- 2764,98300$
$Log. Den. =$	$- 598,12937$	$+ 167,57287$	$- 2587,75407$

§. 542.

Hic iterum prima columna omissa, calculum pro binis sequentibus faciamus:

	$2p-t-2r$	$2p+t+2r$
Log. M	$-0,4943689$	$-9,6357250$
Log. $\frac{2(m+1)}{\mu}$	$-0,9347258$	$+9,7062796$
Log. $\frac{2(m+1)M}{\mu}$	$+1,4290947$	$-9,3420046$
$\frac{2(m+1)M}{\mu}$	$+26,85930$	$-0,21979$
$-\mathfrak{M}$	$-22,02917$	$-0,30423$
Numerat.	$+4,83013$	$-0,52402$
Log. Numerat.	$+0,6839588$	$-9,7193479$
Log. Denom.	$+2,2242038$	$-3,4129220$
L. $\mathfrak{N}$	$+8,4597550$	$+6,3064259$
adde Log. $\frac{2(m+1)}{\mu}$	$-0,9347258$	$+9,7062796$
L. Pars II.	$-9,3944808$	$+6,0127055$
L. M	$-0,4943689$	$-9,6357250$
L. μ^2	$+0,9848000$	$+3,4416924$
L. P. I.	$-9,5095689$	$-6,1940326$
P. I.	$-0,3232$	$-0,0002$
$-$ P. II.	$+0,2480$	$-0,0001$
N	$-0,0752$	$-0,0003$
at $\mathfrak{N}$	$+0,0288$	$+0,0002$

§. 543.

§. 543.

Pro partibus incognitis ponamus:

$$\mathfrak{Z} = \beta.\ \mathrm{cof.}\,(t + 2r) + \gamma.\ \mathrm{cof.}\,(2p - t - 2r)$$
$$+ \delta.\ \mathrm{cof.}\,(2p + t + 2r)$$
$$Z = b.\ \mathrm{fin.}\,(t + 2r) + c.\ \mathrm{fin.}\,(2p - t - 2r)$$
$$+ d.\ \mathrm{fin.}\,(2p + t + 2r)$$

vnde quaeramus litteras $\mathfrak{M}'$ et M':

Pro $\mathfrak{M}'$.

	$\mathrm{cof.}\,(t + 2r)$	$\mathrm{cof.}\,(2p-t-2r)$	$\mathrm{cof.}\,(2p+t+2r)$
$\mathfrak{Z}(-9{,}221.\mathrm{cof.}\,2p)$	$-4{,}6106.(\gamma+\delta)$	$-4{,}6106.\ \beta$	$-4{,}6106.\ \beta$
$\mathfrak{Z}(+0{,}026.)$	$+0{,}0264.\ \beta$	$+0{,}0264.\ \gamma$	$+0{,}0264.\ \delta$
$Z(-3{,}990.\mathrm{fin.}\,2p)$	$-1{,}9953.(c+d)$	$-1{,}9953.\ b$	$-1{,}9953.\ b$

Pro M'.

	$\mathrm{fin.}\,(t + 2r)$	$\mathrm{fin.}\,(2p-t-2r)$	$\mathrm{fin.}\,(2p+t+2r)$
$\mathfrak{Z}(-3{,}990.\mathrm{fin.}\,2p)$	$-1{,}9953.(\gamma-\delta)$	$-1{,}9953.\ \beta$	$-1{,}9953.\ \beta$
$Z(+5{,}360\ \mathrm{cf.}\,2p)$	$-2{,}6803.(c-d)$	$-2{,}6803.\ b$	$+2{,}6803.\ b$
$Z(-0{,}0272)$	$-0{,}0272.\ b$	$-0{,}0272.\ c$	$-0{,}0272.\ d$

§. 544.

Iam calculum litterarum $\mathfrak{N}'$ et N' incipiamus ab angulo $2p-t-2r$:

	β	b	γ	c
Log. M'	− 0,3000187	− 0,4281870		− 8,4351545
L. $\frac{2(m+1)}{\mu}$	− 0,9347258	− 0,9347258		− 0,9347258
L $\frac{2(m+1)M'}{\mu}$	+ 1,2347445	+ 1,3629128		+ 9,3698803
$\frac{2(m+1)M'}{\mu}$	+ 17,1690	+ 23,0629		+ 0,2343
− $\mathfrak{M}'$	+ 4,6106	+ 1,9953	− 0,0264	
Num.	+ 21,7796	+ 25,0582	− 0,0264	+ 0,2343
L. Num.	+ 1,3380498	+ 1,3989498	− 8,4216039	+ 9,3698803
L. Den.	+ 2,2242038	+ 2,2242038	+ 2,2242038	+ 2,2242038
L. $\mathfrak{N}'$	+ 9,1138460	+ 9,1747460	− 6,1974001	+ 7,1456765
L $\frac{2(m+1)}{\mu}$	− 0,9347258	− 0,9347258	− 0,9347258	− 0,9347258
L. P. II.	− 0,0485718	− 0,1094718	+ 7,1321259	− 8,0804023
L. M'	− 0,3000187	− 0,4281870		− 8,4351545
L. μ^2	+ 0,9848000	+ 0,9848000		+ 0,9848000
L. P. I.	− 9,3152187	− 9,4433870		− 7,4503545
P. I.	− 0,2066	− 0,2776		− 0,0028
− P. II.	+ 1,1183	+ 1,2867	− 0,0014	+ 0,0120
N'	+ 0,9117	+ 1,0091	− 0,0014	+ 0,0092
at $\mathfrak{N}'$	+ 0,1300	+ 0,1495	− 0,0002	+ 0,0014

hinc

$$\gamma = +0{,}0288 + 0{,}1300\,\beta + 0{,}1495.b - 0{,}0002.\gamma + 0{,}0014.c$$
$$c = -0{,}0752 + 0{,}9117.\beta + 1{,}0091.b - 0{,}0014.\gamma + 0{,}0092.c$$

vnde

vnde concludimus

$$\gamma = +0,0287 + 0,1313.\beta + 0,1509.b$$
$$c = -0,0757 + 0,9199.\beta + 1,0182.b.$$

§. 545.

Pro tertio angulo binas columnas δ et d praetermittamus, vnde calculus erit:

Pro angulo $2p + t + 2r$

	β	b
Log. M′	− 0,3000187	+ 0,4281870
$L.\frac{2(m+1)}{\mu}$	+ 9,7062796	+ 9,7062796
$L\frac{2(m+1)M'}{\mu}$	− 0,0062983	+ 0,1344666
$\frac{2(m+1)M'}{\mu}$	− 1,0146	+ 1,3629
− $\mathfrak{M}'$	+ 4,6106	+ 1,9953
Numer.	+ 3,5960	+ 3,3582
L. Num.	+ 0,5558197	+ 0,5261066
L. Den.	− 3,4129220	− 3,4129220
Log. $\mathfrak{N}'$	− 7,1428977	− 7,1131846
$L.\frac{2(m+1)}{\mu}$	+ 9,7062796	+ 9,7062796
L. P. II	− 6,8491773	− 6,8194642
Log M′	− 0,3000187	+ 0,4281870
Log. μ^2	+ 3,4416924	+ 3,4416924
L. P. I.	− 6,8583263	+ 6 9864946
P. I.	− 0,0007	+ 0,0009
− P. II.	+ 0,0007	+ 0,0007
N′	− 0,0000	+ 0,0016
$\mathfrak{N}'$	− 0,0014	− 0,0013

hinc

$$\delta = +0,0002 - 0,0014.\beta - 0,0013.b$$
$$d = -0,0003 - 0,0000.\beta + 0,0016.b.$$

§. 546.

Quum igitur nunc habeamus:

$$\gamma + \delta = +0,0289 + 0,1299.\beta + 0,1496.b$$
$$\gamma - \delta = +0,0285 + 0,1327\ \beta + 0,1522.b$$
$$c + d = -0,0760 + 0,9199.\beta + 1,0198.b$$
$$c - d = -0,0754 + 0,9199.\beta + 1,0166.b$$

reperiemus pro angulo primo $t + 2r$

$$\mathfrak{M} = -6,73498 - 2,42600.\beta - 2,72476.b$$
$$M = -0,30032 - 2,73042.\beta - 3,05575.b.$$

§. 547.

§. 547.

Iam igitur pro prima columna sequens instituatur calculus:

Pro angulo $t + 2r$.

	1.	β	b
L. M	− 9,4775843	− 0,4362295	− 0,4851178
$L.\frac{2(m+1)}{\mu}$	+ 9,9823746	+ 9,9823746	+ 9,9823746
$L\frac{2(m+1)M}{\mu}$	− 9,4599589	− 0,4186041	− 0,4674924
$\frac{2(m+1)M}{\mu}$	− 0,28837	− 2,62183	− 2,93422
$-\mathfrak{M}$	+ 6,73498	+ 2,42600	+ 2,72476
Numer.	+ 6,44661	− 0,19583	− 0,20946
L. Num.	+ 0,8093314	− 9,2918792	− 9,3211011
L. Den.	− 2,7767879	− 2,7767879	− 2,7767879
Log. $\mathfrak{N}$	− 8,0325435	+ 6,5150913	+ 6,5413132
$L.\frac{2(m+1)}{\mu}$	+ 9,9823746	+ 9,9823746	+ 9,9823746
L. P. II	− 8,0149181	+ 6,4974659	+ 6,5266878
Log M	− 9,4775843	− 0,4362295	− 0,4851178
Log. μ^2	+ 2,8895024	+ 2,8895024	+ 2,8895024
L. P. I.	− 6,5880819	− 7,5467271	− 7,3746202
P. I	− 0,0004	− 0,0035	− 0,0024
− P. II	+ 0,0103	− 0,0003	− 0,0003
N	+ 0,0099	− 0,0038	− 0,0027
$\mathfrak{N}$	− 0,0108	+ 0,0003	+ 0,0004

hinc

hinc $\beta = -0,0108 + 0,0003 \cdot \beta + 0,0004 \cdot b$
et $b = +0,0099 - 0,0038 \, \beta - 0,0027 \cdot b$
ex priori fit $\beta = -0,0108 + 0,0004 \cdot b$
vnde $b = +0,0099 - 0,0027 \cdot b$; ideoque

$$b = +0,0099;\ \beta = -0,0108;$$
$$\gamma = +0,0288;\ c = -0,0755;$$
$$\delta = +0,0001;\ d = -0,0004;$$

vnde tertia euolutio nobis praebet

$$\mathfrak{Z} = -0,0108 \cdot \text{cof.}(t+2r) + 0,0288 \cdot \text{cof.}(2p-t-2r) + 0,0001 \cdot \text{cof.}(2p+t+2r)$$
$$Z = +0,0099 \cdot \text{fin.}(t+2r) - 0,0755 \cdot \text{fin.}(2p-t-2r) - 0,0004 \cdot \text{fin.}(2p+t+2r).$$

§. 548.

Completi ergo valores nostrarum litterarum $\mathfrak{Z}$ et Z erunt:

$$\begin{aligned}\mathfrak{Z} = {} & +0,0098 \cdot \text{cof.}\, t - 0,0584 \cdot \text{cof.}(2p-t) \\ & + 0,0220 \cdot \text{cof.}(2p+t) \\ & +0,0189 \cdot \text{cof.}(t-2r) - 0,0017 \cdot \text{cof.}(2p-t+2r) \\ & -0,0096 \cdot \text{cof.}(2p+t-2r) \\ & -0,0108 \cdot \text{cof.}(t+2r) + 0,0288 \cdot \text{cof.}(2p-t-2r) \\ & +0,0001 \cdot \text{cof.}(2p+t+2r)\end{aligned}$$

$$\begin{aligned}Z = {} & -0,2496 \cdot \text{fin.}\, t + 0,0697 \cdot \text{fin.}(2p-t) \\ & - 0,0247 \cdot \text{fin.}(2p+t) \\ & +0,0204 \cdot \text{fin.}(t-2r) + 0,0017 \cdot \text{fin.}(2p-t+2r) \\ & +0,1016 \cdot \text{fin.}(2p+t-2r) \\ & +0,0099 \cdot \text{fin.}(t+2r) - 0,0755 \cdot \text{fin.}(2p-t-2r) \\ & -0,0004 \cdot \text{fin.}(2p+t+2r).\end{aligned}$$

NOVAE

NOVAE THEORIAE
MOTVVM LVNAE
LIBER SECVNDVS
CONTINENS ADPLICATIONEM THEORIAE LVNAE AD CALCVLVM ASTRONOMICVM.

PARS PRIMA.
COMPARATIO FORMVLARVM INVENTARVM CVM TABVLIS CEL. DE CLAIRAVLT.

CAPVT I.

RECAPITVLATIO OMNIVM INAEQVALITATVM LVNAE, QVAE IN PARTE SECVNDA ET TERTIA LIBRI PRIMI SVNT INVENTAE.

§. 549.

Omnes inaequalitates Lunae, quas hactenus euoluimus, operae pretium erit, simul obtutui exponere, quo facilius in sequentibus iis vti queamus; ante omnia autem hic meminisse iuuabit, inter quatuor illas quantitates constantes, K, i, a et $\varkappa$, quibus illae inae-

 quali-

qualitates adficiuntur, binas posteriores a et $\varkappa$ iam satis exacte esse cognitas, quum sit $a = \frac{1}{353}$ et $\varkappa =$ 0,01678, dum priorum valores demum ex ipso motu Lunae definiri debent; quamobrem in formulis sequentibus hos valores litterarum a et $\varkappa$ statim introduci conueniet.

§. 550.

Quoniam hic motum Lunae per ternas coordinatas x, y et z repraesentamus, seorsim singulas percurramus. Pro prima igitur earum x, quam secundum ordines ita exhibuimus:

$$x = \mathfrak{O} + K\mathfrak{P} + K^2\mathfrak{Q} + K^3\mathfrak{R} + a\mathfrak{S} + aK\mathfrak{T} + \varkappa\mathfrak{U} + \varkappa K\mathfrak{V} + \varkappa K^2\mathfrak{W} + a\varkappa\mathfrak{w} + ii\mathfrak{X} + iiK\mathfrak{Y} + ii\varkappa\mathfrak{Z}.$$

Singularum harum litterarum valores ita sunt determinati:

Multiplic.

$$\mathfrak{O} = +0,0000240 - 0,0071801.\,\mathrm{cos.}\,2p + 0,0000060.\,\mathrm{cos.}\,4p \qquad (1)$$

$$\mathfrak{P} = +1,000000.\,\mathrm{cos.}\,q + 0,187695.\,\mathrm{cos.}(2p-q) - 0,002703.\,\mathrm{cos.}(2p+q) - 0,000514.\,\mathrm{cos.}(4p-q) - 0,000021.\,\mathrm{cos.}(4p+q) \qquad (K)$$

$\mathfrak{Q} =$

Multiplic.

$$\mathfrak{Q} = -0,53896 + 0,21903.\cos.2p + 0,00195.\cos.4p + 0,50967.\cos.2q - 0,20179.\cos.(2p-2q) + 0,00482.\cos.(2p+2q) + 0,02278.\cos.(4p-2q) + 0,00004.\cos.(4p+2q) \quad (K^2)$$

$$\mathfrak{R} = +0,\cos.q - 0,1908.\cos.(2p-q) - 0,2300.\cos.(2p+q) - 0,0482.\cos.(4p-q) - 0,0058.\cos.(4p+q) - 0,3807.\cos.3q + 0,2623.\cos.(2p-3q) - 0,0068\cos.(2p+3q) - 0,0239.\cos.(4p-3q) + 0,0000.\cos.(4p+3q) \quad (K^3)$$

$$\mathfrak{S} = +0,11419.\cos.p - 0,00289.\cos.3p \quad (a)$$

$$\mathfrak{T} = -0,0813.\cos.(p-q) - 0,0088.\cos.(3p-q) + 0,1205.\cos.(p+q) + 0,0015.\cos.(3p+q) \quad (aK)$$

$$\mathfrak{U} = -0,006829.\cos.t + 0,029397\cos.(2p-t) - 0,003452.\cos(2p+t) + 0,000046.\cos.(4p-t) - 0,000004.\cos.(4p+t) \quad (x)$$

Multiplic.

$$\mathfrak{V} = -0{,}18182.\text{cof.}(q-t)+0{,}03084.\text{cof.}(2p-q+t)$$
$$+0{,}01379.\text{cof.}(2p+q-t)$$
$$+0{,}08427\ \text{cof.}(q+t)-0{,}40759.\text{cof.}(2p-q-t)$$
$$-0{,}00093.\text{cof.}(2p+q+t) \quad (\kappa K)$$

$$\mathfrak{W} = +0{,}1278.\text{cof.}\,t-0{,}6509.\text{cof.}(2p-t)$$
$$+0{,}1436.\text{cof.}(2p+t)$$
$$-0{,}4431.\text{cof.}(2q-t)-0{,}4322.\text{cof.}(2p-2q+t)$$
$$-0{,}0253.\text{cof.}(2p+2q-t)$$
$$+0{,}0642.\text{cof.}(2q+t)+0{,}2528.\text{cof.}(2p-2q-t)$$
$$+0{,}0030.\text{cof.}(2p+2q+t) \quad (\kappa K^2)$$

$$\mathfrak{w} = +0{,}1164.\text{cof.}(p-t)+0{,}6135.\text{cof.}(p+t)$$
$$+0{,}0162.\text{cof.}(3p-t)$$
$$-0{,}0048.\text{cof.}(3p+t) \quad (a\kappa)$$

$$\mathfrak{X} = -0{,}25019+0{,}01928.\text{cof.}\,2p$$
$$+0{,}00002.\text{cof.}\,4p$$
$$+0{,}24728.\text{cof.}\,2r-0{,}01242.\text{cof.}(2p-2r)$$
$$+0{,}00038.\text{cof.}(2p+2r)$$
$$+0{,}00025.\text{cof.}(4p-2r)$$
$$+0{,}00000.\text{cof.}(4p+2r) \quad (ii)$$

$\mathfrak{Y} =$

Multiplic.

$$\mathfrak{Y} = +0,\cos. q - 0,0716.\cos.(2p-q)$$
$$-0,0067.\cos.(2p+q)$$
$$-0,0610.\cos.(q-2r) - 0,0268.\cos.(2p-q+2r)$$
$$-0,1044.\cos.(2p+q-2r)$$
$$-0,1264.\cos.(q+2r) + 0,0258.\cos.(2p-q-2r)$$
$$-0,0009.\cos.(2p+q+2r) \quad (iiK)$$

$$\mathfrak{Z} = +0,0098.\cos. t - 0,0584.\cos.(2p-t)$$
$$+0,0220.\cos.(2p+t)$$
$$+0,0189.\cos.(t-2r) - 0,0017.\cos.(2p-t+2r)$$
$$-0,0096.\cos.(2p+t-2r)$$
$$-0,0108.\cos.(t+2r) + 0,0288.\cos.(2p-t-2r)$$
$$+0,0001.\cos.(2p+t+2r). \quad (ii\kappa)$$

§. 551.

Quum vero sit $a = \frac{1}{390} = \frac{1}{400}(1 + \frac{1}{93})$ et $\kappa = \frac{1}{60}(1 + \frac{1}{139})$ formulae illae, in quibus hae litterae occurrunt, commodius sequenti modo exhibentur:

Multiplic.

$$a\mathfrak{S} = +0,0002928.\cos. p - 0,0000074.\cos. 3p \quad (I)$$

$$a\mathfrak{T} = -0,000208.\cos(p-q)$$
$$+0,000309.\cos(p+q)$$
$$-0,000023.\cos.(3p-q)$$
$$+0,000003.\cos.(3p+q) \quad (K)$$

$\kappa\,\mathfrak{U} =$

Multiplic.

$$\varkappa \mathfrak{U} = -0,0001146.\cos t + 0,0004933.\cos(2p-t) - 0,0000579.\cos(2p+t) + 0,0000007.\cos(4p-t) - 0,0000001.\cos(4p+t) \quad (\mathrm{I})$$

$$\varkappa \mathfrak{V} = -0,003051.\cos(q-t) + 0,000518.\cos(2p-q+t) + 0,000231.\cos(2p+q-t) + 0,001414.\cos(q+t) - 0,006839.\cos(2p-q-t) - 0,000016.\cos(2p+q+t) \quad (\mathrm{K})$$

$$\varkappa \mathfrak{W} = +0,00215\cos t - 0,01162.\cos(2p-t) + 0,00241.\cos(2p+t) - 0,00744.\cos(2q-t) - 0,00726\cos(2p-2q+t) - 0,00042.\cos(2p+2q-t) + 0,00108\cos(2q+t) + 0,00424.\cos(2p-2q-t) + 0,00005.\cos(2p+2q+t) \quad (\mathrm{K}^2)$$

$$a \varkappa w = +0,0000050.\cos(p-t) + 0,0000264.\cos(p+t) - 0,0000007.\cos(3p-t) - 0,0000002.\cos(3p+t) \quad (\mathrm{I})$$

$$\varkappa \mathfrak{Z} = +0,00016.\cos t - 0,00098.\cos(2p-t) + 0,00037.\cos(2p+t) + 0,00032\cos(t-2r) - 0,00003.\cos(2p-t+2r) - 0,00016.\cos(2p+t-2r) - 0,00018.\cos(t+2r) + 0,00048.\cos(2p-t-2r) + 0,00000.\cos(2p+t+2r) \quad (ii)$$

§. 552.

§. 552.

Eodem modo secundae coordinatae y ratio est comparata, postquam enim posuimus:

$$y = O + KP + K^2Q + K^3R + \alpha S + \alpha KT + \varkappa U + \varkappa KV + \varkappa K^2W + \alpha\varkappa w + iiX + iiKY + ii\varkappa Z,$$

sequentes horum ordinum elicuimus valores:

Multiplic.

$$O = +0{,}0102117.\sin.2p + 0{,}0000057.\sin.4p \qquad (1)$$

$$\begin{aligned} P = &-2{,}012639\ \sin.q - 0{,}411247.\sin(2p-q) \\ &-0{,}003212.\sin.(2p+q) \\ &-0{,}000724.\sin.(4p-q) \\ &-0{,}000019.\sin(4p+q) \end{aligned} \qquad (K)$$

$$\begin{aligned} Q = &+0{,}09800.\sin.2p + 0{,}00175.\sin.4p \\ &+0{,}25209.\sin.2q + 0{,}31159.\sin.(2p-2q) \\ &+0{,}00428.\sin.(2p+2q) \\ &+0{,}01183.\sin.(4p-2q) \\ &+0{,}00005.\sin.(4p+2q) \end{aligned} \qquad (K^2)$$

$$\begin{aligned} R = &+1{,}3662.\sin.q + 0{,}4255.\sin.(2p-q) \\ &-0{,}0853.\sin.(2p+q) \\ &-0{,}0377.\sin.(4p-q) \\ &+0{,}0010.\sin.(4p+q) \\ &-0{,}2955.\sin.3q - 0{,}2211.\sin.(2p-3q) \\ &-0{,}0061.\sin.(2p+3q) \\ &-0{,}0071.\sin.(4p-3q) \\ &-0{,}0001.\sin.(4p+3q) \end{aligned} \qquad (K^3)$$

Multiplic.

$$S = -0{,}24035 \cdot \sin. p + 0{,}00285 \cdot \sin. 3p \qquad (a)$$

$$T = +1{,}8056 \cdot \sin.(p-q) + 0{,}0720 \cdot \sin.(3p-q)$$
$$+0{,}0603 \cdot \sin.(p+q) - 0{,}0008 \cdot \sin.(3p+q) \qquad (aK)$$

$$U = +0{,}190587 \cdot \sin. t - 0{,}043312 \sin (2p-t)$$
$$+0{,}005525 \cdot \sin.(2p+t)$$
$$-0{,}000143 \cdot \sin (4p-t)$$
$$+0{,}000005 \cdot \sin.(4p+t) \qquad (\varkappa)$$

$$V = +0{,}68575 \sin.(q-t) - 0{,}13086 \cdot \sin.(2p-q+t)$$
$$+0{,}01518 \cdot \sin.(2p+q-t)$$
$$-0{,}44216 \cdot \sin.(q+t) + 1{,}00824 \cdot \sin (2p-q-t)$$
$$-0{,}00422 \cdot \sin.(2p+q+t) \qquad (\varkappa K)$$

$$W = +2{,}6319 \cdot \sin. t - 0{,}3130 \cdot \sin.(2p-t)$$
$$+0{,}0952 \cdot \sin.(2p+t)$$
$$-0{,}1673 \sin.(2q-t) - 0{,}\ldots \sin.(2p-2q+t)$$
$$-0{,}0233 \cdot \sin.(2p+2q-t)$$
$$+0{,}3349 \cdot \sin.(2q+t) - 1{,}7240 \cdot \sin.(2p-2q-t)$$
$$+0{,}0076 \cdot \sin.(2p+2q+t) \qquad (\varkappa K^2)$$

$$w = -0{,}1093 \cdot \sin.(p-t) - 1{,}2630 \cdot \sin.(p+t)$$
$$-0{,}0167 \cdot \sin.(3p-t)$$
$$+0{,}0015 \cdot \sin.(3p+t) \qquad (a\varkappa)$$

$$X = -0{,}02145 \cdot \sin. 2p - 0{,}00003 \cdot \sin 4p$$
$$-0{,}24645 \cdot \sin. 2r + 0{,}03407 \sin.(2p-2r)$$
$$-0{,}00037 \cdot \sin.(2p+2r)$$
$$-0{,}00014 \cdot \sin (4p-2r)$$
$$+0{,}00000 \cdot \sin.(4p+2r) \qquad (ii)$$

Y =

Multiplic.

$$Y = +0{,}0014.\sin.q + 0{,}1917.\sin.(2p-q)$$
$$+0{,}0095.\sin.(2p+q)$$
$$-0{,}4965.\sin.(q-2r) + 0{,}0289.\sin.(2p-q+2r)$$
$$+0{,}1805.\sin.(2p+q-2r)$$
$$+0{,}1251.\sin.(q+2r) - 0{,}0597.\sin.(2p-q-2r)$$
$$+0{,}0007.\sin.(2p+q+2r) \qquad (iiK)$$

$$Z = -0{,}2496.\sin.t + 0{,}0697.\sin.(2p-t)$$
$$-0{,}0247.\sin.(2p+t)$$
$$+0{,}0204.\sin(t-2r) + 0{,}0017.\sin.(2p-t+2r)$$
$$+0{,}1016.\sin.(2p+t-2r)$$
$$+0{,}0099.\sin.(t+2r) - 0{,}0755.\sin.(2p-t-2r)$$
$$-0{,}0004.\sin.(2p+t+2r) \qquad (ii\varkappa)$$

§. 553.

Subſtituendo hic etiam valores litterarum α et $\varkappa$, impetrabimus expreſſiones ſequentes:

Multiplic.

$$\alpha S = -0{,}0006163.\sin.p + 0{,}0000073.\sin.3p \qquad (1)$$

$$\alpha T = +0{,}004630.\sin.(p-q)$$
$$+0{,}000155.\sin.(p+q)$$
$$+0{,}000185.\sin.(3p-q)$$
$$-0{,}000002.\sin.(3p+q) \qquad (K)$$

Multiplic.

$$\varkappa U = +0{,}0031981.\sin t - 0{,}0007268.\sin(2p-t)$$
$$+0{,}0000928.\sin(2p+t)$$
$$-0{,}0000024.\sin(4p-t)$$
$$+0{,}0000001.\sin(4p+t) \qquad \text{(I)}$$

$$\varkappa V = +0{,}011507.\sin(q-t) - 0{,}002196.\sin(2p-q+t)$$
$$+0{,}000255.\sin(2p+q-t)$$
$$-0{,}007420.\sin(q+t) + 0{,}016918.\sin(2p-q-t)$$
$$-0{,}000071.\sin(2p+q+t) \text{(K)}$$

$$\varkappa W = +0{,}04418.\sin t - 0{,}00526.\sin(2p-t)$$
$$+0{,}00160.\sin(2p+t)$$
$$-0{,}00281.\sin(2q-t) \;\ldots\ldots\; \sin(2p-2q+t)$$
$$-0{,}00039.\sin(2p+2q-t)$$
$$+0{,}00562.\sin(2q+t) - 0{,}02893.\sin(2p-2q-t)$$
$$+0{,}00013.\sin(2p+2q+t) \text{ (K}^{\circ}\text{)}$$

$$q\varkappa w = -0{,}0000047.\sin(p-t) - 0{,}0000544.\sin(p+t)$$
$$-0{,}0000007.\sin(3p-t)$$
$$+0{,}0000001.\sin(3p+t) \qquad \text{(i)}$$

$$\varkappa Z = -0{,}00419.\sin t + 0{,}00117.\sin(2p-t)$$
$$-0{,}00041.\sin(2p+t)$$
$$+0{,}00034.\sin(t-2r) + 0{,}00003.\sin(2p-t+2r)$$
$$+0{,}00171.\sin(2p+t-2r)$$
$$+0{,}00017.\sin(t+2r) - 0{,}00127.\sin(2p-t-2r)$$
$$-0{,}00001.\sin(2p+t+2r). \qquad (ii)$$

§. 554.

§. 554.

Tertia autem coordinata z plerumque per alios angulos determinatur, quum enim posuissemus:

$$z = i\mathfrak{p} + iK\mathfrak{q} + iKK\mathfrak{r} + i\varkappa\mathfrak{s} + i^3\lambda\mathfrak{t} + ia\mathfrak{u}.$$

sequentes adepti sumus determinationes:

Multiplic.

$$\begin{aligned}\mathfrak{p} = \text{sin.}\, r &+ 0{,}036982.\ \text{sin.}\,(2p - r)\\ &+ 0{,}001513.\ \text{sin.}\,(2p + r)\\ &+ 0{,}000047.\ \text{sin.}\,(4p - r)\\ &+ 0{,}000006.\ \text{sin.}\,(4p + r) \qquad (i)\end{aligned}$$

$$\begin{aligned}\mathfrak{q} = &-1{,}48323.\text{sin.}(q-r) - 0{,}11149.\text{sin.}(2p-q+r)\\ &\qquad -0{,}01634.\text{sin}(2p+q-r)\\ &-0{,}50497.\text{sin.}(q+r) - 0{,}24129.\text{sin.}(2p-q-r)\\ &\qquad -0{,}00296.\text{sin.}(2p+q+r)\\ &-0{,}00063.\text{sin.}(4p-q+r) - 0{,}00364.\text{sin.}(4p-q-r)\\ &-0{,}00008.\text{sin.}(4p+q-r) - 0{,}00002.\text{sin.}(4p+q+r) \quad (iK)\end{aligned}$$

$$\begin{aligned}\mathfrak{r} = +0, \text{sin.}\, r &+ 0{,}0363.\ \text{sin.}\,(2p - r)\\ &+ 0{,}1589.\ \text{sin.}\,(2p + r)\\ +0{,}3425.\text{sin.}(2q-r) &+ 0{,}0498.\text{sin.}(2p-2q+r)\\ &+ 0{,}0125.\text{sin.}(2p+2q-r)\\ +0{,}3799.\text{sin.}(2q+r) &+ 0{,}1701.\text{sin.}(2p-2q-r)\\ &+ 0{,}0045.\text{sin.}(2p+2q+r) \quad (iKK)\end{aligned}$$

Multiplic.

$$\begin{aligned}\mathfrak{s} = -0{,}02010.\text{fin.}(r-t) &+ 0{,}03994\ \text{fin.}(2p-r+t)\\ &-0{,}00686.\text{fin.}(2p+r-t)\\ +0{,}01675.\text{fin.}(r+t) &- 0{,}10960.\text{fin.}(2p-r-t)\\ &+0{,}00090.\text{fin.}(2p+r+t) \qquad (ix)\end{aligned}$$

fiue

$$\begin{aligned}\kappa\mathfrak{s} = -0{,}000337.\text{fin.}(r-t) &+ 0{,}000670.\text{fin.}(2p-r+t)\\ &-0{,}000115.\text{fin.}(2p+r-t)\\ +0{,}000281.\text{fin.}(r+t) &- 0{,}001839\ \text{fin.}(2p-r-t)\\ &+0{,}000015.\text{fin.}(2p+r+t) \qquad (i)\end{aligned}$$

$$\begin{aligned}\mathfrak{t} = 0,\ \text{fin.}\ r &+ 0{,}0023.\ \text{fin.}\ (2p-r)\\ &-0{,}0007.\ \text{fin.}\ (2p+r)\\ &+0{,}0000.\ \text{fin.}\ (4p-r)\\ &+0{,}0000.\ \text{fin.}\ (4p+r)\\ +0{,}0004.\ \text{fin.}\ 3r &+ 0{,}0175.\ \text{fin.}\ (2p-3r)\\ &+0{,}0000.\ \text{fin.}\ (2p+3r)\\ &+0{,}0018.\ \text{fin.}\ (4p-3r)\\ &+0{,}0000.\ \text{fin.}\ (4p+3q) \qquad (i^{\mathfrak{r}})\end{aligned}$$

$$\begin{aligned}\mathfrak{u} = -0{,}1583.\text{fin.}(p-r) &- 0{,}0604.\ \text{fin.}(p+r)\\ &-0{,}0037.\ \text{fin.}(3p-r)\\ &-0{,}0000.\text{fin.}(3p+r) \qquad (ia)\end{aligned}$$

fiue

$$\begin{aligned}a\mathfrak{u} = -0{,}000405\ \text{fin.}(p-r) &- 0{,}000155.\text{fin.}(p+r)\\ &-0{,}000009.\text{fin.}(3p-r)\\ &-0{,}000000.\text{fin.}(3p+r) \qquad (i)\end{aligned}$$

§. 555.

§. 555.

Quo nunc has formulas commodius ad vſum noſtrum accommodare queamus, elementa initio conſtituta aliquantillum immutari conueniet. Ibi enim diſtantiam mediam telluris a Sole, vnitate deſignauimus, vnde ternae coordinatae pro motu Lunae fuerant $a(1+x)$, ay et az; nunc autem quoniam omnia in ſola ratione ſubſiſtunt, vnitate vtemur ad diſtantiam mediam Lunae a Terra deſignandam, ita vt iam $\frac{1}{a}$ exprimat mediam diſtantiam Terrae a Sole, ternae autem coordinatae pro loco Lunae futurae ſint: I°. $1+x$; II°. y et III°. z.

§. 556.

Quo haec clarius perſpiciantur, ſit pro tempore quopiam propoſito, ♁ centrum Terrae et recta ♁LM referat longitudinem mediam Lunae in ecliptica, arcu Mm ſignante ordinem ſignorum coeleſtium; tum vero centrum Lunae verſetur in ☽, vnde ad planum eclipticae demiſſo perpendiculo ☽l, hincque ad ♁M ducta normali lL, ita vt locus Lunae, his tribus coordinatis ♁L, Ll, et l☽ determinetur, atque nunc habebimus I°. ♁L $= 1+x$, II°. L$l = y$ et III°. l☽ $= z$, vbi x, y, z eas ipſas quantitates denotant, quas hactenus definiuimus. Ductis igitur rectis ♁l et ♁☽, illa ♁l exhibebit longitudinem Lunae veram, haec vero ♁☽ dabit verum locum Lunae in coelo, eiusque

que magnitudo veram distantiam Lunae a Terra cui parallaxis Lunae reciproce est proportionalis.

§. 557.

Quod si ergo ponamus angulum $M \male l = \Phi$, statim habemus Tang. $\Phi = \frac{y}{1+x}$, vnde ex cognitis x et y innotescet angulus Φ, qui additus longitudini mediae statim praebet longitudinem Lunae veram. Porro posito angulo $l \male ☽ = \psi$, hic ipsam Lunae latitudinem definit et quia distantia $\male l = \frac{1+x}{cos. \Phi}$, hinc colligimus Tang. $\psi = \frac{z\, cos. \Phi}{1+x}$, vnde patet, quam facili negotio ex cognitis ternis quantitatibus x, y et z, verus Lunae locus in coelo assignari possit, scilicet quaerantur duo anguli Φ et ψ, vt sit Tang. $\Phi = \frac{y}{1+x}$ et Tang. $\psi = \frac{z\, cos. \Phi}{1+x}$, atque si longitudo media Lunae fuerit $= \zeta$, statim innotescit longitudo vera $= \zeta + \Phi$ et latitudo $= \psi$.

§. 558.

Denique quum ipsa distantia Lunae a Terra sit $☽ \male = \frac{1+x}{cos. \Phi . cos. \psi}$, si vocemus parallaxin Lunae aequatoream, quae eius distantiae mediae $= 1$ conuenit, $= \pi$, pro tempore obseruationis, hinc statim colligitur vera parallaxis aequatorea $\frac{\pi\, cos. \Phi . cos. \psi}{1+x}$, ad quam quum diameter Lunae horizontalis datam teneat rationem ex ipsis obseruationibus cognitam, hinc expedite diameter Lunae apparens definiri poterit.

CAPVT II.

CAPVT II.

DESCRIPTIO TABVLARVM LVNARIVM AB INSIGNI GEOMETRA CLAIRALTIO EDITARVM.

§. 559.

Antequam nostram Theoriam cum ipsis obseruationibus comparare liceat, conueniet eam cum aliis Tabulis Lunaribus melioris notae conferri, quae scilicet a coelo quam minime aberrent. Atque hic quidem statim occurrebant celebratae Tabulae Mayerianae, quae a coelo nunquam vltra vnum minutum primum recedere, perhiberi solent; verum quia inter argumenta harum Tabularum, non tam media elongatio Lunae a Sole, quam vera inducitur, comparationem cum nostra Theoria non sine taedioso labore instituere liceret, quocirca hic Tabulas Clairaltianas praeferimus, in quibus perpetuo ipsa elongatio media Lunae a Sole vsurpatur, ita vt eius argumenta cum noſtris apprime conſentiant, verum etiam harum tabularum error tam exiguus deprehenditur, vt nunquam ad duo minuta prima aſſurgere videatur.

§. 560.

Ante omnia igitur huius Auctoris denominationes, ad noſtras reuocari oportet, quemadmodum ſubiecta Tabella declarat

Denominationes:

	Clairalti	Noſtrae
I. Elongatio media Lunae a Sole	t	p
II. Anomalia media Lunae	y	q
III. Argumentum latitud. medium	s	r
IV. Anomalia media Solis	z	t

atque hinc eius formulas, quibus ſuas Tabulas ſuperſtruxit facile per noſtra elementa exprimere licebit.

§. 561.

Ex his vero elementis iſte Auctor non ſtatim longitudinem Lunae veram, ſed potius eius locum in propria orbita definit, hoc modo, vt ſi longitudo Lunae media ponatur $= \zeta$ eius vero locus in orbita $= \theta$, differentia $\theta - \zeta$, per formulam ſequentem exprimatur:

$\theta -$

$$
\begin{array}{rlrl}
\theta-\zeta = & -1'.\,58'',3\,\sin.p & - & 6^{\circ}.\,17'.\,21'',0.\,\sin.q \\
& +39'.52'',9.\sin.2p & + & 12.\,57,\,8.\,\sin.2q \\
& +\;27'',0.\sin.4p & - & 37.\,1.\,\sin.3q \\
\hline
& +\;6,\,1.\sin.(p+q) & + & 11'.25'',0.\sin.t \\
& +18,2.\sin(2p+2q) & + & 23,3.\sin.(2p+t) \\
& +23,\,1.\sin.(p-q) & - & 2.\,39,6.\sin(2p-t) \\
& +3'.18,8.\sin.(2p-2q) & - & 1.\,48,2.\sin(q+t) \\
& -3.\,19,9\,\sin.(2p+q) & + & 2.\,23,7.\sin.(q-t) \\
& -1^{\circ}.16'.15,7.\sin(2p-q) & - & 11,0.\sin.(2q-t) \\
& +\;33,2.\sin.(4p-2q) & - & 26,8.\sin.(2p-q+t) \\
& -\;17,9.\sin.(2p-3q) & + & 20,3.\sin.(2p+q-t) \\
& -1'.\;8'',1.\sin.(4p-q) & + & 3'.\,18,3.\sin.(2p-q-t) \\
& & - & 11,8.\sin.(2p-2q-t) \\
\hline
& +1'.\,1,7.\sin.(2p-2r) & & \\
& -1.\,27,8.\sin.(q-2r) & &
\end{array}
$$

§. 562.

Perſpicuum autem eſt hanc formulam nondum conuenire cum angulo noſtro Φ, ſed illi inſuper adjici debere illam particulam, quae ab Aſtronomis vocari ſolet reductio ad eclipticam ita vt poſita hac reductione $=\varrho$, tum demum habeamus $\Phi=\theta-\zeta+\varrho$, verum hanc ipſam reductionem Celeb. Clairaut ſe-

 quen-

quenti modo definiuit. Primo scilicet quaerit angulum r', ita vt sit

$$r' = r + 2'.5''.\,\text{fin.}\,q - 10'.23''.\,\text{fin.}\,t + (\theta - \zeta)$$

quo inuento reductionem ad eclipticam ita exhibet

$$\begin{aligned}\rho = -6'.57'',0.\,\text{fin.}\,2r' - 21'',5.\,\text{fin.}\,2p - 1'',0.\,\text{fin.}\,2q \\ + 2'',1.\,\text{fin.}\,(2p+q) \\ - 2,6.\,\text{fin.}\,(2p-q) \\ - 0'',9.\,\text{fin.}\,(2p-t) - 1''.8.\,\text{fin.}\,(2p-2t) \\ - 0'',5.\,\text{fin.}\,(2p+t).\end{aligned}$$

§. 563.

Denique ope eiusdem anguli r', latitudinem Lunae ψ, sequenti modo determinat: postquam angulum s inuestigauit, ita vt sit fin. $s = 5°.8'.38''.\,\text{fin.}\,r'$, pro latitudine dat hanc formulam:

$$\begin{aligned}\psi = s + 8'.46''.\,\text{fin.}\,(r'+2p-2r) - 10'',1.\,\text{fin.}\,(r'-q) \\ - 21,8.\,\text{fin.}\,(r'-2q) \\ + 2,6.\,\text{fin.}\,(r'-3q) \\ - 4,2.\,\text{fin.}\,(r'+q) \\ + 17'',2.\,\text{fin.}\,(r'+2p-q-2r) \\ + 9,5.\,\text{fin.}\,(r'+2p+q-2r) \\ - 1,7.\,\text{fin.}\,(r'+2p-2q-2r) \\ - 23'',4.\,\text{fin.}\,(r'+2p-t-2r) \\ + 10,5.\,\text{fin.}\,(r'+2p+t-2r) \\ - 4,0.\,\text{fin.}\,(r'+2p+2t-2r).\end{aligned}$$

§. 564.

Parallaxin autem Lunae Celeb. Auctor sequenti modo determinare docet; definit autem Parallaxin Lunae horizontalem pro obseruatorio Parisino, quandoquidem facile est inde Parallaxin aequatoream cognoscere. Quantitatem scilicet huius Parallaxis exprimit sequenti formula:

$$\begin{aligned}
57'.3''.&+28'',1.\cos.2p-3'.5'',3.\cos q-34'',0.\cos.(2p-q)\\
&+0,3.\cos.4p+10,3.\cos.2q-0,4.\cos.(4p-2q)\\
&\qquad-10,6.\cos.3q-3,0.\cos.(2p+q)\\
&\qquad\qquad-0,7.\cos.(4p-q)\\
&\qquad\qquad-1,7.\cos.(2p-t)\\
&\qquad\qquad+1,2.\cos.(q-t)\\
&\qquad\qquad-0,9.\cos.(q+t)\\
&\qquad\qquad+1,6.\cos.(2p-q-t).
\end{aligned}$$

CAPVT III.

CONSIDERATIO INAEQVALITATVM LVNAE CIRCA CONIVNCTIONEM SOLIS, VBI $p = 0$.

§. 565.

Quum ergo hic sit $p = 0$, inueſtigemus valores noſtrarum trium coordinatarum ac pro prima quidem quaeramus valores ſingulorum ordinum:

	Multiplic.
$\mathfrak{O} = -0,0071501.$	(1)
$\mathfrak{P} = +1,184457.\ \text{coſ.}\ q$	(K)
$\mathfrak{Q} = -0,31798 + 0,33552.\ \text{coſ.}\ 2q$	(K^2)
$\mathfrak{R} = -0,4748.\ \text{coſ.}\ q - 0,1491.\ \text{coſ.}\ 3q$	(K^3)
$a\,\mathfrak{S} = +0,0002854.$	(1)
$a\,\mathfrak{T} = +0,000081.\ \text{coſ.}\ q$	(K)
$\varkappa\,\mathfrak{U} = +0,0003214.\ \text{coſ.}\ t$	(1)

$\varkappa\,\mathfrak{V} =$

Multiplic.

$$\varkappa\mathfrak{B} = -0,002302.\,\text{cof.}(q-t)$$
$$-0,005441.\,\text{cof.}(q+t) \qquad (K)$$

$$\varkappa\mathfrak{W} = -0,00706.\,\text{cof.}\,t - 0,01512.\,\text{cof.}(2q-t)$$
$$+0,00537.\,\text{cof.}(2q+t) \qquad (K^2)$$

$$a\varkappa\mathfrak{w} = +0,0000305.\,\text{cof.}\,t. \qquad (1)$$

$$\mathfrak{X} = -0,23089 + 0,23549.\,\text{cof.}\,2r \qquad (ii)$$

$$\mathfrak{Y} = -0,0783.\,\text{cof.}\,q - 0,1922.\,\text{cof.}(q-2r)$$
$$-0,1015.\,\text{cof.}(q+2r) \qquad (iiK)$$

$$\varkappa\mathfrak{Z} = -0,00045.\,\text{cof.}\,t + 0,00013.\,\text{cof.}(t-2r)$$
$$+0,00030.\,\text{cof.}(t+2r) \qquad (ii)$$

Hinc ergo prima noſtra quantitas x, ſequenti modo ſatis ſuccincte repraeſentari poterit:

$$x = -0,0068647 + 0,0003519.\,\text{cof.}\,t$$
$$+\left\{\begin{array}{r}1,184538.\text{cof.}q - 0,002302.\text{cof.}(q-t)\\ -0,005441.\text{cof.}(q+t)\end{array}\right\} \quad (K)$$
$$+\left\{\begin{array}{l}-0,31798+0,33522.\text{cof.}2q-0,00706.\text{cof.}t\\ -0,01512.\text{cof.}(2q-t)+0,00537.\text{cof.}(2q+t)\end{array}\right\} K^2$$
$$+(-0,4748.\,\text{cof.}\,q - 0,1491.\,\text{cof.}\,3q) \qquad (K^3)$$
$$+\left\{\begin{array}{l}-0,23089-0,00045.\text{cof.}t+0,23549.\text{cof.}2r\\ +0,00013.\text{cof.}(t-2r)+0,00030.\text{cof.}(t+2r)\end{array}\right\} (ii)$$
$$+\left\{\begin{array}{r}-0,0783.\text{cof.}q-0,1922.\text{cof.}(q-2r)\\ -0,1015.\text{cof.}(q+2r)\end{array}\right\} \quad (iiK)$$

§. 566.

§. 566.

Simili modo pro secunda coordinata y, euoluamus primo singulos ordines sequenti modo:

Multiplic.

$O = 0.$

$P = -1,603899.\ \text{fin.}\ q$ (K)

$Q = -0,06700.\ \text{fin.}\ 2q$ (K²)

$R = +0,9441.\ \text{fin.}\ q - 0,0785.\ \text{fin.}\ 3q$ (K²)

$aS = 0$

$aT = -0,004662.\ \text{fin.}\ q$ (K)

$\varkappa U = +0,0040202.\ \text{fin.}\ t$ (1)

$\varkappa V = +0,013958.\ \text{fin.}\ (q-t)$
$- 0,024409.\ \text{fin.}\ (q+t)$ (K)

$\varkappa W = +0,05104.\ \text{fin.}\ t - 0,00320.\ \text{fin.}\ (2q-t)$
$+ 0,03468\ \text{fin.}\ (2q+t)$ (K²)

$a\varkappa w = -0,0000489.\ \text{fin.}\ t$ (1)

$X = -0,28075.\ \text{fin.}\ 2r$ (ii)

$Y = -0,1808.\ \text{fin.}\ q - 0,3449.\ \text{fin.}\ (q-2r)$
$+ 0,1855.\ \text{fin.}\ (q+2r)$ (iiK)

$\varkappa Z = -0,00577.\ \text{fin.}\ t + 0,00202.\ \text{fin.}\ (t-2r)$
$+ 0,00143.\ \text{fin.}\ (t+2r)$ (ii)

atque

atque hinc consequimur:

Multiplic.

$$
\begin{aligned}
\mathfrak{x} = &+ 0,0039713.\,\sin.\,t \\
&+ \left\{\begin{array}{l} -1,608561.\sin.\,q + 0,013958.\sin.(q-t) \\ \qquad\qquad -0,024409\,\sin.(q+t) \end{array}\right\} \quad (K) \\
&+ \left\{\begin{array}{l} -0,06700.\sin.\,2q + 0,05104.\sin.\,t \\ -0,00320.\sin.(2q-t) + 0,03468.\sin.(2q+t) \end{array}\right\} (K^2) \\
&+ (+0,9441.\,\sin.\,q - 0,0735.\,\sin.\,3\,q) \quad (K^2) \\
&+ \left\{\begin{array}{l} -0,28075.\sin.\,2r - 0,00577.\sin.\,t \\ +0,00202.\sin.(t-2r) + 0,00143\,\sin.(t+2r) \end{array}\right\} (ii) \\
&+ \left\{\begin{array}{l} -0,1808\,\sin.\,q - 0,3449.\sin.(q-2r) \\ \qquad\qquad +0,1855.\sin.(q+2r) \end{array}\right\} (iiK)
\end{aligned}
$$

§. 567.

Pro tertia denique coordinata $\mathfrak{z}$ euolutis his ordinibus:

$$\mathfrak{p} = +0,964490.\,\sin.\,r. \qquad (i)$$

$$\mathfrak{q} = -1,38753\,\sin.(q-r) \\ -0,26302.\,\sin.(q+r) \qquad (iK)$$

$$\mathfrak{r} = +0,1226.\,\sin.\,r + 0,3052.\,\sin.(2q-r) \\ +0,2143.\,\sin.(2q+r) \quad (iKK)$$

$$\varkappa\,\delta = -0,001122.\,\sin.(r-t) \\ +0,002135.\,\sin.(r+t) \qquad (i)$$

Multiplic.

$$t = -0,0030.\ \text{fin.}\ r - 0,0189.\ \text{fin.}\ r \qquad (i^3)$$

$$a\,u = +0,000259.\ \text{fin.}\ r \qquad (i)$$

nanciſcimur hanc formulam

$$z = \left\{ \begin{array}{l} +0,964749.\ \text{fin.}\ r \\ -0,001122.\text{fin.}(r-t)+0,002135.\text{fin.}(r+t) \end{array} \right\} (i)$$

$$+\left(-1,38753.\text{fin.}(q-r)-0,26302.\text{fin.}(q+r)\right) \qquad (iK)$$

$$+\left\{ \begin{array}{l} +0,1226.\ \text{fin.}\ r+0,3052.\ \text{fin.}(2q-r) \\ \qquad\qquad +0,2143.\ \text{fin.}(2q+r) \end{array} \right\} (iKK)$$

$$+(-0,0030.\ \text{fin.}\ r - 0,0189.\ \text{fin.}\ 3\,r). \qquad (i^3)$$

§. 568.

Pro eadem igitur poſitione $p = 0$, euoluamus formulas a Celeb. Clairaut datas, ac prima quidem hoc modo contrahetur:

$$\begin{array}{lll} \phi - \zeta = -5^\circ.\ 3'.\ 34''.\ 1.\ \text{fin.}\ q & +14'.\ 27'',9.\ \text{fin.}\ t & \\ \qquad +9.\ 24.\ 0\ \text{fin.}\ 2q & -\ 5.\ \ 6,5.\ \text{fin.}(q+t) & \\ \qquad -19.\ 2.\ \text{fin.}\ 3q & +\ 3.\ 10,8.\ \text{fin.}(q-t) & \\ & \qquad +11,8.\ \text{fin.}(2q+t) & \\ & \qquad -11,0.\ \text{fin.}(2q-t) & \\ \qquad -1'.\ \ 1'',7.\ \text{fin.}\ 2\,r & & \\ \qquad -1.\ 27,8.\ \text{fin.}\ (q-2\,r). & & \end{array}$$

Deinde

Deinde capere iubet

$$r' = r + 2'.\,5''.\,\text{fin.}\,q - 10'.\,23''.\,\text{fin.}\,t + (\theta - \zeta),$$

hincque reductionem ita definiuit, vt fiat

$$\varrho = -6'.\,57'',\text{fin.}\,2\,r' + 4'',7.\,\text{fin.}\,q - 1'',0.\,\text{fin.}\,2\,q + 0'',4.\,\text{fin.}\,t + 1''.\,8.\,\text{fin.}\,2\,r$$

ita vt fit $\theta - \zeta + \varrho = \Phi$. Deinde pro Latitudine ψ postquam inuentus fuerit angulus s, ita vt fit fin. $s = 5^\circ.\,8'.\,38''.\text{fin.}\,r''$. feu potius fin. $s = +0{,}0897779.\text{fin.}\,r''$, formula Auctoris fit

$$\begin{aligned}\psi = s + 8'.\,46''.\,\text{fin.}\,(r' - 2\,r) &- 10''.\,1.\,\text{fin.}\,(r' - q)\\ &- 21.\,\ 8.\,\text{fin.}\,(r' - 2\,q)\\ &+ \ 2.\,\ 6.\,\text{fin.}\,(r' - 3\,q)\\ &- \ 4.\,\ 2.\,\text{fin.}\,(r' + q)\\ + 17'',2.\,\text{fin.}\,(r' - q - 2\,r)&\\ + \ 9,5.\,\text{fin.}\,(r' + q - 2\,r)&\\ - \ 1,7.\,\text{fin.}\,(r' - 2\,q - 2\,r)&\\ - 23'',4.\,\text{fin.}\,(r' - t - 2\,r)&\\ + 10,5.\,\text{fin.}\,(r' + t - 2\,r)&\\ - \ 4,0.\,\text{fin.}\,(r' + 2\,t - 2\,r).&\end{aligned}$$

Denique Parallaxis horizontalis Parifina euadet:

$$\begin{aligned}57'.\,3''. - 3'.\,43'',0.\,\text{cof.}\,q &- 1'',7.\,\text{cof}\,t\\ + \ 9,9.\,\text{cof.}\,2\,q &+ 1,2.\,\text{cof.}\,(q - t)\\ - 10,6.\,\text{cof.}\,3\,q &+ 0,7.\,\text{cof.}\,(q + t).\end{aligned}$$

§. 569.

Pro hac iam Lunae positione, duos casus principales statuamus ponentes pro altero $q = 0$, pro altero vero $q = 90°$, quos ita seorsim euoluamus, vt vtrique casus magis speciales, ac prae reliquis memorabiles subiiciamus.

I. Casus Principalis quo $p = 0$ et $q = 0$.

§. 570.

Secundum nostram ergo Theoriam, ternae coordinatae sequenti modo definientur:

$$x$$

$$\begin{aligned}
&- 0,0068647 + 0,0003519 \cdot \text{cos}\, t \\
&+ (+ 1,184538 - 0,007743 \cdot \text{cos}\, t) \;\ldots\; K \\
&+ (+ 0,01724 - 0,01681 \cdot \text{cos}\, t) \;\ldots\; K^2 \\
&\qquad - 0,6239 \;\ldots\ldots\; K^3 \\
&+ \left\{ \begin{array}{l} - 0,23089 - 0,00045 \cdot \text{cos}\, t \\ + 0,23549 \cdot \text{cos}\, 2r + 0,00013 \cdot \text{cos}\, (t - 2r) \\ \qquad + 0,00030 \cdot \text{cos}\, (t + 2r) \end{array} \right\} ii \\
&+ (- 0,0783 - 0,2937 \cdot \text{cos}\, 2r) \;\ldots\; iiK
\end{aligned}$$

y

$$
\begin{aligned}
&+0,0039713.\ \text{fin}.\ t\\
&-0,038367.\ \text{fin}.\ t\ \ldots\ K\\
&+0,\ \ldots\ldots\ \text{fin}.\ t\ \ldots\ K^2\\
&+0,00000.\ \ldots\ldots\ K^3\\
&+\left.\begin{cases}-0,28075.\ \text{fin}.\ 2r\\ -0,00577.\ \text{fin}.\ t\\ +0,00202.\ \text{fin}.\ (t-2r)\\ +0,00143.\ \text{fin}.\ (t+2r)\end{cases}\right\}ii\\
&+0,5304.\ \text{fin}.\ 2r\ \ldots\ iiK
\end{aligned}
$$

z

$$
\begin{aligned}
&+0,964749.\ \text{fin}.\ r\ \ldots\ i\\
&+1,12451.\ \text{fin}.\ r\ \ldots\ iK\\
&+0,0317.\ \text{fin}.\ r\ \ldots\ iKK\\
&\left.\begin{matrix}-0,001122.\ \text{fin}.\ (r-t)\\ +0,002135.\ \text{fin}.\ (r+t)\end{matrix}\right\}i\\
&\left.\begin{matrix}-0,0030.\ \text{fin}.\ r\\ -0,0189.\ \text{fin}.\ 3r\end{matrix}\right\}i^3
\end{aligned}
$$

Pro hoc autem casu formulae Clairaltianae in sequentes contrahentur:

$$\theta - \zeta = 6'.\ 33'',4.\ \text{fin}.\ t + 26'',1.\ \text{fin}.\ 2r$$

existente

$$r' = r - 3'.\ 49'',6.\ \text{fin}.\ t + 26'',1.\ \text{fin}.\ 2r,$$

vnde fit

$$\varsigma = -6'.\ 57'',0.\ \text{fin}.\ 2r'' + 0'',4.\ \text{fin}.\ t + 1'',8.\ \text{fin}.\ 2r.$$

Denique sumto sin. $s = +0,0897779 \cdot \sin. r''$ prodit Latitudo:

$$\psi = s + 9'. 11'', 0. \sin.(r' - 2r) - 33, 5. \sin. r'$$
$$- 23'', 4. \sin.(r' - t - 2r)$$
$$+ 10, \ 5. \sin.(r' + t - 2r)$$
$$- \ 4, \ 0. \sin.(r' + 2t - 2r)$$

et Parallaxis $= +53', 19'', 3 + 0'', 2 \cos. t$, hunc iam casum principalem in duos particulares subdiuidamus, ponendo vel $r = 0$, vel $= 90$.

I. Casus particularis, quo $p = 0$, $q = 0$ et $r = 0$.

Pro hoc casu nostrae formulae sequentes induent formas:

$$x$$
$$-0,0068647 + 0,0003519. \cos. t$$
$$+(1,184538 - 0,007743. \cos. t) \ . \ . \ K$$
$$+(+0,01724 - 0,01681. \cos. t) \ . \ . \ K^2$$
$$-0,6239. \ . \ . \ . \ K^3$$
$$(+0,00460 - 0,00002. \cos. t) \ . \ . \ ii$$
$$-0,3720. \ . \ . \ . \ iiK$$

$$y$$
$$+0,0039713. \sin. t$$
$$-0,038367. \sin. t \ . \ . \ . \ K$$
$$+0, \ . \ . \ \sin. t. \ . \ . \ K^2$$
$$+0,00000. \ . \ . \ . \ K^3$$
$$-0,00232. \sin. t. \ . \ . \ ii$$
$$+0,00000. \ . \ . \ . \ iiK$$

z

$+0,003257.\sin. t. \quad . \quad . \quad . \quad i.$

Clairaltianae vero formulae erunt:

$$\theta - \zeta = 6'. 33'', 4. \sin. t;$$

$$r' = -3, 49'', 6. \sin. t;$$

$$\varrho = -6'. 57''. 0. \sin. 2r' + 0'', 4 \sin. t$$

tum vero

$$\sin. s = +0,0897779. \sin. r'' \text{ et}$$

$$\psi = s + 8'. 37'', 5. \sin. r' - 23'', 4. \sin. (r' - t)$$
$$+ 10, 5. \sin. (r' + t)$$
$$- 4, 0. \sin. (r' + 2t)$$

et parallaxis $= +53'. 19'', 3 + 0'', 2 \cos. t$. Hinc igitur sequentes casus speciales consideremus.

§. 571.

I°. $p = 0$; $q = 0$; $r = 0$; $t = 0$.

Hoc ergo casu nostrae formulae erunt:

$$x = -0,0065128 + 1,176795. K + 0,00043. K^2$$
$$-0,6239. K^3 + 0,00458. ii - 0,3720. iiK.$$

$y = 0$; $z = 0$.

Clairaltianae vero ita erunt comparatae

$\theta - \zeta = 0$; $r' = 0$; $\varrho = 0$; $\psi = 0$ et parallaxis $= 53'. 19'', 5$, vnde patet circa locum Lunae perfectum esse consensum, pro parallaxi autem ob $\Phi = 0$ et et $\psi = 0$, nostra parallaxis media π, si etiam Parisios referatur, capi deberet $\pi = 53'.\ 19'',\ 5.\ (1+x)$.

§. 572.

II°. $p = 0;\ q = 0;\ r = 0;\ t = 180.$

Nostrae formulae erunt

$$x = -0,0072166 + 1,192281.\ K + 0,03405.\ K^2 - 0,6239.\ K^3 + 0,00462.\ ii - 0,3720.\ iiK$$

$y = 0,\ z = 0.$

Clairaltianae vero ita se habebunt:

$$\theta - \zeta = 0,\ r' = 0,\ \varrho = 0,\ \psi = 0$$

et parallaxis $53'.\ 19''.\ 1$, vnde aeque parum super consensu concludere licet, atque ex casu praecedente.

II. Casus particularis $p = 0,\ q = 0,\ r = 90°.$

§. 573.

Hoc casu nostrae formulae in sequentes abibunt:

$$\begin{aligned} x \\ = & -0,0068647 + 0,0003519.\ \text{cos}.\ t \\ & + (1,184538 - 0,007743.\ \text{cos}.\ t)\ .\ .\ K \\ & + (0,01724 - 0,01681.\ \text{cos}.\ t)\ .\ .\ K^2 \\ & \qquad - 0,6239\ .\ .\ .\ K^3 \\ & (-0,46638 - 0,00088.\ \text{cos}.\ t)\ .\ .\ ii \\ & \qquad + 0,2154\ .\ .\ .\ iiK \end{aligned}$$

y

y

$+0,0039713.\ \text{fin.}\ t$

$-0,038367.\ \text{fin.}\ t.\ \ldots\ K$

$+0,\ \ .\ \ .\ \ \text{fin.}\ t.\ \ldots\ K^2$

$+0,0000.\ \ldots\ K^2$

$-0,00922.\ \text{fin.}\ t.\ \ldots\ ii$

z

$\left.\begin{array}{l}+0,964749.\\ +0,001013.\ \text{cof.}\ t\end{array}\right\}\ldots i$

$+1,12451.\ \ldots\ iK$

$+0,0317.\ \ldots\ iKK$

$+0,0159.\ \ldots\ i^2$

Formulae autem Clairaltianae euadent:

$\theta - \zeta = 6'.\ 33'',4.\ \text{fin.}\ t;$

$r' = 90° - 3'.\ 49'',6.\ \text{fin.}\ t;$

$\varrho = -6'.\ 57'',0.\ \text{fin.}\ 2\,r' + 0'',4.\ \text{fin.}\ t;$

$\psi = s - 9'.\ 44'',5.\ \text{fin.}\ r' + 23'',4.\ \text{fin.}\ (r' - t)$
$\qquad - 10'',5.\ \text{fin.}\ (r' + t)$
$\qquad + 4,0.\ \text{fin.}\ (r' + 2\,t)$

et parallaxis $= 53'.\ 19'',3 + 0'',2.\ \text{cof.}\ t$, vnde sequentes duos casus speciales accuratius perpendamus:

§. 574.

I°. $p = 0$, $q = 0$, $r = 90$, $t = 0$.

Noſtrae igitur formulae ita ſe habebunt:

$$x = -0,0065128 + 1,176795.K + 0,00043.K^2 - 0,6239.K^3 - 0,46726.ii + 0,2154.iiK$$

$$y = 0.$$

$$z = +0,965762.i + 1,12451.iK + 0,0317.iKK + 0,0159.i^3.$$

Clairaltianae vero formulae hic dabunt:

$\theta - \zeta = 0$; $r' = 90°$, vnde colligimus $\varrho = 0$; tum vero reperitur $s = 5°.9'.3''$ ex quo conficitur Latitudo $\psi = 5°.9'.3'' - 9'.27'',6 = 4°.59'.35''4$. ac denique parallaxis $= 53'.19'',5$.

§. 575.

Quoniam $\Phi = 0$ et $y = 0$, primae aequationi Tang. $\Phi = \frac{y}{1+x}$ ſponte ſatisfit, altera vero,

$$\text{Tang.}\ \psi = \frac{z\ \text{coſ.}\ \Phi}{1+x},\ \text{ſiue}\ z - (1+x)\ \text{Tang.}\ \psi = 0,$$

nobis ſuppeditat hanc aequationem

$$+0,965762.i + 1,12451.iK + 0,0317.iKK + 0,0159.i^3 = +0,0867975 + 0,102813.K + 0,00004.K^2 - 0,0545.K^3 - 0,64082.ii + 0,0188.iiK.$$

De

De Parallaxi hic non adeo ſumus ſolliciti, quoniam noſtra Theoria eam multo certiorem exhibere debet, ſi modo elementa rite fuerint conſtituta.

§. 576.

II°. $p = 0$, $q = 0$, $r = 90$, $t = 180°$.

Pro hoc ergo caſu noſtrae formulae erunt:

$$x = -0,0072166 + 1,192281.\,K + 0,03405.\,K^2 - 0,6239.\,K^3 - 0,46550.\,ii + 0,2154.\,iiK$$

$$y = 0.$$

$$z = +0,963736.\,i + 1,12451.\,iK + 0,0317.\,iKK + 0,0159.\,i^3.$$

Formulae autem Clairaltianae praebent

$$\theta - \zeta = 0;\ r' = 90°;\ \varrho = 0;$$

ex quo reperitur

$s = 5°.\,9'.\,3''.$ et Latitudo

$$\psi = 5°.\,9'.\,3'' - 9'.\,53'',4 = 4°.\,59',9''.\,9$$

et parallaxis $= 53'.\,19''.\,1.$

§. 577.

Quum hic quoque ſit tam $y = 0$, quam $\Phi = 0$, altera aequatio $z = (1 + x)$ Tang. ψ erit:

$$+0,963736.\,i + 1,12451.\,iK + 0,0317.\,iKK + 0,0159.\,i^3 = +0,0866148 + 0,104019.\,K + 0,00297.\,K^2 - 0,0544.\,K^3 - 0,04061.\,ii + 0,0188.\,iiK.$$

 II. Ca-

II. Casus Principalis quo $p = 0$ et $q = 90°$.

§. 578.

Secundum nostram Theoriam, ternae coordinatae sequenti modo definientur:

$$x$$

$$-0,0068647 + 0,0003519.\,\text{cos.}\,t$$

$$+0,003139.\,\text{sin.}\,t \;\ldots\; K$$

$$(-0,65320 + 0,00269.\,\text{cos.}\,t) \ldots K^2$$

$$0, \;\ldots\; K^3$$

$$\left\{\begin{array}{l} -0,23089 + 0,23549.\,\text{cos.}\,2r \\ -0,00045.\,\text{cos.}\,t \\ +0,00013.\,\text{cos.}\,(t-2r) \\ +0,00030.\,\text{cos.}\,(t+2r) \end{array}\right\} ii$$

$$-0,0907.\,\text{sin.}\,2r \;\ldots\; iiK$$

$$y$$

$$+0,0039713.\,\text{sin.}\,t$$

$$(-1,608561 - 0,010451.\,\text{cos.}\,t) \ldots K$$

$$\ldots\ldots\; \text{sin.}\,t. \;\ldots\; K^2$$

$$+1,0176. \;\ldots\; K^3$$

$$\left\{\begin{array}{l} -0,28075.\,\text{sin.}\,2r - 0,00577.\,\text{sin.}\,t \\ +0,00202.\,\text{sin.}\,(t-2r) \\ +0,00143.\,\text{sin.}\,(t+2r) \end{array}\right\} ii$$

$$(-0,1808 - 0,1594.\,\text{cos.}\,2r) \ldots iiK$$

z

$$z$$

$$\left.\begin{array}{l} +0{,}964749.\ \text{sin.}\ r \\ -0{,}001122.\ \text{sin.}\ (r-t) \\ +0{,}002135.\ \text{sin.}\ (r+t) \end{array}\right\}\ i$$

$$-1{,}65055.\ \text{cos.}\ r.\ \ .\ \ .\ \ iK$$

$$+0{,}2135.\ \ .\ \ .\ \ .\ \ iKK$$

$$(-0{,}0030.\ \text{sin.}\ r - 0{,}0189.\ \text{sin.}\ 3\,r)\ .\ .\ i^{2}$$

§. 579.

Pro hoc autem casu formulae Clairaltianae in sequentes contrahentur:

$$\begin{aligned} \theta - \zeta = -5^{\circ}.\ 3'.\ 14'',9 + 14'.\ 5'',1.\ &\text{sin.}\ t \\ -1'.\ 55'',7.\ &\text{cos.}\ t \\ -1'.\ 1'',7.\ &\text{sin}\ 2\,r \\ -1'.\ 27'',8.\ &\text{cos.}\ 2\,r \end{aligned}$$

Deinde capiatur

$$r' = r + 2'.\ 5'' - 10'.\ 23''.\ \text{sin.}\ t + (\theta - \zeta),$$

hinc fit

$$\begin{aligned} \varrho = -6'.\ 57'',0.\ \text{sin.}\ 2\,r' + 4'',7. + 0'',4.\ &\text{sin.}\ t \\ + 1'',8.\ &\text{sin.}\ 2\,r. \end{aligned}$$

Deinde sumto

$$\text{sin.}\ s = 0{,}0897779.\ \text{sin.}\ r',$$

fit Latitudo

$$\psi = s + 8'. 47'', 7. \sin.(r' - 2r) + 8'', 5. \cos. r' + 21''. 8. \sin. r'$$
$$- 7''. 7. \cos.(r' - 2r)$$
$$- 23, 4. \sin.(r' - t - 2r)$$
$$+ 10. 5. \sin.(r' + t - 2r)$$
$$- 4, 0. \sin.(r' + 2t - 2r)$$

et parallaxis

$$= 56'. 53'', 1 - 1''. 7. \cos. t + 0, 5. \sin. t.$$

I. Casus particularis $p = 0$, $q = 90$, $r = 0$.

Hoc igitur casu nostrae formulae euadunt:

$$x$$
$$- 0, 0068647 + 0, 0003519. \cos. t$$
$$+ 0, 003139. \sin. t. \quad . \quad . \quad K$$
$$(- 0, 65320 + 0, 00269. \cos. t) \quad . \quad . \quad K^2$$
$$+ 0, \quad . \quad . \quad K^3$$
$$\left.\begin{matrix} + 0, 00460. \\ + 0, 00002. \cos. t. \end{matrix}\right\} ii$$

$$y$$
$$+ 0, 0030713. \sin. t$$
$$(+ 1, 608561 - 0, 010451. \cos. t) K$$
$$. \quad . \quad . \quad . \quad \sin. t. \quad . \quad . \quad . \quad K^2$$
$$+ 1, 0176. \quad . \quad . \quad . \quad K^3$$
$$- 0, 00232. \sin. t \quad . \quad . \quad . \quad ii$$
$$- 0, 3402. \quad . \quad . \quad . \quad iiK$$

$+0,083257.\ \text{fin.}\ t \quad . \quad . \quad . \quad . \quad i$
$-1,65055 \quad . \quad . \quad . \quad . \quad iK$
$+0,2135. \quad . \quad . \quad . \quad . \quad iKK$
$+0, \quad . \quad . \quad . \quad . \quad i^3$

Formulae autem Clairaltianae erunt:

$$\theta - \zeta = -5^\circ.\ 4'.\ 42'',7 + 14'.\ 5'',1.\ \text{fin.}\ t$$
$$-1'.\ 55'',7.\ \text{cof.}\ t$$

vnde

$$r' = -5^\circ.\ 2'.\ 37''.\ 7 + 3'.\ 42'',1.\ \text{fin.}\ t$$
$$-1'.\ 55'',7.\ \text{cof.}\ t$$

hinc autem

$$\varrho = -6'\ 57'',0.\ \text{fin.}\ 2\ r' + 4'',7 + 0'',4.\ \text{fin.}\ t.$$

Tum vero posito

$$\text{fin.}\ s = +0,0897779.\ \text{fin.}\ 2\ r'$$

prodit Latitudo

$$\psi = s + 9'.\ 9'',5.\ \text{fin.}\ r' + 0'',8.\ \text{cof.}\ r'$$
$$-23'',4.\ \text{fin.}\ (r' - t)$$
$$+10,\ 5.\ \text{fin.}\ (r' + t)$$
$$-4,\ 0.\ \text{fin.}\ (r' + 2\ t)$$

et parallaxis $56'.\ 53'',1 - 1''.\ 7.\ \text{cof.}\ t + 0,5.\ \text{fin.}\ t.$

§. 580.

P. $p = 0,\ q = 90,\ r = 0,\ t = 0.$

Hic nostrae formulae fiunt:

$$x = -0,0065128 + 0,000.\ K - 0,65051.\ K^2 -$$
$$+0,K^3 + 0,00458.\ ii - 0,iiK$$

$y =$

$y = + 0, \ldots - 1,619012. K \ldots + 0. K^2$
$+ 1,0176. K^3 - 0,3402. iiK$
$z = - 1,65055. iK + 0,2135. iKK + 0. i^3$

formulae autem Clairaltianae . .

$\theta - \zeta = - 5^\circ. 6'. 38'',4$; $r' = - 5^\circ. 4'. 33'',4$,

hinc

$\varrho = + 6'. 57'',0. \sin. (10^\circ. 9'. 7'') + 4'',7 = + 1'. 18''.2.$

Porro colligitur $s = - 27'. 18'',4$. ideoque Latitudo $\psi = - 28'. 5'',4$, denique Parallaxis $= 56'.51'',4$.

§. 581.

Hoc ergo casu fit noster angulus

$\Phi = \theta - \zeta + \varrho = - 5^\circ. 5'. 20'',2$,

quare quum esse debeat $(1 + x)$ Tang. $\Phi = y$, hinc deducimus istam aequationem:

$$- 1,619012. K + 1,0176. K^3 - 0,3402. iiK$$
$$= + 0,0884731 + 0,57930. K^2 - 0,00041. ii,$$

siue

$$+ 0,0884731 - 1,619012. K - 0,057930. K^2$$
$$+ 1,0176. K^3 + 0,00041. ii - 0,3402. ii K = 0.$$

§. 582.

Porro vero altera aequatio $z = (1 + x)$ Tang. ψ. Sec. Φ; valoribus substitutis suppeditat:

$$- 0,0081501 + 0,00534. K^2 - 0,00004. ii =$$
$$- 1,65055. iK + 0,2135. iKK.$$

§. 583.

§. 583.

II°. $p = 0$, $q = 90$, $r = 0$, $t = 180°$.

Hoc casu nostrae formulae erunt:

$$x = -0,0072166 - 0,65589.K^2 + 0,00462.ii$$
$$y = -1,598110.K + 1,0176.K^3 - 0,3416.iiK$$
$$z = -1,65055.iK + 0,2135.iKK$$

formulae autem Clairaltianae

$\theta - \zeta = -5°.2'.47'',0$, $r' = -5°.0'.42'',0$. hinc $\varrho = +1'.17'',3$. atque $s = -26',57''.7$. et Latitudo $\psi = -27'.45''.6$, tandem parallaxis $56'.54'',8$.

§. 584.

Hinc ergo nanciscimur

$$\Phi = \theta - \zeta + \varrho = -5°.1'.29''.7,$$

hinc aequatio $y = (1 + x)$ Tang. Φ fiet

$$-0,0872924 + 0,057670.K^2 - 0,00041.ii =$$
$$-1,598110.K + 1,0176.K^3 - 0,3402.iiK$$

siue

$$+0,0872924 - 1,598110.K - 0,057670.K^2$$
$$+1,0176.K^3 + 0,00041.ii - 0,3402.iiK = 0.$$

Altera vero aequatio $z = (1 + x)$ Tang. ψ. Sec. Φ, valoribus substitutis dat

$$-0,0080478 + 0,00532.K^2 - 0,00004.ii =$$
$$-1,65055.iK + 0,2135.iKK.$$

II. Ca-

§. 585.

II°. Casus particularis $p = 0$, $q = 90$, $r = 90$.

Pro hoc casu nostrae formulae erunt:

x

$$
\begin{aligned}
& -0,0068647 + 0,0003519.\cos.t \\
& \qquad + 0,003139.\sin.t \ldots K \\
& (-0,65320 + 0,00269.\cos.t) \ldots K^2 \\
& \quad + 0. \ldots K^3 \\
& (-0,46638 - 0,000088.\cos.t) \ldots ii
\end{aligned}
$$

y

$$
\begin{aligned}
& + 0,0039713.\sin.t \\
& (-1,608561 - 0,010451.\cos.t) \ldots K \\
& \quad \ldots \sin.t \ldots K^2 \\
& \quad + 1,0176 \ldots K^3 \\
& \quad - 0,00922.\sin.t \ldots ii \\
& \quad - 0,0214 \ldots iiK
\end{aligned}
$$

z

$$
\begin{aligned}
& \left.\begin{aligned} & + 0,964749. \\ & + 0,001013.\cos.t \end{aligned}\right\} i \\
& + 0,2135 \ldots iKK \\
& + 0,0159 \ldots i^3.
\end{aligned}
$$

§. 586.

§. 586.

Pro hoc casu formulae Clairaltianae in sequentes contrahuntur:

$$\theta - \zeta = -5^{\circ}.\ 1'.\ 47'',1 + 14'.\ 5'',1.\ \text{sin.}\ t$$
$$- 1'.\ 55'',7.\ \text{cos}\ t,$$

$$r' = 85^{\circ}.\ 0'.\ 17'',9 + 3'.\ 42'',1.\ \text{sin.}\ t$$
$$- 1.\ 55'',7.\ \text{cos.}\ t.$$

Porro fit

$$\varrho = -6'.\ 57'',0.\ \text{sin.}\ 2\ r' + 4'',7 + 0'',4.\ \text{sin.}\ t$$

atque Latitudo

$$\psi = s - 8'.\ 25'',9.\ \text{sin.}\ r' + 16'',2.\ \text{cos.}\ r'$$
$$+ 23''.\ 4.\ \text{sin.}\ (r' - t)$$
$$- 10.\ 5.\ \text{sin.}\ (r' + t)$$
$$+ 4.\ 0.\ \text{sin.}\ (r' + 2\ t)$$

et parallaxis

$$56'.\ 53''.\ 1 - 1,7\ \text{cos.}\ t + 0,5.\ \text{sin.}\ t.$$

§. 587.

I°. $p = 0;\ q = 90;\ r = 90;\ t = 0.$

Hic nostrae formulae fiunt:

$$x = -0,0065128 - 0,65051.\ K^2 - 0,46726.\ ii$$
$$y = -1,619012.\ K + 1,0176.\ K^3 - 0,0214.\ iiK$$
$$z = +0,965762.\ i + 0,2135.\ iKK + 0,0159.\ i^3$$

formulae autem Clairaltianae

$$\theta - \zeta = -5^\circ.\ 3'.\ 42'',8;\ r' = 84^\circ.\ 58'.\ 22'',2.$$

porro fit

$$\varrho = -6'.\ 57''.\ 0.\ \text{fin.}\ 2r' + 4'',7 = -1'.\ 8'',1.$$

atque ob

$$\text{fin.}\ s = +0,0897779.\ \text{fin.}\ r' = \text{fin.}\ (5^\circ.\ 7'.\ 51'',4),$$

Latitudo $\psi = 4^\circ.\ 59'.\ 45'',7$ et parallaxis $56'.\ 51'',4$.

§. 588.

Quum igitur fit

$$\Phi = \theta - \zeta + \varrho = -5^\circ.\ 4'.\ 50'',9,$$

aequatio nostra prior $(1 + x)$ Tang. $\Phi = y$ datit:

$$-0,0883310 + 0,057837.\ K^2 + 0,04154.\ ii =$$
$$-1,619012.\ K + 1,0176\ K^3 - 0,0214.\ iiK,\ \text{fiue}$$

$$+0,0883310 - 1,619012.\ K - 0,057837.\ K^2$$
$$+ 1,0176.\ K^3 - 0,04154.\ ii - 0,0214\ iiK$$

$= 0$. Altera vero aequatio $(1 + x)$ Tang. ψ Sec. Φ $= z$ dat

$$+0,0871943 - 0,057092.\ K^2 - 0,04101.\ ii =$$
$$+0,965762.\ i + 0,2135.\ iKK + 0,0159.\ i^3,\ \text{fiue}$$

$$+0,0871943 - 0,057092.\ K^2 - 0,965762.\ i$$
$$-0,2135.\ iKK - 0,04101.\ ii - 0,0159.\ i^3 = 0.$$

§. 589.

§. 589.

II°. $p = 0$, $q = 90$, $r = 90$, $t = 180°$.

Nostrae formulae hoc casu fiunt:

$$x = -0,0072166 - 0,65589\, K^2 - 0,46550.\, ii$$
$$y = -1,598110.\, K + 1,0176.\, K^3 - 0,0214.\, iiK$$
$$z = +0,963736.\, iK + 0,2135.\, iKK$$
$$+ 0,0159.\, i^3$$

formulae autem Clairaltianae:

$\theta - \zeta = -4°.\, 59'.\, 51'', 4$;

$r' = 85°.\, 2'.\, 13'', 6$, hinc

$\varrho = -6', 57''.\ \text{fin.}\ 2r' + 4'', 7 = -1'.\, 7'', 2.$

atque ob fin. $s =$ fin. $5°.\, 7'.\, 53''$, Latit. $\psi = 4°.\, 59'.\, 20'', 2$. et parallaxis $56', 54''.\, 8$.

§. 590.

Quum proinde fit $\Phi = \theta - \zeta + \varrho = -5°.\, 0'.\, 58'', 6$ aequatio nostra prior $(1 + x)$ Tang. $\Phi = y$ fiet:

$$-0,0871415 + 0,057571.\, K^2 + 0,04086.\, ii =$$
$$-1,598110.\, K + 1,0176.\, K^3 - 0,0214.\, iiK.$$

Altera vero aequatio $(1 + x)$ Tang. ψ Sec. $\Phi = z$ dat

$$+0,0869994 - 0,057476.\, K^2 - 0,04079.\, ii =$$
$$+0,963736 + 0,2135.\, iKK + 0,0159.\, i^3.$$

CAPVT IV.

CONSIDERATIO INAEQVALITATVM LVNAE CIRCA QVADRATVRAM MEDIAM VBI $p = 90°$.

§. 591.

Quum igitur hic sit $p = 90°$, primo quaeramus valores singulorum ordinum pro prima coordinata x, qui ita se habebunt:

	Multiplic.
$\mathfrak{O} = + 0,0072101.$	(1)
$\mathfrak{P} = + 0,814473. \cos. q$	(K)
$\mathfrak{Q} = - 0,75604 + 0,72946. \cos. 2q$	(K^2)
$\mathfrak{R} = + 0,3668. \cos. q - 0,6601. \cos. 3q$	(K^3)
$a\mathfrak{S} = + 0,$	(1)
$a\mathfrak{T} = - 0,000491. \sin. q$	(K)
$\varkappa\mathfrak{U} = - 0,0005494. \cos. t$	(1)

$\varkappa\mathfrak{V} =$

Multiplic.

$$\kappa\mathfrak{V} = -0,003800.\ \text{cof.}\,(q-t) + 0,008269.\ \text{cof.}\,(q+t) \qquad (K)$$

$$\kappa\mathfrak{W} = +0,01136.\ \text{cof.}\,t + 0,00024.\ \text{cof.}\,(2q-t) - 0,00321.\ \text{cof.}\,(2q+t) \qquad (K^2)$$

$$a\kappa\mathfrak{w} = -0,0000207.\ \text{fin.}\,t. \qquad (1)$$

$$\mathfrak{X} = -0,26945 + 0,25907.\ \text{cof.}\,2r \qquad (ii)$$

$$\mathfrak{Y} = +0,0783.\ \text{cof.}\,q + 0,0702.\ \text{cof.}\,(q-2r) - 0,1513.\ \text{cof.}\,(q+2r) \qquad (iiK)$$

$$\kappa\mathfrak{Z} = +0,00077.\ \text{cof.}\,t + 0,00051.\ \text{cof.}\,(t-2r) - 0,00066.\ \text{cof.}\,(t+2r) \qquad (ii)$$

Hinc ergo prima coordinata x, fequenti modo fatis fuccincte repraefentari poterit:

$$x = \left\{ \begin{array}{l} +0,0072101 - 0,0005494.\ \text{cof.}\,t \\ \qquad - 0,0000207.\ \text{fin.}\,t \end{array} \right\} (1)$$

$$\left\{ \begin{array}{l} +0,814473.\ \text{cof.}\,q - 0,000491.\ \text{fin.}\,q \\ -0,003800.\ \text{cof.}\,(q-t) + 0,008269.\ \text{cof.}\,(q+t) \end{array} \right\} (K)$$

$$\left\{ \begin{array}{l} -0,75604 + 0,72946.\ \text{cof.}\,2q + 0,01136.\ \text{cof.}\,t \\ +0,00024.\ \text{cof.}\,(2q-t) - 0,00321.\ \text{cof.}\,(2q+t) \end{array} \right\} (K^2)$$

$$(+0,3668.\ \text{cof.}\,q - 0,6601.\ \text{cof.}\,3q) \qquad (K^3)$$

$$\left\{ \begin{array}{l} -0,26945 + 0,25907.\ \text{cof.}\,2r + 0,00077.\ \text{cof.}\,t \\ +0,00051.\ \text{cof.}\,(t-2r) - 0,00066.\ \text{cof.}\,(t+2r) \end{array} \right\} (ii)$$

$$\left\{ \begin{array}{l} +0,0783.\ \text{cof.}\,q + 0,0702.\ \text{cof.}\,(q-2r) \\ \qquad - 0,1513.\ \text{cof.}\,(q+2r) \end{array} \right\} (iiK)$$

§. 592.

§. 592.

Simili modo pro altera coordinata y, formulae singulorum ordinum ita prodeunt:

Multiplic.

$O = 0.$

$P = -2,419969. \sin. q$ (K)

$Q = +0,54762. \sin 2q$ (K²)

$R = +1,8657. \sin. q - 0,5035. \sin. 3q$ (K³)

$a S = -0,0006236.$ (1)

$a T = +0,004602. \cos. q$ (K)

$\varkappa U = +0,0023810. \sin. t$ (1)

$\varkappa V = +0,009056. \sin. (q - t)$
$\quad + 0,009569. \sin. (q + t)$ (K)

$\varkappa W = +0,03732. \sin. t - 0,00000. \sin. (2q - t)$
$\quad + 0,02344. \sin. (2q + t)$ (K²)

$a \varkappa w = -0,0000585. \cos. t$ (1)

$X = -0,21187. \sin. 2r$ (*ii*)

$Y = +0,1836. \sin. q - 0,6481. \sin. (q - 2r)$
$\quad + 0,0647. \sin. (q + 2r)$ (*ii*K)

$\varkappa Z = -0,00261. \sin t - 0,00134. \sin. (t - 2r)$
$\quad - 0,00111. \sin. (t + 2r)$ (*ii*)

Quae

Quae formulae in sequentem formam contrahuntur:

Multiplic.

$$y = -0,0006236 - 0,0000585.\cos. t + 0,0023810.\sin. t$$

$$\left\{\begin{matrix} -2,419969.\sin. q + 0,004602.\cos. q \\ +0,009056.\sin.(q-t) + 0,009569\ \sin.(q+t) \end{matrix}\right\} \quad (K)$$

$$\left\{\begin{matrix} +0,54762.\sin. 2q + 0,03732.\sin. t \\ - \ldots .\sin.(2q-t) + 0,02344.\sin.(2q+t) \end{matrix}\right\} \quad (K^2)$$

$$(+1,8657.\sin. q - 0,5035.\sin. 3q) \quad (K^3)$$

$$\left\{\begin{matrix} -0,21187.\sin. 2r - 0,00261.\sin. t \\ -0,00134.\sin.(t-2r) - 0,00111.\sin.(t+2r) \end{matrix}\right\} \quad (ii)$$

$$\left\{\begin{matrix} +0,1836.\sin. q - 0,6481.\sin.(q-2r) \\ +0,0647.\sin.(q+2r) \end{matrix}\right\} \quad (iiK)$$

§. 593.

Tandem singuli ordines pro tertia coordinata z euoluti dant:

$$\mathfrak{p} = +1,035428.\sin. r. \quad (i)$$

$$\mathfrak{q} = -1,57783\ \sin.(q-r) - 0,73968.\sin.(q+r) \quad (iK)$$

$$\mathfrak{r} = -0,1226.\sin. r + 0,3798.\sin.(2q-r) + 0,5455.\sin.(2q+r) \quad (iKK)$$

$$\kappa\vartheta = +0,000448.\sin.(r-t) - 0,001573.\sin.(r+t) \quad (i)$$

L l l l t =

Multiplic.

$t = +0,0030 . \sin. r + 0,0161 . \sin. 3r$ $\quad (i^{3})$

$a u = -0,000551 . \cos. r$ $\quad (\iota)$

vnde colligimus hanc formam

$$
\begin{aligned}
z = &+1,035428 . \sin. r - 0,000551 . \cos. r \\
&+0,000448 . \sin. (r-t) - 0,001573 . \sin (r+t) \quad (i) \\
&-1,57783 . \sin. (q-r) - 0,73968 . \sin. (q+r) \quad (iK) \\
&-0,1226 . \sin. r + 0,3798 . \sin. (2q-r) \\
&\qquad + 0,5455 . \sin. (2q+r) \quad (iKK) \\
&+0,0030 . \sin. r + 0,0161 . \sin. 3r \quad (i^{3})
\end{aligned}
$$

§. 594.

His nostris formulis euolutis, videamus quomodo formulae Clairaltianae sint prodituræ, ac primo quidem pro hoc casu:

$$
\begin{aligned}
\phi - \zeta = &- 1'.\ 58'',3 - 7^{\circ}.\ 29'.\ 8'',7 . \sin. q + 29''.2 . \cos. q \\
&+ 15.\ 25.\ 2 . \sin. 2q \\
&- 55.\ 0 . \sin. 3q \\
&+ 8'.\ 22'',1 . \sin. t \qquad + 1'.\ 1'',7 . \sin. 2r \\
&+ 1.\ 30,\ 1 . \sin. (q+t) \qquad - 1.\ 27,\ 8 . \sin. (q-2r) \\
&+ 1.\ 36,\ 6 . \sin. (q-t) \\
&- 11,\ 8 . \sin. (2q+t) \\
&- 11,\ 0 . \sin. (2q-t)
\end{aligned}
$$

tum

tum vero capiatur

$$r' = r + 2'.\ 5''.\ \text{fin.}\ q - 10'.\ 23''.\ \text{fin.}\ t + (\theta - \zeta),$$

fequitur reductio ad eclipticam

$$\varrho = -6'.\ 57''.\ \text{fin.}\ 2\,r' - 4'',7.\ \text{fin.}\ q - 1'',0.\ \text{fin.}\ 2\,q$$
$$-\ 0'',4.\ \text{fin.}\ t - 1''.\ 8.\ \text{fin.}\ 2\,r$$

Deinde fumto angulo s ita, vt fit

$$\text{fin.}\ s = +\ 0,0897779.\ \text{fin.}\ r'$$

prodit ipfa Latitudo

$$\begin{aligned}\psi = s - 8'.\ 46''.\ \text{fin.}\ (r' - 2\,r) &- 10''.\ 1.\ \text{fin.}\ (r' - q)\\
&- 21.\ \ 8.\ \text{fin.}\ (r' - 2\,q)\\
&+\ \ 2.\ \ 6.\ \text{fin.}\ (r' - 3\,q)\\
&-\ \ 4.\ \ 2.\ \text{fin.}\ (r' + q)\end{aligned}$$

$$\begin{aligned}&- 17'',2.\ \text{fin.}\ (r' - q - 2\,r)\\
&-\ \ 9,\ 5.\ \text{fin.}\ (r' + q - 2\,r)\\
&+\ \ 1,\ 7.\ \text{fin.}\ (r' - 2\,q - 2\,r)\\
&+ 23'',4.\ \text{fin.}\ (r' - t - 2\,r)\\
&- 10,\ 5.\ \text{fin.}\ (r' + t - 2\,r)\\
&+\ \ 4,\ 0.\ \text{fin.}\ (r' + 2\,t - 2\,r).\end{aligned}$$

Deinde Parallaxis horizontalis Parifina exhibetur:

$$\begin{aligned}= 56'.35''.2 - 2'.29'',0.\ \text{cof.}\ q &+ 1'',7.\ \text{cof}\ t\\
+\ \ 9,\ 9.\ \text{cof.}\ 2\,q &+ 1,\ 2.\ \text{cof.}\ (q - t)\\
-\ 10,\ 6.\ \text{cof.}\ 3\,q &- 2,\ 5.\ \text{cof.}\ (q + t).\end{aligned}$$

 I. Cafus

§. 595.

I. Casus Principalis, quo $p = 90°$ et $q = 0$.

Primo igitur nostrae tres coordinatae ita repraesentari poterunt:

x

$$\left\{\begin{array}{l} +0{,}0072101 - 0{,}0005494.\ \text{cos}.\ t \\ \qquad\qquad\quad - 0{,}0000207.\ \text{sin}.\ t \end{array}\right\} 1$$

$$(+0{,}814473 + 0{,}004469.\ \text{cos}.\ t) \ldots K$$

$$(-0{,}02658 + 0{,}00839.\ \text{cos}\ t) \ldots K^2$$

$$-0{,}2933. \ldots\ldots K^3$$

$$\left\{\begin{array}{l} -0{,}26945 \qquad + 0{,}00077.\ \text{cos}.\ t \\ +0{,}25907.\text{cos}.2r + 0{,}00051.\text{cos}.(t-2r) \\ \qquad\qquad\qquad - 0{,}00066.\text{cos}.(t+2r) \end{array}\right\} ii$$

$$(+0{,}0783 - 0{,}0811.\ \text{cos}.\ 2r) \ldots iiK$$

y

$$\left\{\begin{array}{l} -0{,}0006236 + 0{,}0023810.\ \text{sin}.\ t \\ \qquad\qquad - 0{,}0000585.\ \text{cos}.\ t \end{array}\right\} 1$$

$$+0{,}004602.\ \text{cos}.\ q + 0{,}000513.\ \text{sin}.\ t \ldots K$$

$$+0, \ldots\ldots \text{sin}.\ t \ldots K^2$$

$$+0{,}00000. \ldots\ldots K^3$$

(— 0,

$$\left.\begin{cases}-0{,}21187.\sin.2r\\-0{,}00261.\sin.t\\-0{,}00134.\sin.(t-2r)\\-0{,}00111.\sin.(t+2r)\end{cases}\right\}ii$$

$$+0{,}7128.\sin.2r\quad .\quad .\quad iiK$$

z

$$\left.\begin{matrix}+1{,}035428.\sin.r\ .\ .\\-0{,}000551.\cos.r\ .\ .\end{matrix}\right\}i$$

$$\left.\begin{matrix}+0{,}000448.\sin.(r-t)\\-0{,}001573.\sin.(r+t)\end{matrix}\right\}i$$

$$+0{,}83815.\sin.r\ .\ .\ .\ iK$$

$$+0{,}0431.\sin.r\ .\ .\ .\ iKK$$

$$\left.\begin{matrix}+0{,}0030.\sin.r\\+0{,}0161.\sin.3r\end{matrix}\right\}i^{1}$$

Pro hoc autem casu formulae Clairaltianae, ita se habent:

$$\theta-\zeta=-1'.\,29'',1+8'.\,14'',8.\sin.t$$
$$+2'.\,29'',5.\sin.2r$$

$$r'=r-1'.\,29'',1-2'.\,8'',2.\sin.t$$
$$+2'.\,29'',5.\sin.2r$$

$$\varrho=-6'.\,57'',0.\sin.2r'-0'',4.\sin.t-1'',8.\sin.2r.$$

$$\sin.s=+0{,}0897779.\sin.r'.$$

 $\psi=$

$$\psi = s - 9'.\ 11'',0.\ \text{fin.}(r' - 2r) - 33'',6.\ \text{fin.}\ r'$$
$$+\ 23'',4.\ \text{fin.}(r' - t - 2r)$$
$$-\ 10,\ 5.\ \text{fin.}(r' + t - 2r)$$
$$+\ 4,\ 0.\ \text{fin.}(r' + 2t - 2r)$$

Parallaxis $= 54', 5'', 5 + 0'', 4\ \text{cof.}\ t.$

§. 596.

I. Cafus particularis, quo $p = 90$, $q = 0$ et $r = 0$.

Noftrae ergo formulae contrahentur in fequentes:

x

$$\left\{\begin{array}{l} +0,0072101 - 0,0005494.\ \text{cof.}\ t \\ \qquad\qquad\qquad - 0,0000207.\ \text{fin.}\ t \end{array}\right\} \ 1$$

$(+0,814473 + 0,004469.\ \text{cof.}\ t)\ .\ .\ K$

$(-0,02658 + 0,00839.\ \text{cof.}\ t)\ .\ .\ K^2$

$-0,2933.\ .\ .\ .\ K^3$

$(-0,01038 + 0,00062.\ \text{cof.}\ t)\ .\ .\ ii$

$-0,0028.\ .\ .\ .\ iiK$

y

$$\left\{\begin{array}{l} -0,0006236 + 0,0023810.\ \text{fin.}\ t \\ \qquad\qquad\qquad - 0,0000585.\ \text{cof.}\ t \end{array}\right\} \ 1$$

$(+0,004602 + 0,000513.\ \text{fin.}\ t)\ .\ .\ .\ K$

$+0,\ .\ .\ \text{fin.}\ t.\ .\ .\ K^2$

$+0,00000.\ .\ .\ .\ K^3$

$-0,00506.\ \text{fin.}\ t.\ .\ .\ ii$

$+0,00000.\ .\ .\ .\ iiK$

z

z

$$\left.\begin{array}{l} -0,000551. \\ -0,002021.\,\text{fin.}\,t. \end{array}\right\} i$$

Formulae autem Clairaltianae euadunt:

$$\theta - \zeta = -1'.\,29'',1 + 8'.\,14'',8.\ \text{fin.}\ t;$$

$$r' = -1,\,29'',1 - 2'.\,8'',2.\ \text{fin.}\ t;$$

$$\varrho = -6'.\,57''.0.\ \text{fin}\ 2\,r' - 0'',4\ \text{fin.}\ t;$$

$$\text{fin.}\ s = +0,0897779.\ \text{fin.}\ r'\ \text{et}$$

$$\begin{aligned} \psi = s - 9'.\,44'',6.\ \text{fin.}\ r' &+ 23'',4.\ \text{fin.}\,(r' - t) \\ &- 10,\ 5.\ \text{fin.}\,(r' + t) \\ &+ \ \ 4,\ 0.\ \text{fin.}\,(r' + 2\,t) \end{aligned}$$

Parallaxis $= 54'.\,5'',5 + 0'',4\ \text{cof.}\ t.$

§. 597.

I°. $p = 90°$; $q = 0$; $r = 0$; $t = 0$.

Noftrae formulae iam euadent:

$$x = +0,0066607 + 0,818942.\,K - 0,01819.\,K^2 - 0,2933.\,K^3 - 0,00976.\,ii - 0,0028.\,iiK.$$

$$y = -0,0006821 + 0,004602.\,K$$

$$z = -0,000551.\,i.$$

Formulae autem Clairaltianae

$$\theta - \zeta = -1'.\,29'',1;\quad r' = -1'.\,29'',1;$$

$$\varrho = +0'',4;\quad s = -7'',5\ \text{et}\ \psi = -7'',0;$$

Paral-

Parallaxis $= 54'. 5''. 8$, vnde fit $\Phi = - 1'. 28'', 7$, quae cum nostris formulis satis exacte conueniunt:

§. 598.

II°. $p = 90$; $q = 0$; $r = 0$; $t = 180°$.

Nostrae formulae erunt

$x = + 0,0077595 + 0,810004. K - 0,03497. K^2$
$\quad - 0,2933. K^3 - 0,01100. ii - 0,0028. iiK$

$y = - 0,0005651 + 0,004602. K$

$z = - 0,000551. i.$

Formulae autem Clairaltianae

$\theta - \zeta = - 1'. 29'', 1$; $r' = - 1'. 29'', 1$;

$\varrho = + 0'', 4$; $s = - 7', 5$ et $\psi = - 7'', 3$.

Parallaxis $= 54'. 5'', 1$ atque $\Phi = - 1'. 28'', 7$.

§. 599.

II. Casus particularis $p = 90$, $q = 0$, $r = 90°$.

Pro hoc casu nostrae formulae erunt:

$$x$$

$$\left\{ \begin{array}{l} + 0,0072101 - 0,0005494. \text{cos}. t \\ \qquad\qquad\quad - 0,0000207. \text{sin}. t \end{array} \right\} 1$$

$$(+ 0,814473 + 0,004469. \text{cos}. t)\,.\,.\, K$$

$$(- 0,02658 + 0,00839. \text{cos}. t)\,.\,.\, K^2$$

$$- 0,2933\,.\,.\,.\, K^3$$

$$(- 0,52852 + 0,00092. \text{cos}. t)\,.\,.\, ii$$

$$+ 0,1594\,.\,.\,.\, iiK$$

y

$$\left\{\begin{array}{l}-0,0006236+0,0023819.\ \text{fin.}\ t\\ \qquad\qquad\qquad -0,0000585.\ \text{cof.}\ t\end{array}\right\}\ \text{I}$$

$$(+0,004602+0,000513.\ \text{fin.}\ t)\quad \text{K}$$
$$+0\ \ldots\ \text{fin.}\ \text{K}^2$$
$$+0\ \ldots\ \text{K}^3$$
$$-0,00016.\ \text{fin.}\ t\ \ldots\ ii$$
$$+0,\ \ldots\ ii\text{K}$$

z

$$\left\{\begin{array}{l}1,035428.\\ -0,001125.\ \text{cof.}\ t\end{array}\right\}\ i$$
$$+0,83815.\quad i\text{K}$$
$$+0,0431.\quad i\text{K}^2$$
$$-0,0131.\quad i^3$$

Formulae autem Clairaltianae erunt

$$\vartheta-\zeta=-1'.\ 29'',1+8',\ 14'',8.\ \text{fin.}\ t;$$
$$r'=89^\circ.\ 58'.\ 30'',9-2'.\ 8'',2.\ \text{fin.}\ t;$$
$$\varrho=-6'.\ 57''.\ \text{fin.}\ 2\,r'-0'',4.\ \text{fin.}\ t;$$
$$\text{fin.}\ s=+0,0897779.\ \text{fin.}\ r';$$
$$\psi=s+8'.\ 37'',4.\ \text{fin.}\ r'-23'',4.\ \text{fin.}\ (r'-t)$$
$$+10'',5.\ \text{fin.}\ (r'+t)$$
$$-4'',0.\ \text{fin.}\ (r'+2\,t)$$

Parallaxis $54'.\ 5'',5+0,4.\ \text{cof.}\ t$.

§. 600.

I°. $p = 90°$, $q = 0$, $r = 90$, $t = 0$.

Nostrae formulae:

$$x = +0,0066607 + 0,818942.\,K - 0,01819.\,K^2$$
$$- 0,2933.\,K^3 - 0,52760.\,ii + 0,1594.\,iiK$$
$$y = -0,0006821 + 0,004602.\,K$$
$$z = 1,034303.\,i + 0,83815.\,iK + 0,0431.\,iKK$$
$$- 0,0131.\,i^3.$$

Formulae autem Clairaltianae

$\theta - \zeta = -1'.\,29'',1$; $r' = 89°.\,58'.\,30'',9$;

$\varrho = -6'.\,57''$, sin. $2\,r'$; $s = 5°.\,9'.\,3''$;

$\psi = 5°.\,17'.\,23'',5$. Parallaxis $= 54'.\,5'',9$.

§. 601.

Hinc ergo fiet $\Phi = -1'.\,29'',5$, quod satis convenit cum nostra formula Tang. $\Phi = \frac{y}{1+x}$, verum latitudo ψ nobis suppeditat sequentem aequationem $z = (1+x)$ Tang ψ Sec. Φ, quae praebet:

$$+1,034303.\,i + 0,83815.\,iK + 0,0431.\,iKK$$
$$- 0,0131.\,i^3 = +0,0932266 + 0,075850.\,K$$
$$- 0,00168.\,K^2 - 0,0272.\,K^3 - 0,04886.\,ii$$
$$+ 0,0147.\,iiK.$$

§. 602.

§. 602.

II°. $p = 90°$, $q = 0$, $r = 90$, $t = 180°$.

Nostrae formulae:

$$x = +0,0077595 + 0,810004.K - 0,03497.K^2 - 0,2933.K^3 - 0,52944.ii + 0,1594.iiK$$

$$y = -0,0005651 + 0,004602.K$$

$$z = +1,036553.i + 0,83815.iK + 0,0431.iKK - 0,0131.i^3.$$

Formulae autem Clairaltianae erunt

$\theta - \zeta = 1'.29''.1$; $r' = 89°.58'.30'',9$;

$\varrho = -0'',4$; $s = 5°.9'.3''$;

$\psi = 5°.17'.49''.3$, parallaxis $54'.5''.1$.

§. 603.

Circa Φ consensus est vt ante, at ex Latitudine elicimus hanc aequationem

$$z = (1 + x)\ \text{Tang.}\ \psi.\ \text{Sec.}\ \Phi$$
$$+ 1,036553.i + 0,83815.iK + 0,0431.iKK - 0,0131.i^3 = +0,0934655 + 0,075124.K - 0,00324.K^2 - 0,0272.K^3 - 0,04910.ii + 0,0148.iiK.$$

§. 604.

II°. Casus Principalis, quo $p = 90°$ et $q = 90°$.

Pro hoc ergo casu tres nostrae coordinatae ita se habebunt:

x

$$\left\{\begin{array}{l} +0{,}0072101 - 0{,}0005494.\,\text{cos}\,t \\ \qquad\qquad\quad - 0{,}0000207.\,\text{sin}.\,t \end{array}\right\} 1$$
$$(-0{,}000491 - 0{,}012069.\,\text{sin}.\,t)\ K$$
$$(-1{,}48550 + 0{,}01433.\,\text{cos}.\,t)\ K^2$$
$$+ 0.\ K^3$$
$$\left\{\begin{array}{l} -0{,}26945 \qquad\qquad + 0{,}00077.\,\text{cos}.\,t \\ +0{,}25907.\,\text{cos}.\,2r + 0{,}00051.\,\text{cos}.\,(t-2r) \\ \qquad\qquad\qquad\quad -0{,}00066.\,\text{cos}.\,(t+2r) \end{array}\right\} ii$$
$$+ 0{,}2215.\,\text{sin}.\,2r\ iiK$$

y

$$\left\{\begin{array}{l} -0{,}0006236 + 0{,}0023810.\,\text{sin}.\,t \\ \qquad\qquad\quad - 0{,}0000585.\,\text{cos}.\,t \end{array}\right\} 1$$
$$(-2{,}419969 + 0{,}018625.\,\text{cos}.\,t)\ K$$
$$+ 0 \ldots\ \text{sin}.\,t\ K^2$$
$$+ 2{,}3692.\ K^3$$
$$\left\{\begin{array}{l} -0{,}21187.\,\text{sin}.\,2r - 0{,}00261.\,\text{sin}.\,t \\ \qquad\qquad\qquad\quad -0{,}00134.\,\text{sin}.\,(t-2r) \\ \qquad\qquad\qquad\quad -0{,}00111.\,\text{sin}.\,(t+2r) \end{array}\right\} ii$$
$$(+0{,}1836 - 0{,}5834.\,\text{cos}.\,2r)\ iiK$$

z

$$z$$

$$\left\{\begin{aligned} &+ 1,035428.\ \text{sin.}\ r \\ &- 0,000551.\ \text{cos.}\ r \\ &+ 0,000448.\ \text{sin.}\ (r-t) \\ &- 0,001573.\ \text{sin.}\ (r+t) \end{aligned}\right\} i$$

$$- 2,31751.\ \text{cos.}\ r\ iK$$

$$(- 0,2883.\ \text{sin.}\ r\ iKK$$

$$(+ 0,0030.\ \text{sin.}\ r + 0,0161.\ \text{sin.}\ 3r)\ i^3$$

§. 605.

Formulae autem Clairaltianae fiunt:

$$\theta - \zeta = -7^\circ.\ 30'.\ 12'',0 + 8'.\ 22'',9.\ \text{sin.}\ t$$
$$+ 3.\ \ 6,\ 7.\ \text{cos.}\ t$$
$$+ 1,\ \ 1,\ 7.\ \text{sin.}\ 2r$$
$$- 1,\ 27,\ 8.\ \text{cos.}\ 2r$$

$$r' = r + 2'.\ 5''. - 10'.\ 23'',\ \text{sin.}\ t + (\theta - \zeta)$$

$$\varrho = - 6'.\ 57'',0.\ \text{sin.}\ 2r' - 4'',7 - 0''.\ 4.\ \text{sin.}\ t$$
$$- 1'',8.\ \text{sin.}\ 2r$$

$$\text{sin.}\ s = + 0,0897779.\ \text{sin.}\ r'\ \text{et}$$

$$\psi = s - 9'.0''.3.\ \text{sin.}(r' - 2r) + 7'',7.\ \text{cos.}(r' - 2r)$$
$$+ 21,8.\ \text{sin.}\ r' + 8'',5.\ \text{cos.}\ r'$$
$$+ 23,4.\ \text{sin.}\ (r' - t - 2r)$$
$$- 10,5.\ \text{sin.}(r' + t - 2r)$$
$$+ \ 4,0.\ \text{sin.}(r' + 2t - 2r)$$

Parallaxis autem horizontalis

$$= 56'.\ 25'',3 + 1'',7.\ \text{cos.}\ t + 3'',7.\ \text{sin.}\ t.$$

§. 606.

I. Casus particularis, quo $p = 90$, $q = 90$, $r = 0$.

Hoc casu nostrae formulae sunt:

x

$$\left\{\begin{array}{r} +0,0072101 - 0,0005494.\ \text{cos}.\ t \\ -0,0000207.\ \text{sin}.\ t \end{array}\right\} \text{I}$$

$$\begin{array}{l} (-0,000491 - 0,012069.\ \text{sin}.\ t.)\ \text{K} \\ (-1,48550 + 0,01433.\ \text{cos}.\ t)\ .\ .\ \text{K}^2 \\ \qquad\qquad +0,\ .\ .\ \text{K}^3 \\ (-0,01038 + 0,00062.\ \text{cos}.\ t)\quad ii \\ +0,\ .\ .\ .\ ii\text{K} \end{array}$$

y

$$\left\{\begin{array}{r} -0,0006236 + 0,0023810.\ \text{sin}.\ t \\ -0,0000585.\ \text{cos}.\ t \end{array}\right\} \text{I}$$

$$\begin{array}{l} (-2,419969 + 0,018625.\ \text{cos}.\ t)\ \text{K} \\ \qquad .\ .\ .\ .\ \text{sin}.\ t.\ .\ .\ .\ \text{K}^2 \\ \qquad +2,3692.\ .\ .\ .\ \text{K}^3 \\ \qquad -0,00506.\ \text{sin}.\ t\ .\ .\ .\ ii \\ \qquad -0,3998.\ .\ .\ .\ ii\text{K} \end{array}$$

z

$$\begin{array}{l} -0,000551 - 0,002021.\ \text{sin}.\ t.\quad i \\ -2,31751.\ .\ .\ .\ i\text{K} \\ +0,\ .\ .\ .\ i\text{KK} \\ +0,\ .\ .\ .\ i^3 \end{array}$$

For-

Formulae autem Clairaltianae sunt:

$$\theta - \zeta = -7^\circ.\ 31'.\ 39'',8 + 8'.\ 22'',9.\ \text{sin.}\ t$$
$$+ 3'.\ 6'',7.\ \text{cos.}\ t$$

$$r' = -7^\circ.\ 29'.\ 34''.\ 8 - 2'.\ 0'',1.\ \text{sin.}\ t$$
$$- 3'.\ 6'',7.\ \text{cos.}\ t$$

$$\varrho = -6'.\ 57'',0.\ \text{sin.}\ 2\,r' - 4'',7 - 0'',4.\ \text{sin.}\ t.$$

$$\text{sin.}\ s = +0,0897779.\ \text{sin.}\ r'$$

$$\psi = s - 8'.\ 38'',5.\ \text{sin.}\ r' + 16'',2.\ \text{cos.}\ r'$$
$$+ 23'',4.\ \text{sin.}\ (r' - t)$$
$$- 10,\ 5.\ \text{sin.}\ (r' + t)$$
$$+ 4,\ 0.\ \text{sin.}\ (r' + 2\,t)$$

et parallaxis horizontalis

$$= 56'.\ 25'',3 + 1'',7.\ \text{cos.}\ t + 3'',7.\ \text{sin.}\ t.$$

§. 607.

I°. $p = 90^\circ$, $q = 90$, $r = 0$, $t = 0$.

Pro hoc casu nostrae formulae:

$$x = +0,0066607 - 0,000491.\ K - 1,47117.\ K^2$$
$$- 0,00976.\ ii$$

$$y = -0,0006821 - 2,401344.\ K + 2,3692.\ K^2$$
$$- 0,3998.\ iiK$$

$$z = -0,000551 - 2,31751.\ iK$$

Formu-

Formulae Clairaltianae

$$\theta - \zeta = -7^\circ.\ 28'.\ 33'',1;$$
$$\eta' = -7^\circ.\ 26'.\ 28'',1;$$
$$\varrho = -6'.\ 57''.\ \text{sin}.\ 2\eta' - 4'',7 = +1'.\ 42'',4;$$
$$s = -39'.\ 58''; \quad \psi = -38'.\ 31''.$$

Parallaxis $56'.\ 27''.\ 0$.

§. 608.

Quum igitur hinc sit

$$\Phi = \theta - \zeta + \varrho = -7^\circ.\ 26'.\ 50''.\ 7,$$

aequatio nostra prior $y - (1 + x)\,\text{Tang.}\,\Phi = 0$ euadit:

$$+0,1809090 - 2,401408.\,K - 0,19231.\,K^2$$
$$+2,3692.\,K^3 - 0,00128.\,ii - 0,3998.\,ii\,K = 0.$$

Quod ad Latitudinem attinet, quia valde est parua ex ea nihil pro nostro instituto concludi potest.

§. 609.

II°. $p = 90^\circ$, $q = 90^\circ$, $r = 0$, $t = 180^\circ$.

Pro hoc casu nostrae formulae sunt:

$$x = +0,0077595 - 0,000491.\,K - 1,49983.\,K^2 - 0,01100.\,ii$$
$$y = -0,0005651 - 2,438594.\,K + 2,3692.\,K^3 - 0,3998.\,iiK$$
$$z = -0,000551 - 2,31751.\,iK.$$

For-

Formulae Clairaltianae

$\theta - \zeta = - 7°. 34'. 46'', 5;$
$r' = - 7°. 32'. 41'', 5;$
$\varrho = + 1' 43'', 8; s = - 40'. 33'';$
$\psi = - 39'. 14''$. Parallaxis $56'. 23'', 6.$

§. 610.

Quum hic sit $\Phi = - 7°. 33'. 2'', 7$, orietur haec aequatio:

$$+ 0,1330193 - 2,438659. K - 0,19881. K^2$$
$$+ 2,3692. K^3 - 0,00146. ii - 0,3998. ii K = 0.$$

Latitudo vero ad vſum noſtrum nihil confert, vnde aequationem inde natam non euoluimus.

§. 611.

II°. Caſus particularis, quo $p=90°$; $q=90$; $r=90°$.

Pro hoc caſu noſtrae formulae reperiuntur:

$$x$$

$$\left\{ \begin{array}{r} +0,0072101 - 0,0005494. \cos t \\ -0,0000207. \sin t \end{array} \right\} 1$$
$$(-0,000491 - 0,012069. \sin t)\ K$$
$$(-1,48550 + 0,01433. \cos t)\ K^2$$
$$+ 0.\ K^3$$
$$(-0,52852 + 0,00092. \cos t)\ ii$$
$$+ 0.\ ii K$$

y

$$\left.\begin{array}{r} -0{,}0006236+0{,}0023810.\,\text{sin.}\,t \\ -0{,}0000585.\,\text{cos.}\,t \end{array}\right\} i$$

$$\begin{array}{l} (-2{,}419969+0{,}018625.\,\text{cos.}\,t)\ K \\ \qquad +0\ \ldots\ \text{sin.}\,t.\ K^2 \\ \qquad +2{,}3692.\ K^3 \\ \qquad -0{,}00016.\,\text{sin.}\,t.\ ii \\ \qquad +0{,}7670.\ iiK \end{array}$$

z

$$\left.\begin{array}{l} +1{,}035428. \\ -0{,}001125.\,\text{cos.}\,t \end{array}\right\} i$$

$$\begin{array}{l} -0{,}2883.\ iKK \\ -0{,}0131.\ i^3. \end{array}$$

Formulae autem Clairaltianae prodeunt:

$$\theta-\zeta=-7^{\circ}.\ 28'.\ 44'',4+8'.\ 22'',9.\ \text{sin.}\ t$$
$$+\ 3.\ \ 6,\ \ 7.\ \text{cos.}\ t$$

$$r'=82^{\circ}.\ 33'.\ 20''.\ 6-2'.\ 0'',1.\ \text{sin.}\ t$$
$$+\ 3.\ \ 6,\ \ 7.\ \text{cos.}\ t$$

$$\varrho=-6'.\ 57'',0.\ \text{sin.}\ 2\,r'-4''.\ 7-0'',4.\ \text{sin.}\ t$$

$$\text{sin.}\ s=+0{,}0897779.\ \text{sin.}\ r'$$

$$\begin{array}{l} \psi=s+9'.\ 22'',1.\ \text{sin.}\ r'+0'',8.\ \text{cos.}\ r' \\ \qquad\qquad -\ 23{,}4.\ \text{sin.}\ (r'-t) \\ \qquad\qquad +\ 10{,}5.\ \text{sin.}\ (r'+t) \\ \qquad\qquad -\ 4{,}0.\ \text{sin.}\ (r'+2\,t) \end{array}$$

Parallaxis horizontalis

$$56'.\ 25'',3+1'',7.\ \text{cos.}\ t+3''.\ 7.\ \text{sin.}\ t.$$

§. 612.

§. 612.

I°. $p = 90$; $q = 90$; $r = 90$; $t = 0$.

Pro hoc casu nostrae formulae prodeunt:

$$x = +0,0066607 - 0,000491.K - 1,47117.K^2 - 0,52762.ii$$

$$y = -0,0006821 - 2,401344.K + 2,3692.K^3 + 0,7670.iiK$$

$$z = +1,034303 - 0,2883.iKK - 0,0131.i^3.$$

Formulae vero Clairaltianae:

$$\theta - \zeta = -7°.\ 25'.\ 38'',0;$$

$$r' = 82°.\ 36'.\ 27'',3;\ \varrho = -1'.\ 51'',1.$$

$$s = 5°.\ 6'.\ 28'';\ \psi = 5°.\ 15'.\ 29''.$$

§. 613.

Quum hinc sit $\Phi = -7°.\ 27'.\ 29''$. prior aequatio nostra $y - (1 + x)$ Tang. Φ praebebit:

$$+0,1310976 - 2,401408.K - 0,19259.K^2 + 2,3692.K^3 - 0,06906.ii + 0,7670\ iiK = 0.$$

Altera vero aequatio $z = (1 + x)$ Tang. ψ Sec. Φ abibit in hanc:

$$+1,0343031.i - 0,2883.iKK - 0,0131.i^3 = +0,0934373 - 0,000045.K - 0,13656.K^2 - 0,04897.ii.$$

§. 614.

II°. $p = 90^\circ$, $q = 90^\circ$, $r = 90^\circ$, $t = 180^\circ$.

Pro hoc casu nostrae formulae fiunt:

$$x = + 0,0077595 - 0,000491 . K - 1,49983 . K^2 - 0,52944 \, ii.$$

$$y = - 0,0005651 - 2,438594 . K + 2,3692 . K^2 + 0,7670 . iiK.$$

$$z = + 1,036553 . i - 0,2883 . iKK - 0,0131 . i^3$$

Formulae autem Clairaltianae

$\theta - \zeta = - 7^\circ . 31' . 51'' , 3;$

$r' = 82^\circ . 30' . 13'' , 9;$ $\varrho = - 1' . 52'' , 5;$

$s = 5^\circ . 6' . 24'';$ $\psi = 5^\circ . 15' . 49'';$

Parallaxis $56' . 23'' , 6.$

§. 615.

Quum igitur hic sit $\Phi = - 7^\circ . 33' . 43'' , 8$, prior nostra aequatio praebebit:

$$+ 0,1332224 - 2,438659 . K - 0,19911 . K^2 + 2,3692 . K^3 - 0,07028 . ii + 0,7670 . iiK.$$

Altera vero aequatio ita repraesentabitur:

$$1,036553 . i - 0,2883 \, iKK - 0,0131 . i^3 = + 0,0936654 - 0,000045 . K - 0,13939 . K^2 - 0,04920 . ii.$$

CAPVT V.

CAPVT V.

CONIECTVRA CIRCA VALORES LITTERARVM K ET *i*.

§. 616.

Quum Tabulae Clairaltianae in loco Lunae, nunquam vltra duo minuta prima aberrare perhibeantur, in latitudine vero error adhuc sit minor, videamus vtrum litteris K et *i* eiusmodi valores tribui queant quibus aequationibus ante exhibitis satisfiat. Plures autem huiusmodi aequationes inuestigauimus, vt ex earum vel consensu vel dissensu tutius iudicium super his litteris ferre possimus.

§. 617.

Euoluimus autem in Capitibus praecedentibus eiusmodi Lunae positiones pro quibus siue differentia inter longitudinem mediam et veram Lunae, siue Latitudo quasi maxima prodit, propterea quod aequationes prioris generis imprimis aptae sint litterae K definiendae, posteriores vero alteri litterae *i*. Deinde

 pro

pro qualibet positione angulis p, q et r determinata, duos casus euoluimus prouti ponitur $t = 0$, vel $t = 180°$, qui quoniam inter se parum discrepant binos huiusmodi casus coniungamus, quoniam hoc modo vtrique simul satisfit.

§. 618.

Incipiamus igitur ab aequationibus pro determinatione litterae K supra inuentis, quas propterea sequenti modo repraesentabimus:

Casus quo $p = 0$; $q = 90$; $r = 0$.

si $t = 0$; $0 = +0,0884731 - 1,619012.K - 0,057930.K^2 + 1,0176.K^3 + 0,00041.ii - 0,3402.iiK$

si $t = 180°$; $0 = +0,0872924 - 1,598110.K - 0,057670.K^2 + 1,0176.K^3 + 0,00041.ii - 0,3402.iiK$

I. $0 = +0,1757655 - 3,217122.K - 0,115600.K^2 + 2,0352.K^3 + 0,00082.ii - 0,6804.iiK$

Casus quo $p = 0$; $q = 90$; $r = 90$.

si $t = 0$; $0 = +0,0883310 - 1,619012.K - 0,057837.K^2 + 1,0176.K^3 - 0,04154.ii - 0,0214.iiK$

si $t =$

si $t = 180$; $0 = +0{,}0871415 - 1{,}598110.K$
$-0{,}057571.K^2 + 1{,}0176.K^3 - 0{,}04086.ii$
$-0{,}0214.iiK$

II. $0 = +0{,}1754725 - 3{,}217122.K - 0{,}115408.K^2$
$+2{,}0352.K^3 - 0{,}08240.ii - 0{,}0428.iiK$

Casus quo $p = 90$; $q = 90$; $r = 0$.

si $t = 0$; $0 = +0{,}1309090 - 2{,}401408.K$
$-0{,}19231.K^2 + 2{,}3692.K^3 - 0{,}00128.ii$
$-0{,}3998.iiK$

si $t = 180°$; $0 = +0{,}1330193 - 2{,}438659.K$
$-0{,}19881.K^2 + 2{,}3692.K^3 - 0{,}00146.ii$
$-0{,}3998.iiK$

III. $0 = +0{,}2639283 - 4{,}840067.K - 0{,}39112.K^2$
$+4{,}7384.K^3 - 0{,}00274.ii - 0{,}7996.iiK$

Casus quo $p = 90$; $q = 90$; $r = 90$.

si $t = 0$; $0 = +0{,}1310976 - 2{,}401408.K$
$-0{,}19259.K^2 + 2{,}3692.K^3 - 0{,}06906.ii$
$+0{,}7670.iiK$

si $t = 180°$; $0 = +0{,}1332224 - 2{,}438659.K$
$-0{,}19911.K^2 + 2{,}3692.K^3 - 0{,}07028.ii$
$+0{,}7670.iiK$

IV. $0 = 0{,}2643200 - 4{,}840067.K - 0{,}39170.K^2$
$+4{,}7384.K^3 - 0{,}13934.ii + 1{,}5340.iiK$.

§. 619

§. 619.

Quoniam his aequationibus etiam altera incognita i adhuc inest eam neutiquam per solitas eliminationes expellere licet, siquidem conclusiones inde deductae maxime futurae essent incertae, sed omnino necesse habemus valorem huius quantitatis i proxime saltem nosse, id quod omnino sufficit, cum termini in quibus occurrit sint valde exigui, leui autem tentaminis instituto comperimus proxime esse $ii = \frac{1}{1553}$; hunc ergo valorem in istis quatuor aequationibus substituamus, quae hinc sequentes formas induent:

$$\text{I.} \quad 0{,}1757721 - 3{,}222565.K - 0{,}115600.K^2 + 2{,}0352.K^3 = 0$$

$$\text{II.} \quad 0{,}1748133 - 3{,}217464.K - 0{,}115408.K^2 + 2{,}0352.K^3 = 0$$

$$\text{III.} \quad 0{,}2639064 - 4{,}846464.K - 0{,}391120.K^2 + 4{,}7384.K^3 = 0$$

$$\text{IV.} \quad 0{,}2632054 - 4{,}827795.K - 0{,}391700.K^2 + 4{,}7384.K^3 = 0.$$

§. 620.

Ad has aequationes resoluendas iterum necesse est, valorem incognitae K proxime nosse, quem aliquot institutis tentaminibus facile reperimus, aliquantillum superare 0,654 statuamus propterea $K = 0{,}654(1+\omega)$ et quia ω est fractio valde parua sumere licet

KK =

$KK = 0,054^2.(1+2\omega)$ et $K^3 = 0,054^3(1+3\omega)$; his igitur valoribus substitutis nostrae aequationes transformabuntur in sequentes simplices:

I. $0 = +0,0017369 - 0,1737215.\omega$, hinc
$\omega = 0,009998$; $K = 0,054540$.

II. $0 = +0,0010541 - 0,1734549.\omega$, hinc
$\omega = 0,006077$; $K = 0,054328$.

III. $0 = +0,0018229 - 0,2617518.\omega$ hinc
$\omega = 0,006964$; $K = 0,054376$.

IV. $0 = +0,0021082 - 0,2607472.\omega$ hinc
$\omega = 0,008085$; $K = 0,054436$.

§. 621.

Consensus inter has quatuor conclusiones tantus est, vt maiorem vix sperare ausi fuissemus, ad quod ostendendum notasse iuuabit vni parti decies millesimae in littera K, tantum quadraginta minuta secunda in loco Lunae respondere. Neque tamen consultum iudicamus, verum litterae K valorem hinc concludere, quippe quod negotium ipsis obseruationibus reseruamus, interea vero numero rotundiori acquiescentes statuamus $K = 0,0544$.

§. 622.

Definito igitur valore litterae K, ſimili modo inquiramus in litteram i, quem in finem vt ante binas aequationes, tantum ratione anguli t differentes in vnam contrahamus, atque ſequentes quatuor aequationes conſequemur.

Caſus quo $p = 0$, $q = 0$, $r = 90$.

I. $+0,1734123 - 1,929498.i - 0,08143.ii - 0,0318.i^3 = 0$
$+0,206832.K - 2,24902.iK + 0,0376.iiK$
$+0,00301.K^2 - 0,0634.iKK$
$-0,1089.K^3$

Caſus quo $p = 0$, $q = 90$, $r = 90$.

II. $+0,1741937 - 1,929498.i - 0,08180.ii - 0,0318.i^3 = 0$
$-0,114568.K^2 - 0,4270.iK^2$

Caſus quo $p = 90$, $q = 0$, $r = 90$.

III. $+0,1866921 - 2,070856.i - 0,09796.ii + 0,0262.i^3 = 0$
$+0,150974.K - 1,67630.iK + 0,0295.iiK$
$-0,00492.K^2 - 0,0862.iK^2$
$-0,0544.K^3$

Caſus

Casus quo $p = 90$, $q = 90$, $r = 90$.

IV. $+0,1871027 - 2,070856.i - 0,09817.ii + 0,0262.i^3 = 0$
$-0,000090.K + 0,5766.iK^2$
$-0,27595.K^2$.

§. 623.

In his aequationibus nunc substituamus loco K valorem modo aestimatum $K = 0,0544$, vt adipiscamur sequentes aequationes solam incognitam i inuoluentes:

I. $0 = +0,1846553 - 2,052032.i - 0,07939.ii$
$-0,0318.i^3 = 0$

II. $0 = +0,1738547 - 1,930762.i - 0,08180.ii$
$-0,0318.i^3 = 0$

III. $0 = +0,1948817 - 2,162302.i - 0,09636.ii$
$+0,0262.i^3 = 0$

IV. $0 = +0,1862812 - 2,069150.i - 0,09817.ii$
$+0,0262.i^3 = 0$.

§. 624.

Ad has aequationes resoluendas pariter necesse est, vt valor saltem prope verus litterae i innotescat, leui autem attentione adhibita mox patet hunc valorem paulisper excedere $0,089$, quocirca ponamus $i = 0,089(1+\omega)$, ita vt ω sit fractio quam mi-

 nima,

nima, eiusque potestates propterea tuto negligi queant, hoc autem valore substituto aequationes illae transmutabuntur in sequentes simplices:

I. $0 = +0,0013732 - 0,1839557.\omega$, hinc
$\omega = 0,007465$; $i = 0,089664$.
II. $0 = +0,0013465 - 0,1732009.\omega$, hinc
$\omega = 0,007774$; $i = 0,089692$.
III. $0 = +0,0016920 - 0,1939160.\omega$, hinc
$\omega = 0,008725$; $i = 0,089776$.
IV. $0 = +0,0013677 - 0,1856541.\omega$, hinc
$\omega = 0,007367$; $i = 0,089656$.

§. 625.

Hic quidem differentia inter maximum et minimum valorem ipsius y est $= 0,00012$. sed notari oportet hinc in Latitudine Lunae tantum discrimen $24''$ oriri, quodsi ergo medium rotundius accipiamus statuentes $i = 0,0897$, totum discrimen vix semiminutum primum superabit, qui egregius consensus sine dubio omni attentione est dignus; quocirca in posterum vtemur his valoribus $K = +0,0544$ et $i = +0,0897$ quoad nobis licuerit has litteras accuratius per obseruationes assignare.

§. 626.

Quum igitur sit $i = 0,0897$, hinc fit $ii = 0,00804$, quare quum supra posuerimus $ii = \frac{8}{1030}$, haec

haec positio iam ita erat exacta, vt vlteriore correctione non indigeat.

§. 627.

Antequam autem hos valores ad vsum nostrum accommodemus, circa gradum praecisionis singularum formarum nostrarum, quaedam imprimis annotanda occurrunt. Primo enim iam supra obseruauimus ordinem $iiKK$, ita esse comparatum, vt etiamsi quis laborem calculi immensum suscipere vellet, tamen leuissimos errores in praecedentibus ordinibus commissos, quamuis ne minutum quidem secundum producant, hunc calculum plane irritum esse reddituros; quamobrem etiam hunc ordinem penitus omisimus, idque eo magis quod omnes anguli huius ordinis vnico $2q-2r$ excepto, nihil plane ad locum Lunae conferant, atque hic angulus tantum in coordinata y alicuius momenti esse possit; quamobrem ibi iam consilium cepimus, sinum huius anguli cum coefficiente incognito, in calculum nostrum introducendi.

§. 628.

Deinde vero re attentius perpensa, simile fere incommodum in ordine iKK locum habere deprehendimus, ita vt inaequalitates huius ordinis, maxime suspectae sint habendae, quatenus quidem coordinatam y affectant, in coordinata enim x nihil plane

 effice-

efficere valent; id autem neutiquam vſu venit ob vitium quodqiam Theoriae, ſed potius ob ſummas Calculi difficultates, ad quas ſuperandas omnes praecedentium ordinum formulas non ſolum ad vnum minutum ſecundum exactas cognoſcere deberemus, ſed etiam error vnius tantum centeſimae partis, totum negotium maxime perturbare poſſet, quam ob cauſſam hunc ordinem cui reſpondent litterae 𝔚 et W penitus rejicere cogimur et quia facile intelligere licet, ex hoc ordine tantum angulos t et $2p-2q+t$, aliquid ad quantitatem y conferre poſſe, etiam ſinus horum angulorum cum incognitis coefficientibus iſti quantitati adiungemus.

§. 629.

Quum igitur non leuis ſuſpicio habeatur angulos $2p-2q+t$ et $2q-2r$ forſitan motum Lunae afficere poſſe, valori ipſius y, inſuper hos terminos indefinitos adiungamus γ ſin. $(2p-2q+t)$ $+\delta$. ſin. $(2q-2r)$ vti iam ſupra obſeruauimus.

§. 630.

Poſtquam autem litteris K et i, iſtos valores tribuentes, plures obſeruationes examinauimus, exiguas quidem aberrationes vnum minutum primum vix ſuperantes deprehendimus, quas vero intra multo arctiores terminos reduci poſſe agnouimus, ſi litteris K et

K et i sequentes valores tribuamus $K = 0,0545$ et $i = 0,08964$, quibus ergo in posterum vtemur, donec ex plurimis obseruationibus eas accuratius definire licuerit, circa coefficientes autem indefinitos γ et δ annotauimus, haud contemnendam errorum diminationem obtineri, si in partibus 10000000 vnitatis statuamus $\gamma = -250$ et $\delta = +250$ quantum quidem hactenus colligere licuit.

CAPVT VI.

CAPVT VI.

DE GRADV PRAECISIONIS, QVO SINGVLAE INAEQVALITATES INVENTAE SVNT PRAEDITAE.

§. 631.

Omnes numeros, quibus ternas nostras coordinatas x, y et z definiuimus, tantum pro vero proximis haberi posse manifestum est; cum enim litterae, m, n et l, quibus illi innituntur, tantum ad certum gradum sint exactae; qui quidem partes millionesimas excedere videtur; perspicuum est, gradum praecisionis in numeris a nobis inuentis non vltra partes millionesimas extendi posse; atque hoc gradu formulae priorum ordinum litteris O et P signatae sunt forte exactae habendae, id quod ad theoriam Lunae abunde sufficit; quandoquidem quod ad coordinatas y et z attinet, vna pars millionesima vnitatis; tantum vni circiter parti quintae minuti secundi in loco Lunae respondet. Pro coordinata vero x effectus adhuc decies minor

minor eſt cenſendus. Quanquam autem quantitates priorum ordinum O^2, P^2 ad ſex figuras decimales ſunt exactae; tamen in ſequentibus ordinibus aberratio a veritate eo maior fieri poteſt, quo magis characteres ordinum fuerint complicati, propterea quod multae formulae in numeros praegrandes, veluti 537 atque adeo 1035 ſunt ducendi; quo fit, vt errores, qui forte in partibus millioneſimis inerant, ad partes milleſimas adeo excreſcant, ac ſi tales formulae in ordinem ſequentem ingrediantur, error inde denuo quaſi millies maior reſultare poteſt. Ob hanc cauſſam iam in tertio ordine litterarum $\mathfrak{Q}$ et Q coefficientes non vltra quatuor figuras decimales exactae haberi poſſunt; praecipue ille, qui adficit angulum $2p - 2q$.

§. 632.

Atque hinc in formulis $\mathfrak{R}$ et R error fere in partes decimas aſcendere poterit; quia autem in hoc ordine, cuius character K^3, integra vnitas propemodum ſemiminuto primo in loco Lunae reſpondet, talis error vtique non eſt magni momenti; pari fere gradu ſequentes ordines S, T, U, X, Y exacti ſunt habendi; verum ordo litterarum $\mathfrak{V}$ et V, prouti ſupra eſt tactatus, maioribus erroribus obnoxius cenſeri debet; propterea quod ibi calculo conſulentes terminos, in quibus $4p$ ineſt, negligere coepimus. Quare cum ordo ſequens litterarum $\mathfrak{W}$ et W formulis illius ordinis innitatur, valores harum litterarum traditi merito maxime ſuſpecti habentur.

§. 633.

Operam igitur hic dabimus, vt iſtas litteras 𝔙 et V adcuratius determinemus, quem in finem ſequens ſupplementum hic ſubiungimus.

SVPPLEMENTVM

ad Caput VIII. de valoribus 𝔙 et V.

Quoniam in hoc capite nimiam calculi prolixitatem vitaturi particulas, quae ex angulis $4p$ naſcerentur, neglеximus; operae pretium iudicauimus, hunc calculum omni rigore inſtituere; quo clarius adpareat, quantum diſcrimen inde in valoribus litterarum 𝔙 et V exoriatur. Quamobrem has correctiones ſecundum paragraphos ibi notatos adiungamus:

ad §. 315.

Formulae huius paragraphi adcuratius euolutae ita prodierunt:

I. $2\mathfrak{P}\mathfrak{U}$ dat

$$\begin{aligned}
-0{,}001302.\ \text{coſ.}\ q-t - 0{,}004748.\ \text{coſ.}\ 2p-q+t \\
+0{,}029416.\ \text{coſ.}\ 2p+2q-t \\
-0{,}000649.\ \text{coſ.}\ 4p-q+t \\
-0{,}000033.\ \text{coſ.}\ 4p+q-t \\
-0{,}
\end{aligned}$$

$$-0{,}007556.\cos.\,q+t+0{,}028117.\cos.\,2p-q-t$$
$$-0{,}003435.\cos.\,2p+q+t$$
$$+0{,}005567.\cos.\,4p-q-t$$
$$+0{,}000005.\cos.\,4p+q+t.$$

Eius multiplicator pro

prima $\left(+537{,}63359+15{,}44258.\cos.\,2p \atop +0{,}26587.\cos.\,4p\right.$

secunda $(+10{,}98139.\sin.\,2p+0{,}20324.\sin.\,4p.$

II. $\mathfrak{P}$ U + P $\mathfrak{U}$ dat

$$-0{,}086434.\sin.\,q-t+0{,}018557.\sin.\,2p-q+t$$
$$-0{,}050974.\sin.\,2p+q-t$$
$$+0{,}001179.\sin.\,4p-q+t$$
$$-0{,}000107.\sin.\,4p+q-t$$
$$+0{,}101869.\sin.\,q+t-0{,}008553.\sin.\,2p-q-t$$
$$+0{,}005989.\sin.\,2p+q+t$$
$$-0{,}010089.\sin.\,4p-q-t$$
$$+0{,}000003.\sin.\,4p+q+t.$$

Eius multiplicator pro

prima $(+21{,}96278.\sin.\,2p+0{,}40648.\sin.\,4p$

secunda $\left(-537{,}56350-15{,}44258.\cos.\,2p \atop -0{,}33595.\cos.\,4p\right.$

III. 2 PU dat

$$\begin{aligned}
&- 0,365789.\ \cos. q - t + 0,067290.\ \cos. 2p - q + t \\
&\qquad - 0,087725.\ \cos. 2p + q - t \\
&\qquad + 0,002400.\ \cos. 4p - q + t \\
&\qquad - 0,000431.\ \cos. 4p + q - t \\
&+ 0,381450.\ \cos. q + t + 0,008790.\ \cos. 2p - q - t \\
&\qquad + 0,001174.\ \cos. 2p + q + t \\
&\qquad - 0,017662.\ \cos. 4p - q - t \\
&\qquad + 0,000034.\ \cos. 4p + q + t.
\end{aligned}$$

Eius multiplicator pro

prima $\left(- 268,78175 - 7,72129.\ \cos. 2p \atop \qquad\qquad - 0,16797.\ \cos. 4p \right.$

secunda $(- 8,23604\ \sin. 2p - 0,15243.\ \sin. 4p$

Partes vltimae aequationis prioris siue Ň. IV'. sunt:

$$\begin{aligned}
&+ 2,32200.\ \cos. q - t - 0,98841.\ \cos. 2p - q + t \\
&\qquad - 2,66020.\ \cos. 2p + q - t \\
&\qquad + 0,08345.\ \cos. 4p - q + t \\
&\qquad - 0,01552.\ \cos. 4p + q - t \\
&+ 0,52676.\ \cos. q + t + 8,04887.\ \cos. 2p - q - t \\
&\qquad + 0,37768.\ \cos. 2p + q + t \\
&\qquad - 0,58719.\ \cos. 4p - q - t \\
&\qquad + 0,00223.\ \cos. 4p + q + t
\end{aligned}$$

Eius multiplicator $= 1$.

Poste-

Posteriores siue N. IV''. sunt:

$$
\begin{aligned}
&-3,08187.\ \text{sin.}\ q-t + 0,82186.\ \text{sin.}\ 2p-q+t\\
&\qquad\qquad\qquad + 2,65577.\ \text{sin.}\ 2p+q-t\\
&\qquad\qquad\qquad - 0,08437.\ \text{sin.}\ 4p-q+t\\
&\qquad\qquad\qquad + 0,01549.\ \text{sin.}\ 4p+q-t\\
&-1,28351.\ \text{sin.}\ q+t - 8,21669.\ \text{sin.}\ 2p-q-t\\
&\qquad\qquad\qquad - 0,38218.\ \text{sin.}\ 2p+q+t\\
&\qquad\qquad\qquad + 0,58627.\ \text{sin.}\ 4p-q-t\\
&\qquad\qquad\qquad - 0,00219.\ \text{sin.}\ 4p+q+t
\end{aligned}
$$

Cuius multiplicator $= 1$.

Priorum formularum vtrique aequationi communium singulos terminos ad sex figuras decimales supputauimus; quoniam vltimae demum figurae incertae esse videntur. Quamobrem, quia eos per tantos numeros multiplicare oportet, valores litterarum 𝔐 et M vltra quartam figuram producere nefas foret:

Pro euolutione prima.

Ad §. 316 et 317.

Hinc igitur valores litterarum 𝔐 et M calculo instituto sequentes sumus adepti:

pro $q-t$	𝔐 $=+99,5247$; M $=+43,0901$.
pro $2p-q+t$	𝔐 $=-21,1366$; M $=-\ 8,3899$.
pro $2p+q-t$	𝔐 $=+39,0774$; M $=+32,2447$.

Angulos autem $4p-q+t$ et $4p+q-t$ hic plane omisimus, quia inaequalitates iis respondentes ne ad quintam quidem figuram decimalem assurgerent.

Ad §. 319.

Quanquam valores litterarum $\mathfrak{M}$ et M tantum ad quatuor figuras decimales erant certi; tamen in valoribus litterarum $\mathfrak{N}$ et N incertitudo demum in quintam figuram incidit; qui ita se habere sunt deprehensi:

pro $2p-q+t$	$\mathfrak{N}=+0,14757$; $N=-0,36998$.
pro $2p+q-t$	$\mathfrak{N}=+0,01324$; $N=+0,01399$.

Ad §. 321. et §. 322.

His autem valoribus in calculum introductis collegimus:

$$\gamma=+0,14829+0,01580.\beta-0,17550.b.$$
$$c=-0,37168-0,04660.\beta+0,35870.b.$$
$$\delta=+0,01324-0,00266.\beta+0,00005.b.$$
$$d=+0,01399+0,00047.\beta+0,00193.b.$$

Ad §. 323. et 324.

Cum nunc sit

$$\gamma+\delta=+0,16153+0,01314.\beta-0,17545.b.$$
$$c+d=-0,35769-0,04613.\beta+0,36063.b.$$
$$\gamma-\delta=+0,13505+0,01846.\beta-0,17555\ b.$$
$$c-d=-0,38567-0,04707.\beta+0,35677.b.$$

inue-

inuenimus

Pro angulo $q - t$

$$\mathfrak{M} + \mathfrak{M}' = + 99,4936 + 0,05790.\beta + 0,08937.b$$
$$= \mathfrak{M} \text{ simpliciter.}$$
$$M + M' = + 43,8544 + 0,0893.\beta - 0,6332.b.$$
$$= M \text{ simpliciter.}$$

ex quibus deducuntur isti valores

$$\beta = - 0,14142 + 0,00507.\beta - 0,04324.b.$$
$$b = + 0,60046 - 0,01046.\beta + 0,09013.b.$$

Ex hac colligitur

$$b = + 0,65994 - 0,01150.\beta$$

hincque ex illa

$$\beta = - 0,16996 + 0,00557.\beta$$

ergo $\beta = - 0,17091.$

et $\quad b = + 0,66190.$

Tum vero porro

$$\gamma = + 0,02943 \quad \Big| \quad c = - 0,12631.$$
$$\delta = + 0,01372 \quad \Big| \quad d = + 0,01509.$$

ita, vt prima euolutio hanc praebeat

Conclusionem:

$$\mathfrak{V} = - 0,17091.\text{cof.}\, q - t + 0,02943.\text{cof.}\, 2p - q + t$$
$$+ 0,01372.\text{cof.}\, 2p + q - t$$
$$V = + 0,66190.\text{fin.}\, q - t - 0,12631.\text{fin.}\, 2p - q + t$$
$$+ 0,01372.\text{fin.}\, 2p + q - t.$$

Discri-

Discrimen autem, quod hinc in locum Lunae redundat, ob angulum $q-t$ tantum ad quatuor minuta secunda assurgit.

Pro euolutione secunda.

Ad §. 326. et 329.

Absoluto calculo pro litteris $\mathfrak{M}$ et M inuenimus:

pro $q+t$	$\mathfrak{M} = -105,9313$;	$M = -56,0238$
pro $2p-q-t$	$\mathfrak{M} = +20,3234$;	$M = -4,4269$
pro $2p+q+t$	$\mathfrak{M} = -4,4329$;	$M = -5,9452$.

Hinc reperiemus

pro $2p-q-t$	$N = +1,15699$;	$\mathfrak{N} = -0,46934$
pro $2p+q+t$	$N = -0,00373$;	$\mathfrak{N} = -0,00026$.

Ad §. 331. et 332.

Ex his autem valoribus litterarum N et $\mathfrak{N}$ deducimus sequentes valores:

$$\gamma = -0,47064 - 0,00711.\beta - 0,07201.b.$$

$$c = +1,15826 - 0,00003.\beta + 0,15926.b.$$

$$\delta = -0,00026 - 0,00241\,\beta + 0,00012.b.$$

$$d = -0,00373 + 0,00034\,\beta + 0,00168.b.$$

Ad

Ad §. 333.

Cum igitur hinc fiat

$\gamma + \delta = -0,47090 - 0,00952.\beta - 0,07189.b.$

$\gamma - \delta = -0,47038 - 0,00470.\beta - 0,07213.b.$

$c + d = +1,15453 + 0,00031.\beta + 0,16094.b.$

$c - d = +1,16199 - 0,00037.\beta + 0,15758.b.$

reperiemus pro $q + t$,

$\mathfrak{M} + \mathfrak{M}' = -106,0785 + 0,06971.\beta + 0,01033.b$

et

$M + M' = -58,1998 + 0,01036.\beta - 0,30568.b$

ex quibus deducuntur isti valores:

$\beta = +0,11815 + 0,00193.\beta + 0,02244.b.$

$b = -0,50797 - 0,00358.\beta - 0,04359.b.$

vnde consequimur

$b = -0,48675 - 0,00343.\beta$ et

$\beta = +0,10723 + 0,00185.\beta.$

Atque hinc concluditur

$\beta = +0,10743$ et $b = -0,48712.$

vnde sequentes obtinemus valores

$\gamma = -0,43632$	$c = +1,08068.$
$\delta = -0,00037$	$d = -0,00441.$

Omnino igitur valores litterarum $\mathfrak{V}$ et V ita prodierunt correcti:

$$\mathfrak{V} = +0,17091.\cos. q - t + 0,02943.\cos. 2p - q + t$$
$$+ 0,01372.\cos. 2p + q - t$$
$$+ 0,10743.\cos. q + t - 0,43632.\cos. 2p - q - t$$
$$- 0,00037.\cos. 2p + q + t.$$

 V =

$$V = +0,66190.\,\text{fin.}\,q-t - 0,12631.\,\text{fin.}\,2p-q+t + 0,01372.\,\text{fin.}\,2p+q-t.$$
$$-0,48712.\,\text{fin.}\,q+t + 1,08068.\,\text{fin}\,2p-q-t - 0,00441.\,\text{fin.}\,2p+q+t.$$

vbi errores demum quartam figuram decimalem adficere sunt censendi.

§. 634.

His valoribus litterarum 𝔙 et V inuentis meliore successu determinatio litterarum 𝔚 et W suscipi posset; verum quia hic labor calculos immensos postularet, qualium quidem nobis tot taediosis calculis iam satis defessis nunc quidem maxime taedet, huic labori eo magis supersedemus, quod inaequalitates Lunae inde oriundae non solum sint valde exiguae, sed etiam perpaucae, dum hi duo tantum anguli t et $2p-2q+t$ cuiuspiam momenti in loco Lunae esse possint. Hos ergo duos angulos praestabit tanquam terminos incognitos in nostras formulas introducere earunque valores ex comparatione cum obseruationibus concludere. Praeterea vero etiam ordo characteris $iiKK$, quem ob multo maiores calculi molestias ne suscipere quidem voluimus, continet memorabilem angulum $2q-2r$, cuius ratio in loco Lunae alicuius momenti esse posset; quem ergo etiam perinde ac praecedentes tanquam terminum incognitum nostris formulis annectemus; id quod tantum in coordinata

dinata y fieri opus est, cum in priore x omnes isti anguli certe nihil efficere valeant.

§. 635.

Etiamsi autem hoc immani calculo non deterreremur; tamen vehementer vereri debemus, ne iste labor incassum susciperetur. Ante quidem innuimus, terminos litterarum 𝔔 et Q; 𝔘, U et 𝔙, V, quibus in hoc calculo est vtendum, ad quinque figuras decimales iustos esse habendos; quod vtique de plerisque statui potest; verum tamen non nulli eorum tam sunt lubrici, vt vix vltra tertiam figuram certi esse queamus, veluti in littera Q coefficiens termini sin. $2p-2q$ prima adproximatione inuentus est $+0,31041$ qui deinde correctione adhibita euasit $+0,31159$ qui ergo ob vlteriores correctiones neglectas facile iam in quarta figura errare posset; qualem errorem tum non spectauimus, quia in loco Lunae ne vnum quidem minutum secundum producere valet; verum inde error notabilis in litteris 𝔚 et W esset metuendus; praeterea vero in littera U similis ratio deprehendebatur in termino sin. t, qui prima operatione tantum prodiit $0,175950$; correctione autem exereuit in $0,190587$, quo quidem acquieuimus, etiamsi hic iam tertia figura pro suspecta haberi potuisset; vnde quoque leuiusculi errores in litteras 𝔙 et V irrepere potuerunt, per se quidem contemnendi, sed qui in calculo litterarum 𝔚 et W maximi momenti essent futuri.

§. 636.

Quocirca si quis hunc laborem suscipere voluerit; is ante omnia hos memoratos terminos multo maiori cura determinare debebit; quod opus praecipue in eo consistet, vt introductas decem litteras incognitas β, γ, δ, ε, ζ et b, c, d, e, f multo maiori studio accuratissime inuestiget; tum vero denuo inde litteras 𝔙 et V adcuratius definiat; ante quam felicem successum in litteris 𝔚 et W exspectare posset. Tantos autem labores nunc quidem reformidantes inhaeremus resolutioni iam sumtae, qua terminos horum ordinum tanquam quantitates incognitas formulis nostris subiungemus, in quarum valores demum ex ipsis obseruationibus inquiramus. Calculo autem omni cura instituto reperimus:

$$\begin{aligned}
\mathfrak{U} = -0,006823.\cos. t &+ 0,029397.\cos. 2p - t \\
&- 0,003452.\cos. 2p + t \\
&+ 0,000046.\cos. 4p - t \\
&- 0,000004\ \cos. 4p + t.
\end{aligned}$$

$$\begin{aligned}
U = +0,190453.\sin. t &- 0,043312.\sin. 2p - t \\
&+ 0,005525.\sin. 2p + t \\
&- 0,000143.\sin. 4p - t \\
&+ 0,000005.\sin. 4p + t.
\end{aligned}$$

Vnde vtique leuissima correctio in litteras 𝔙 et V esset redundatura.

NOVAE

NOVAE THEORIAE MOTVVM LVNAE

LIBER SECVNDVS

CONTINENS APPLICATIONEM THEORIAE LVNAE AD CALCVLVM ASTRONOMICVM.

PARS SECVNDA.

CONSTRVCTIO TABVLARVM ASTRONOMICARVM EX FORMVLIS SVPRA INVENTIS.

CAPVT I.

EVOLVTIO VALORVM COORDINATARVM x, y ET z IN NVMERIS ABSOLVTIS.

§. 637.

Tam ex comparatione cum tabulis Cel. Clairaut quam ex ipsis obseruationibus satis perspeximus, litterarum K et i valores statui oportere

$$K = 0,0545000; \quad i = 0,0896400;$$

existente

$$a = \tfrac{1}{230} \text{ et } \varkappa = 0,0167800.$$

Ex

Ex his igitur valoribus omnes formulas, quibus ternae nostrae coordinatae supra sunt definitae, inuestigemus, quem in finem omnium coefficientium logarithmos notasse iuuabit:

Log. K = 8, 7363965.	Log. i K = 7, 6888983.
Log. K^2 = 7, 4727930.	Log. $i K^2$ = 6, 4252948.
Log. K^3 = 6, 2091895.	Log. i^2 K = 6, 6414001.
Log. i = 8, 9525018.	Log. a K = 6, 1453318.
Log. i^2 = 7, 9050036.	Log. $a\varkappa$ = 5, 6337273.
Log. i^3 = 6, 8575054.	Log. $\varkappa$ K = 6, 9611885.
Log. a = 7, 4089353.	Log. $i\varkappa$ = 7, 1772938.
Log. $\varkappa$ = 8, 2247920.	Log. $a i$ = 6, 3614371.
	Log. $i^2\varkappa$ = 6, 1297956.

§. 638.

Hinc igitur primum euoluamus formulas pro nostra quantitate x, ac ne tot fractiones decimales turbent, singulos coefficientes in partibus decies millionesimis exprimamus sequenti modo:

$$\mathfrak{Q} = + 240 - 71801.\ \text{cos.}\ 2p + 60.\ \text{cos.}\ 4p.$$

$$\begin{aligned} K\mathfrak{P} = &+ 545000.\ \text{cos.}\ q + 102296.\ \text{cos.}\ 2p - q \\ &- 1473.\ \text{cos.}\ 2p + q \\ &\quad 280.\ \text{cos.}\ 4p - q \\ &- 11.\ \text{cos.}\ 4p + q. \end{aligned}$$

$$K^2 \mathfrak{Q} = -16008. \quad +6506.\cos.2p + 58.\cos.4p$$
$$+15139.\cos.2q - 5994.\cos.2p-2q$$
$$+143.\cos.2p+2q$$
$$+677.\cos.4p-2q$$
$$+1.\cos.4p+2q$$

$$K^3.\mathfrak{R} = -0.\cos.q - 308.\cos.2p-q$$
$$-372.\cos.2p+q$$
$$-78.\cos.4p-q$$
$$-9.\cos.4p+q$$
$$-616.\cos.3q + 424.\cos.2p-3q$$
$$-11.\cos.2p+3q$$
$$-38.\cos.4p-3q$$
$$+0.\cos.4p+3q$$

$$a\mathfrak{S} = +2928.\cos.p - 74.\cos.3p.$$

$$aK\mathfrak{T} = -114.\cos.p-q - 12.\cos.3p-q$$
$$+168.\cos.p+q + 2.\cos.3p+q.$$

$$\varkappa\mathfrak{U} = -1146.\cos.t + 4933.\cos.2p-t$$
$$-579.\cos.2p+t$$
$$+7.\cos.4p-t$$
$$-1.\cos.4p+t.$$

$$\varkappa K\mathfrak{V} = -1663.\cos.q-t + 282.\cos.2p-q+t$$
$$+126.\cos.2p+q-t$$
$$+771.\cos.q+t - 3727.\cos.2p-q-t$$
$$-8.\cos.2p+q+t.$$

$$a\varkappa\mathfrak{w} = +50.\cos.p-t + 7.\cos.3p-t$$
$$+264.\cos.p+t - 2.\cos.3p+t.$$

$$i^2\mathfrak{X} = -20103 \quad + 1549.\cos.2p + 2.\cos.4p$$
$$+19870.\cos.2r - 1008.\cos.2p-2r$$
$$+ \quad 30.\cos.2p+2r$$
$$+ \quad 4.\cos.4p-2r.$$

$$i^2 K\mathfrak{Y} = -313\ \cos.2p-q - 267.\cos.q-2r$$
$$- \quad 30.\cos.2p+q - 553.\cos.q+2r$$
$$-118\ \cos.2p-q+2r - 457\ \cos.2p+q-2r$$
$$+113.\cos.2p-q-2r - \quad 4.\cos.2p+q+2r.$$

$$i^2\varkappa\mathfrak{Z} = +13.\cos.t \quad -79.\cos.2p-t$$
$$+30.\cos.2p+t$$
$$+25.\cos.t-2r - 2.\cos.2p-t+2r$$
$$-13.\cos.2p+t-2r$$
$$-14.\cos.t+2r + 38.\cos.2p-t-2r$$
$$+ \quad 1.\cos.2p+t+2r.$$

§. 639.

§. 639.

Hos cosinus in ordinem redactos in subiuncta hac tabula ita repraesentamus:

Valor Primae coordinatae x.

$- 35871.$
$+ 2928. \cos. p$
$- 63746. \cos. 2p$
$- 74. \cos. 3p$
$+ 120. \cos. 4p$
$+ 545000. \cos. q$
$+ 15139. \cos. 2q$
$- 616. \cos. 3q$
$+ 168. \cos. p+q$
$+ 143. \cos. 2p+2q$
$- 114. \cos. p-q$
$+ 5994. \cos. 2p-2q$
$+ 1875. \cos. 2p+q$
$+ 101675. \cos. 2p-q$
$+ 677. \cos. 4p-2q$
$- 20. \cos. 4p+q$
$- 358. \cos. 4p-q$
$- 11. \cos. 2p+3q$
$+ 424. \cos. 2p-3q$
$- 38. \cos. 4p-3q$
$- 1133. \cos. t$
$+ 264. \cos. p+t$
$+ 50. \cos. p-t$
$- 549. \cos. 2p+t$
$+ 4854. \cos. 2p-t$
$- 2. \cos. 3p+t$
$+ 7. \cos. 3p-t$
$- 1. \cos. 4p+t$
$+ 7. \cos. 4p-t$

$+ 771. \cos. q+t$
$- 1663. \cos. q-t$
$- 8. \cos. 2p+q+t$
$- 3727. \cos. 2p-q-t$
$+ 126. \cos. 2p+q-t$
$+ 282. \cos. 2p-q+t$
$+ 19870. \cos. 2r$
$+ 30. \cos. 2p+2r$
$- 1008. \cos. 2p-2r$
$+ 4. \cos. 4p-2r$
$- 553. \cos. q+2r$
$- 267. \cos. q-2r$
$- 4. \cos. 2p+q+2r$
$+ 113. \cos. 2p-q-2r$
$- 457. \cos. 2p+q-2r$
$- 118. \cos. 2p-q+2r$
$- 14. \cos. t+2r$
$+ 25. \cos. t-2r$
$+ 1. \cos. 2p+t+2r$
$+ 38. \cos. 2p-t-2r$
$- 13. \cos. 2p+t-2r$
$- 2. \cos. 2p-t+2r$

 §. 640.

§. 640.

Eodem modo singulos ordines pro altera coordinata y ad numeros absolutos reuocemus:

$$O = +102117.\sin.2p + 57.\sin.4p$$

$$\begin{aligned} KP = -1096890.\sin.q &- 224129.\sin.2p-q \\ &- 1750.\sin.2p+q \\ &- 394.\sin.4p-q \\ &- 10.\sin.4p+q \end{aligned}$$

$$\begin{aligned} K^2Q = +2910.\sin.2p &+ 52.\sin.4p \\ +7488.\sin.2q &+ 92551.\sin.2p-2q \\ &+ 127.\sin.2p+2q \\ &+ 351.\sin.4p-2q \\ &+ 1.\sin.4p+2q \end{aligned}$$

$$\begin{aligned} K^3R = +2206.\sin.q &+ 689.\sin.2p-q - 77.\sin.4p-q \\ &- 57.\sin.2p+q + 2.\sin.4p+q \\ -478.\sin.3q &- 358.\sin.2p-3q \\ &- 9.\sin.2p+3q \\ &- 11.\sin.4p-3q \\ &- 1.\sin.4p+3q \end{aligned}$$

$$aS = -6162.\sin.p + 73.\sin.3p$$

$$\begin{aligned} aKT = +2523.\sin.p-q &+ 101.\sin.3p-q \\ + 84.\sin.p+q &- 1.\sin.3p+q \end{aligned}$$

$$\begin{aligned} \varkappa U = +31979.\sin.t - 7268.\sin.2p-t &- 23.\sin.4p-t \\ + 927.\sin.2p+t &+ 1.\sin.4p+t \end{aligned}$$

$\varkappa KV =$

$$\varkappa KV = +6053.\sin.q-t-1152.\sin.2p-q+t$$
$$+\ 125.\sin.2p+q-t$$
$$-4455\ \sin.q+t+9883.\sin.2p-q-t$$
$$-\ 40.\sin.2p+q+t$$

$$a\varkappa w = -\ 47.\sin.p-t-7.\sin.3p-t$$
$$-543.\sin.p+t+1.\sin.3p+t$$

$$iiX = -1723.\sin.2p\ -\ 2.\sin.4p-19803.\sin.2r$$
$$+2738.\sin.2p-2r-11.\sin.4p-2r$$
$$-\ 30.\sin.2p+2r$$

$$iiK\dot{Y} = +6.\sin.q+839.\sin.2p-q$$
$$+\ 41.\sin.2p+q$$
$$-2174.\sin.q-2r+126.\sin.2p-q+2r$$
$$+790\ \sin.2p+q-2r$$
$$+\ 548.\sin.q+2r-261.\sin.2p-q-2r$$
$$+\ 3.\sin.2p+q+2r.$$

$$ii\varkappa Z = -336.\sin.t+94.\sin.2p-t-33.\sin.2p+t$$
$$+\ 28.\sin.t-2r+\ 2.\sin.2p-t+2r$$
$$+137.\sin.2p+t-2r$$
$$+\ 18\ \sin.t+2r-102.\sin.2p-t-2r$$
$$-\ 1.\sin.2p+t+2r.$$

§. 641.

His sinibus pariter in ordinem dispositis orietur

Valor Secundae coordinatae y.

$-\ 6162.\ \text{sin.}\ p$	$-\ 4455.\ \text{sin.}\ q+t$
$+\ 103304.\ \text{sin.}\ 2p$	$+\ 6053.\ \text{sin.}\ q-t$
$+\ 73.\ \text{sin.}\ 3p$	$-\ 40.\ \text{sin.}\ 2p+q+t$
$+\ 107.\ \text{sin.}\ 4p$	$+\ 9883.\ \text{sin.}\ 2p-q-t$
$-\ 1094678.\ \text{sin.}\ q$	$+\ 125.\ \text{sin.}\ 2p+q-t$
$+\ 7488.\ \text{sin.}\ 2q$	$-\ 1152.\ \text{sin.}\ 2p-q+t$
$-\ 478.\ \text{sin.}\ 3q$	$-\ 19803.\ \text{sin.}\ 2r$
$+\ 84.\ \text{sin.}\ p+q$	$-\ 30.\ \text{sin.}\ 2p+2r$
$+\ 127.\ \text{sin.}\ 2p+2q$	$+\ 2738.\ \text{sin.}\ 2p-2r$
$+\ 2523\ \text{sin.}\ p-q$	$-\ 11.\ \text{sin.}\ 4p-2r$
$+\ 9255.\ \text{sin.}\ 2p-2q$	$+\ 548.\ \text{sin.}\ q+2r$
$-\ 1766.\ \text{sin.}\ 2p+q$	$-\ 2174.\ \text{sin.}\ q-2r$
$+\ 1.\ \text{sin.}\ 4p+2q$	$+\ 3.\ \text{sin.}\ 2p+q+2r$
$-\ 222561.\ \text{sin.}\ 2p-q$	$-\ 261.\ \text{sin.}\ 2p-q-2r$
$+\ 251.\ \text{sin.}\ 4p-2q$	$+\ 790.\ \text{sin.}\ 2p+q-2r$
$-\ 1.\ \text{sin.}\ 3p+q$	$+\ 126.\ \text{sin.}\ 2p-q+2r$
$+\ 101.\ \text{sin.}\ 3p-q$	$+\ 13.\ \text{sin.}\ t+2r$
$-\ 8.\ \text{sin.}\ 4p+q$	$+\ 28.\ \text{sin.}\ t-2r$
$-\ 471.\ \text{sin.}\ 4p-q$	$-\ 1.\ \text{sin.}\ 2p+t+2r$
$-\ 9.\ \text{sin.}\ 2p+3q$	$-\ 102.\ \text{sin.}\ 2p-t-2r$
$-\ 358.\ \text{sin.}\ 2p-3q$	$+\ 137.\ \text{sin.}\ 2p+t-2r$
$-\ 1.\ \text{sin.}\ 4p+3q$	$+\ 2.\ \text{sin.}\ 2p-t+2r$
$-\ 11.\ \text{sin.}\ 4p-3q$	
$+\ 31643.\ \text{sin.}\ t$	
$-\ 543.\ \text{sin.}\ p+t$	
$-\ 47.\ \text{sin.}\ p-t$	
$+\ 894.\ \text{sin.}\ 2p+t$	
$-\ 7174.\ \text{sin.}\ 2p-t$	
$+\ 1.\ \text{sin.}\ 3p+t$	
$-\ 7\ \text{sin.}\ 3p-t$	
$+\ 1.\ \text{sin.}\ 4p+t$	
$-\ 23.\ \text{sin.}\ 4p-t$	

§. 642.

§. 642.

Denique eodem modo formulas pro tertia coordinata z datas in numeros euoluamus:

$$ip = +896400.\sin.r + 33151.\sin.2p-r + 42.\sin.4p-r$$
$$+ 1356.\sin.2p+r + 5.\sin.4p+r.$$

$$iKq = -72460.\sin.q-r - 5447.\sin.2p-q+r$$
$$- 798.\sin.2p+q-r$$
$$- 31.\sin.4p-q+r$$
$$- 4.\sin.4p+q-r$$
$$-24669.\sin.q+r - 11787.\sin.2p-q-r$$
$$- 145.\sin.2p+q+r$$
$$- 178.\sin.4p-q-r$$
$$- 1.\sin.4p+q+r.$$

$$iK^2r = +97.\sin.2p-r + 423.\sin.2p+r$$
$$+912.\sin.2q-r + 132.\sin.2p-2q+r$$
$$+ 33.\sin.2p+2q-r$$
$$+1012.\sin.2q+r + 453.\sin.2p-2q-r$$
$$+ 12.\sin.2p+2q+r.$$

$$i\varkappa s = -302.\sin.r-t + 601.\sin.2p-r+t$$
$$- 103.\sin.2p+r-t$$
$$+252.\sin.r+t - 1649.\sin.2p-r-t$$
$$+ 13.\sin.2p+r+t.$$

$$i^3t = +17.\sin.2p-r + 3.\sin.3r + 127.\sin.2p-3r$$
$$- 5.\sin.2p+r + 13.\sin.4p-3r.$$

$$ai.u = -363.\sin.p-r - 1.\sin.3p-r$$
$$-139.\sin.p+r.$$

§. 643.

§. 643.

His igitur finibus in ordinem digestis prodit

Valor Tertiae coordinatae z

$+\ 896400.\ \sin. r$	$+\ 1012.\ \sin. 2q+r$
$+\ 3.\ \sin. 3r$	$+\ 912.\ \sin. 2q-r$
$-\ 130.\ \sin. p+r$	$+\ 12.\ \sin. 2p+2q+r$
$-\ 363.\ \sin. p-r$	$+\ 453.\ \sin. 2p-2q-r$
$+\ 1774.\ \sin. 2p+r$	$+\ 33.\ \sin. 2p+2q-r$
$+\ 33265.\ \sin. 2p-r$	$+\ 132.\ \sin. 2p-2q+r$
$-\ 1.\ \sin. 3p-r$	$+\ 252.\ \sin. r+t$
$+\ 5.\ \sin 4p+r$	$-\ 302.\ \sin. r-t$
$+\ 42.\ \sin. 4p-r$	$+\ 13.\ \sin. 2p+r+t$
$+\ 127.\ \sin. 2p-3r$	$-\ 1649.\ \sin. 2p-r-t$
$-\ 24669.\ \sin. q+r$	$-\ 103.\ \sin. 2p+r-t$
$-\ 72460.\ \sin. q-r$	$+\ 601.\ \sin. 2p-r+t.$
$-\ 145.\ \sin. 2p+q+r$	
$-\ 11787.\ \sin. 2p-q-r$	
$-\ 798.\ \sin. 2p+q-r$	
$-\ 5447.\ \sin. 2p-q+r$	

§. 644.

§. 644.

Ex his igitur formulis more solito tabulas construamus, ex quibus pro datis vtcunque angulis p, q, r et t valores nostrarum ternarum coordinatarum supputari queant; vbi quidem per se manifestum est, eiusdem anguli multiplos in vnam tabulam compingi posse; vnde tabularum numerus non adeo magnus euadet; praecipue quia eas formulas tuto omittere possumus, quarum coefficientes infra numerum 250 subsistant; quandoquidem sufficit eas tantum inaequalitates notasse, quae supra quinque minuta secunda exsurgunt. Hic autem probe meminisse oportet valores coordinatarum ex his tabulis deductos non esse numeros integros, sed partes tantum decies millionesimas vnitatis, siue eos diuidi debere per 10000000.

 CAPVT II.

CAPVT II.

NOVAE TABVLAE LVNARES SINGVLARI METHODO CONSTRVCTAE, QVARVM OPE LOCA LVNAE AD QVODVIS TEMPVS EXPEDITE COMPVTARE LICET.

§. 645.

PRAECEPTA PRO VSV HARVM TABVLARVM.

I. Tempus propofitum reducatur ad tempus medium fub meridiano Parifino, atque ex tabulis mediorum motuum Majerianis depromantur fequentia elementa:

1°. Longitudo Lunae media = L

2°. Longitudo Apogei Lunae = P

3°. Longitudo Nodi afcendentis media = N

4°.

4°. Longitudo Solis media $= \lambda$

5°. Longitudo Apogei Solis $= \pi$.

II. Ex his quinque elementis colligantur ſequentes quatuor anguli:

1°. Elongatio media Lunae a Sole $= p = L - \lambda$.

2°. Anomalia Lunae media $= q = L - P$.

3°. Argumentum latitudinis medium $= r = L - N$.

4°. Anomalia media Solis $= t = \lambda - \pi$.

III. Ex his quatuor angulis ſucceſſiue formentur omnia argumenta, quae tituli ſequentium tabularum prae ſe ferunt; atque ex iis omnibus aequationibus debite colligendis innoteſcent valores ternarum coordinatarum x, y et z in partibus decies millioneſimis vnitatis expreſſi.

IV. Quemadmodum autem has coordinatas intelligi conueniat, adiuncta figura declarabit, vbi ♁ denotat locum centri Terrae indeque educta recta ♁M longitudinem Lunae mediam, ſagitta Mm ordinem ſignorum indicante; tum vero ſit centrum Lunae in ☽, vnde ad planum eclipticae demiſſo perpendiculo ☽l, ex puncto l ad rectam ♁M ducatur normalis lL; ac ſumatur interuallum ♁O = diſtantiae mediae Terrae a Luna, quae hic vnitate diſignatur, ſiue 10000000 par- Fig. IV.

 tibus

tibus decies millionefimis, hoc iam notato ternae illae coordinatae in hac figura erunt $OL = x$; $Ll = y$; et $l☽ = z$.

V. Harum autem coordinatarum binae priores $OL = x$; et $Ll = y$ ex communibus argumentis determinantur; quam ob rem fingulae tabulae huc pertinentes binis conftant partibus, quarum priores coordinatam x, pofteriores vero coordinatam y exhibent; tertia vero $l☽ = z$ argumenta peculiaria poftulat; vnde etiam tabulae ei deftinatae feorfim exhibentur.

VI. Cum autem pro quouis tempore valores harum ternarum coordinatarum fuerint excerpti et, prouti figna adfcripta indicant, inuicem collecti, habebuntur earum valores in partibus deciesmillionefimis expreffi, atque fi ad primam $OL = x$ addatur diftantia media $♁O = 1$. feu 10000000, obtinebitur interuallum $♁L = 1 + x$.

VII. Ex his autem tribus quantitatibus $1 + x$, y et z omnia, quae ad locum Lunae pertinent, facile deriuantur; primum enim pro longitudine Lunae vera, quam refert recta $♁l$, computetur angulus $L♁l$, quem figno Φ fignauimus, ex formula tang. $\Phi = \frac{y}{1+x}$, qui angulus, prouti fuerit fiue pofitiuus fiue negatiuus, cum longitudine

dine media L coniunctus statim praebet longitudinem Lunae veram ♉ l.

VIII. Inuento autem angulo $\Phi = L$ ♉ l, latitudo Lunae, quae angulo ☾ ♉ $l = \psi$ exhibetur, ope huius formulae expedite definietur tang. $\psi = \frac{z.\,cos.\,\Phi}{1+x}$ atque hic angulus ψ, prout prodierit siue positiuus siue negatiuus, latitudinem Lunae siue borealem siue australem dabit.

IX. Denique quod ad parallaxin Lunae attinet, cum distantia Lunae a Terra sit ♉ ☽ $= (1+x)$ sec. Φ. sec. ψ parallaxin Lunae horizontalem aequatoream ita in minutis secundis exprimi inuenimus, vt ea sit $= \frac{3408''}{(1+x)\,sec.\,\Phi.\,sec.\,\psi}$ siue quia distantia $1+x$ in partibus decies millionesimis datur, ista parallaxis erit $= 3408''. \frac{10000000.\,cos.\,\Phi.\,cos.\,\psi}{1+x}$.

§. 646.

Quo vsus horum praeceptorum facilius perspiciatur, sequens exemplum speciminis loco subiungamus.

Exemplum.

Quaeratur locus Lunae ad tempus Parisinum medium 1754. April. $12^d.\ 17^h.\ 23'.\ 54''$.

Pro quo tempore exhibita est ex obseruatione

Longitudo Lunae $9^s\ 9^\circ\ 14'\ 51''$.

Latitudo Lunae bor. $5^\circ\ 14'\ 28''$.

N°. I. Ex Tab. Majerinis fiat hic calculus:

	L	P	N	λ	π
1754.	$11^s 28^\circ 59' 58''$	$11^s\ 3^\circ 42' 50''$	$6^s 22^\circ 57'\ 6''$	$9^s 10^\circ\ 2' 33''$	$3^s 8^\circ 41'\ 7''$
Apr. 12.	$8^s 23^\circ 59' 33''$	$11^\circ 21' 49''$	$5^\circ 24'\ 5''$	$3^s 10^\circ 32' 10''$	$18''$
17^h	$9^\circ 20'\ 0''$	$4' 44''$	$2' 15''$	$41' 53''$	
$23'$	$12' 38''$	$6''$	$3''$	$56''$	
$54''$	$30''$			$2''$	
	$9^s\ 2^\circ 32' 39''$	$11^s 15^\circ\ 9' 29''$	$6^s 17^\circ 30' 43''$	$0^s 21^\circ 17' 34''$	$3^s 8^\circ 41' 25''$

N°. II. Hinc deducamus quatuor nostra elementa principalia:

$L = 9^s\ 2^\circ 32' 39''$	$L = 9^s\ 2^\circ 32' 39''$	$L = 9^s\ 2^\circ 32' 39''$	$\lambda = 0^s 21^\circ 17' 34''$
$\lambda = 0^s 21^\circ 17' 34''$	$P = 11^s 15^\circ\ 9' 29''$	$N = 6^s 17^\circ 30' 43''$	$\pi = 3^s\ 8^\circ 41' 25''$
$p = 8^s 11^\circ 15'\ 5''$	$q = 9^s 17^\circ 23' 10''$	$r = 2^s 15^\circ\ 1' 56''$	$t = 9^s 12^\circ 36'\ 9''$

N°. III.

N°. III. Ex his iam primum colligantur omnia argumenta tabularum priorum litteris x et y destinatarum.

$p = 8^s 11^\circ 15' 5''$. I.	$q = 9^s 17^\circ 23' 10''$
$q = 9^s 17^\circ 23' 10''$. II.	$t = 9^s 12^\circ 36' 9''$
$p+q = 5^s 28^\circ 38' 15''$ III.	$q+t = 6^s 29^\circ 59' 19''$. XII.
$p-q = 10^s 23^\circ 51' 55''$. IV.	$q-t = 0^s 4^\circ 47' 1''$. XIII.
$p = 8^s 11^\circ 15' 5''$.	$2p = 4^s 22^\circ 30' 10''$.
$2p+q = 2^s 9^\circ 53' 20''$. V.	$2p-q-t = 9^s 22^\circ 30' 51''$. XIV.
$2p-q = 7^s 5^\circ 7' 0''$. VI.	$2p-q+t = 4^s 17^\circ 43' 9''$. XV.
$2p = 4^s 22^\circ 30' 10''$.	$p+t = 5^s 23^\circ 51' 14''$. XVI.
$4p-q = 11^s 27^\circ 37' 10''$. VII.	$2p = 4^s 22^\circ 30' 10''$.
$2p+2q = 4^s 19^\circ 46' 40''$.	$2r = 5^s 0^\circ 3' 52''$. XVII.
$2p-3q = 7^s 7^\circ 50' 30''$. VIII.	$2p-2r = 11^s 22^\circ 26' 18''$. XVIII.
$t = 9^s 12^\circ 36' 9''$. IX.	$q = 9^s 17^\circ 23' 10''$.
$2p = 4^s 22^\circ 30' 10''$.	$2r = 5^s 0^\circ 3' 52''$.
$2p+t = 2^s 5^\circ 6' 19''$. X.	$q+2r = 2^s 17^\circ 27' 2''$. XIX.
$2p-t = 7^s 9^\circ 54' 1''$. XI.	$q-2r = 4^s 17^\circ 19' 18''$. XX.
	$2p = 4^s 22^\circ 30' 10''$.
	$2p+q-2r = 9^s 9^\circ 49' 28''$. XXI.

N°. IV.

N°. IV. Nunc singulae tabulae priores sequenti modo euoluantur:

	Pro x. +	Pro x. −	Pro y. +	Pro y. −
I. $8^s\ 11^\circ \frac{1}{4} + \frac{1}{720}$	13724		68652	
II. $9^s\ 17^\circ \frac{1}{3} + \frac{1}{20} + \frac{1}{300}$	150892		1040062	
III. $5^s\ 28^\circ \frac{1}{2}$		25		6
IV. $10^s\ 23^\circ \frac{1}{30}$		1750		10412
V. $2^s\ 9^\circ \frac{1}{2} + \frac{1}{3}$		639		1656
VI. $7^s\ 5^\circ \frac{1}{10} + \frac{1}{60}$		82938	128378	
VII. $11^s\ 27^\circ \frac{1}{2} + \frac{1}{10}$		358		20
VIII. $7^s\ 7^\circ \frac{1}{2} + \frac{1}{30}$		336	216	
	164616	86046	1237308	12094
IX. $9^s\ 12^\circ \frac{1}{2} + \frac{1}{10}$		232		30854
X. $2^s\ 5^\circ \frac{1}{10}$		229	811	
XI. $7^s\ 9^\circ \frac{1}{2} + \frac{1}{3} + \frac{1}{15}$		3723	4599	
	164616	90230	1242718	42948
XII. $6^s\ 29^\circ\ 1 - \frac{1}{60}$		665	2218	
XIII. $0^s\ 4^\circ \frac{1}{2} + \frac{1}{60}$		1652	471	
XIV. $9^s\ 22^\circ \frac{1}{2}$		1424		9118
XV. $4^s\ 17^\circ \frac{1}{2} + \frac{1}{6}$		210		779
XVI. $5^s\ 23^\circ \frac{1}{2} + \frac{1}{3}$		263		61
	164616	94444	1245407	52906

XVII.

	Pro x.		Pro y.	
	+	−	+	−
	164616	94444	1245407	52906
XVII. $5^s\, 0^\circ \frac{1}{10}$		17212		9900
XVIII. $11^s\, 22^\circ \frac{1}{3}$		991		371
XIX. $2^s\, 17^\circ \frac{1}{3}$		121	533	
XX. $4^s\, 17^\circ \frac{1}{4}$	197			1476
XXI. $9^s\, 9^\circ \frac{1}{2} + \frac{1}{6}$		72		780
	+ 164813	− 112840	+ 1245940	− 65433
	− 112840		− 65433	
	+ 51973	$y =$	+ 1180507	
	10000000			

$$1 + x = 10051973$$

N°. V. Hinc longitudo vera eruitur hoc modo:

$$\begin{aligned}
\text{a log. } y &= +6,0720685 \\
\text{subtr. log. } (1+x) &= +7,0022509 \\
\text{log. tang. } \Phi &= 9,0698176 \\
\text{ergo } \Phi &= +6^\circ\, 41'\, 51'' \\
\text{long. med. } &= 9^s\, 2^\circ\, 32'\, 39'' \\
\text{long. ver. } &= 9^s\, 9^\circ\, 14'\, 30'' \\
\text{at obseruata erat } &= 9^s\, 9^\circ\, 14'\, 51'' \\
\text{vnde differentia est } & \quad 0^\circ\, 0'\, 22''.
\end{aligned}$$

N°. VI. Simili modo formentur argumenta pro coordinata z secundum titulos harum tabularum.

$$\begin{aligned} 2p &= 4^s\,22^\circ\,30'\,10'' \\ r &= 2^s\,15^\circ\;\;1'\,56''\ \text{I.} \end{aligned}$$

$$\begin{aligned} 2p+r &= 7^s\;\;7^\circ\,32'\;\;6''\ \text{II.} \\ 2p-r &= 2^s\;\;7^\circ\,28'\,14''\ \text{III.} \\ q &= 9^s\,17^\circ\,23'\,10'' \\ r &= 2^s\,15^\circ\;\;1'\,56'' \end{aligned}$$

$$\begin{aligned} q+r &= 0^s\;\;2^\circ\,25'\;\;6''\ \text{IV.} \\ q-r &= 7^s\;\;2^\circ\,21'\,14''\ \text{V.} \\ 2p &= 4^s\,22^\circ\,30'\,10'' \end{aligned}$$

$$\begin{aligned} 2p-q-r &= 4^s\,20^\circ\;\;5'\;\;4''\ \text{VI.} \\ 2p+q-r &= 11^s\,24^\circ\,51'\,24''\ \text{VII.} \\ 2p-q+r &= 9^s\,20^\circ\;\;8'\,56''\ \text{VIII.} \end{aligned}$$

$$\begin{aligned} 2q &= 7^s\;\;4^\circ\,46'\,20'' \\ r &= 2^s\,15^\circ\;\;1'\,56'' \end{aligned}$$

$$\begin{aligned} 2q+r &= 9^s\,19^\circ\,48'\,16''\ \text{IX.} \\ 2q-r &= 4^s\,19^\circ\,44'\,24''\ \text{X.} \\ 2p &= 4^s\,22^\circ\,30'\,10'' \end{aligned}$$

$$\begin{aligned} 2p-2q-r &= 7^s\;\;2^\circ\,41'\,54''\ \text{XI.} \\ r &= 2^s\,15^\circ\;\;1'\,56'' \\ t &= 9^s\,12^\circ\,36'\;\;9'' \end{aligned}$$

$$\begin{aligned} r+t &= 11^s\,27^\circ\,38'\;\;5''\ \text{XII.} \\ r-t &= 5^s\;\;2^\circ\,25'\,47''\ \text{XIII.} \\ 2p &= 4^s\,22^\circ\,30'\,10'' \end{aligned}$$

$$\begin{aligned} 2p-r-t &= 4^s\,24^\circ\,52'\;\;5''\ \text{XIV.} \\ 2p-r+t &= 11^s\,20^\circ\;\;4'\,23''\ \text{XV.} \\ p &= 8^s\,11^\circ\,15'\;\;5'' \\ r &= 2^s\,15^\circ\;\;1'\,56'' \end{aligned}$$

$$p-r = 5^s\,26^\circ\,13'\,15''\ \text{XVI.}$$

N°. VII.

N°. VII. Iam per haec argumenta singulae tabulae euoluantur, vt sequitur:

	+	−
I. $2^s\ 15^\circ\ \frac{1}{30}$	865884	
II. $7^s\ 7^\circ\ \frac{1}{2}+\frac{1}{30}$		1083
III. $2^s\ 7^\circ\ \frac{1}{8}$	30753	
IV. $0^s\ 2^\circ\ \frac{1}{3}$		1003
V. $7^s\ 2^\circ\ \frac{1}{3}$	38753	
VI. $4^s\ 20^\circ\ \frac{1}{10}$		7589
VII. $11^s\ 24^\circ\ \frac{1}{2}+\frac{1}{3}$	73	
VIII. $9^s\ 20^\circ\ \frac{1}{10}$	5108	
IX. $9^s\ 19^\circ\ \frac{5}{6}$		948
X. $4^s\ 19^\circ\ \frac{1}{2}+\frac{1}{6}$	593	
XI. $7^s\ 2^\circ\ \frac{1}{2}+\frac{1}{6}$		241
XII. $11^s\ 27^\circ\ \frac{1}{2}+\frac{1}{10}$		11
XIII. $5^s\ 2^\circ\ \frac{1}{3}$		140
XIV. $4^s\ 24^\circ\ \frac{1}{2}+\frac{1}{3}$		953
XV. $11^s\ 20^\circ\ \frac{1}{15}$		103
XVI. $5^s\ 26^\circ\ \frac{1}{8}$		19
	+ 941164	− 12090
	− 12090	
ergo $z =$	+ 929074	

N°. VIII.

N°. VIII. Hinc ergo ipsa latitudo Lunae sequenti modo definitur:

$$\begin{array}{rl}
\text{ad Log. } z = & +5,9680502 \\
\text{adde Log. cos. } \Phi = & +9,9970263 \\
\hline
& +5,9650765 \\
\text{subtr. Log. } (1+x) = & +7,0022509 \\
\hline
\text{Log. tang. } \psi = & 8,9628256
\end{array}$$

$$\begin{array}{rl}
\text{ergo latit. } \psi = & 5^\circ\ 14'\ 41''.\ \text{Bor.} \\
\text{at observata} = & 5^\circ\ 14'\ 28''. \\
\hline
\text{hinc differ.} = & +\ 13''.
\end{array}$$

N°. IX. Denique parallaxis horizontalis aequatorea pro hoc tempore ita determinabitur:

$$\begin{array}{rl}
\text{Log. num. const.} = & 10,5324996 \\
\text{Log. cos. } \Phi = & 9,9970263 \\
\text{Log. cos. } \psi = & 9,9981782 \\
\hline
& 0,5277041 \\
\text{Log. } (1+x) = & 7,0022509 \\
\hline
& 3,5254532
\end{array}$$

ergo parall. aequatorea $= 3353'' = 55'\ 53''$.

NOVAE

NOVAE

TABVLAE LVNARES.

Tabula Aequationum I^{ma}.

Argumentum angulus *p*.

Pro coordinata *x*.

	o. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	−	−	∓	+	−	−	
0	96643	65268	2517	27995	5599	70340	30
1	96605	63217	− 648	27901	7595	72260	29
2	96491	61399	+ 1178	27729	9626	74140	28
3	96300	59413	2965	27479	11791	75975	27
4	96033	57394	4702	27150	13787	77763	26
5	95691	55348	6393	26744	15933	80502	25
6	95273	53274	8032	26261	18061	81191	24
7	94782	51178	9620	25702	20232	82820	23
8	94215	49040	11153	25069	22425	84407	22
9	93575	45924	12629	24360	24655	85934	21
10	92861	44772	14047	23577	26860	87396	20
11	92077	42609	15405	22729	29097	88802	19
12	91222	40432	16703	21792	31344	90148	18
13	90296	38250	17936	20792	33599	91428	17
14	89302	36056	19102	19723	35856	92644	16
15	88235	33869	20199	18579	38113	93787	15
16	87112	31678	21237	17378	40366	94874	14
17	85920	29487	22202	16108	42624	95886	13
18	84664	27306	23096	14773	44970	96830	12
19	83346	25131	23909	13375	47107	97701	11
20	81968	22968	24667	11917	49332	98499	10
21	80526	20799	25344	10399	52542	99225	9
22	79037	18685	25943	8823	53714	99877	8
23	77494	16570	26470	7194	55908	100454	7
24	75887	14479	26919	5510	58058	100955	6
25	73234	12389	27292	3775	60182	101381	5
26	72529	10369	27588	1990	62278	101729	4
27	70779	8455	27807	+ 159	64345	102002	3
28	68984	6376	27947	− 1718	66379	102197	2
29	67146	4427	28011	3638	68245	102311	1
30	65268	2517	27995	5599	70340	102351	0
	−	−	+	±	−	−	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. Sg.	VII. Sig.	VI. Sig.	

Tabula

Tabula Aequationum I^{ma}.

Argumentum angulus *p*.

Pro coordinata *y*.

	0. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	+	+	+	−	−	−	
0	0	86549	84035	6235	94707	92561	30
1	3509	88201	82117	9832	96402	90615	29
2	7013	89741	80097	13421	97984	88562	28
3	10508	91169	77973	17002	99451	86401	27
4	13991	92482	75748	20566	100802	84137	26
5	17456	93680	73428	24111	102033	81774	25
6	20900	94759	71012	27633	103146	79312	24
7	24318	95719	68505	31127	104139	76756	23
8	27705	96559	65911	34587	105010	74108	22
9	31055	97276	63232	38011	105757	71372	21
10	34368	97874	60471	41396	106382	68551	20
11	37637	98349	57630	44735	106882	65648	19
12	40859	98695	54715	48026	107150	62666	18
13	44029	98926	51729	51263	107501	59608	17
14	47144	99027	48676	54444	107620	56482	16
15	50201	98999	45557	57563	107609	53287	15
16	53196	98854	42376	60618	107479	50028	14
17	56122	98579	39139	63605	107214	46709	13
18	58978	98276	35852	66521	106819	43335	12
19	61760	97660	32515	69362	106309	39909	11
20	64465	97016	29132	72125	105668	36434	10
21	67088	96247	25707	74804	104900	32915	9
22	69628	95360	22249	77397	104011	29359	8
23	72080	94351	18757	79903	102999	25766	7
24	74440	93224	15237	82316	101863	22142	6
25	76708	91977	11693	84634	100606	18492	5
26	78879	90614	8130	86852	99230	14821	4
27	80953	89137	4550	88973	97735	11132	3
28	82922	87546	± 961	90991	96125	7429	2
29	84789	85846	∓ 2638	92903	94399	3717	1
30	86549	84035	6235	94707	92561	0	0
	−	−	∓	+	+	+	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. Sg	VII. Sig	VI. Sig.	

Tabula

Tabula Aequationum II.
Argumentum angulus q.
Pro coordinata x.

	0. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	+	+	+	−	−	−	
0	559523	479553	265547	15139	280685	464415	30
1	559412	474296	256815	24673	288418	468614	29
2	559157	468886	248009	34187	296055	472676	28
3	558700	463330	239136	43675	303595	476604	27
4	558062	457624	230194	53137	311033	480396	26
5	557240	451775	221191	62568	318372	484048	25
6	556236	445782	212128	71966	325607	487562	24
7	555051	439651	203007	81329	332736	490938	23
8	553685	433279	193834	90653	339761	494174	22
9	552139	426972	184610	99935	346676	497271	21
10	550412	420431	175338	109172	353482	500227	20
11	548507	413759	166022	118363	360176	503043	19
12	546423	406958	156664	127504	366756	505716	18
13	544160	400032	147271	136593	373224	508247	17
14	541720	392986	137843	145626	379574	510638	16
15	539105	385809	128381	154603	385809	512883	15
16	536314	378518	118892	163519	391924	514986	14
17	533349	371112	109879	172373	397920	516946	13
18	530212	363592	99844	181160	403794	518763	12
19	526903	355962	90289	189882	409545	520433	11
20	523421	348224	80720	198532	415172	521960	10
21	519771	340382	71189	207110	420678	523343	9
22	515954	332437	61549	215614	426055	524581	8
23	511970	324390	51951	224039	431305	525673	7
24	507822	316257	42350	232388	436426	526620	6
25	503510	308016	32750	240653	441419	527422	5
26	499036	299691	23153	248834	446282	528078	4
27	494400	291279	13563	256932	451014	528588	3
28	489608	282783	+3983	264941	455614	528953	2
29	484658	274204	−5586	272859	460082	529172	1
30	479553	265547	15139	280685	464415	529245	0
	+	+	±	−	−	−	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. S.	VII. Sig.	VI. Sig.	

Tabula

Tabula Aequationum IIda.

Argumentum angulus *q*.

Pro coordinata *y*.

	o. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	—	—	—	—	—	+	
0	00000	541332	941534	1094200	954504	554302	30
1	18869	557667	951097	1094295	944958	537537	29
2	37732	573836	960285	1094058	935120	520603	28
3	56583	589835	969233	1093490	924990	503503	27
4	75413	605661	977890	1092586	914571	486274	26
5	94231	621309	986255	1091349	903868	468830	25
6	113016	636770	994325	1089783	892882	451266	24
7	131769	652043	1002108	1087882	881519	433558	23
8	150481	667122	1009571	1085650	870079	415713	22
9	169148	682005	1016743	1083089	858266	397734	21
10	187767	696686	1023609	1080195	846185	379629	20
11	206329	711159	1030168	1076970	833839	361403	19
12	224831	725424	1036409	1073416	821232	343062	18
13	243267	739470	1042358	1069533	808368	324611	17
14	261632	753299	1047986	1065322	795249	306057	16
15	279917	766904	1053296	1060784	781880	287405	15
16	298121	780283	1058292	1055922	768265	268662	14
17	316237	793428	1062967	1050732	754410	249833	13
18	334260	806338	1067324	1045221	740318	230923	12
19	352183	819009	1071360	1039388	725989	211939	11
20	370003	831437	1075073	1033235	711434	192889	10
21	387714	843618	1078461	1026763	696653	173776	9
22	405309	855547	1081522	1019975	681654	154609	8
23	422786	867223	1084260	1012880	666439	135391	7
24	440136	878640	1086669	1005455	651012	116130	6
25	457358	889796	1088749	997727	635381	96831	5
26	474472	900685	1090502	989692	619547	77502	4
27	491387	911308	1091924	981349	603517	58149	3
28	508187	921660	1093014	972701	587296	38776	2
29	524837	931736	1093773	963707	570889	19391	1
30	541332	941534	1094200	954504	554302	00000	0
	+	+	+	+	+	+	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. Sig.	VII. Sig.	VI. Sig.	

Tabula Aequationum III.

Argumentum angulus $p + q$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	±	−	−	−	
0	311	219	13	143	155	77	30
1	311	214	7	146	155	74	29
2	311	209	+1	149	154	70	28
3	310	202	−7	152	152	67	27
4	310	196	14	155	150	64	26
5	309	189	20	156	147	61	25
6	306	182	26	157	145	58	24
7	305	175	33	159	143	56	23
8	304	168	39	161	141	53	22
9	302	161	45	162	138	51	21
10	300	154	51	163	134	48	20
11	299	148	57	165	131	46	19
12	296	141	62	166	128	45	18
13	293	134	68	166	125	43	17
14	290	127	74	167	122	41	16
15	286	119	80	167	119	39	15
16	282	112	85	167	116	38	14
17	279	105	90	167	113	35	13
18	275	98	96	168	112	34	12
19	272	91	101	169	108	32	11
20	267	84	104	168	104	31	10
21	264	77	109	168	101	29	9
22	260	70	113	166	98	29	8
23	256	62	117	167	95	28	7
24	251	54	121	166	92	27	6
25	246	47	125	163	90	27	5
26	242	41	129	163	88	26	4
27	236	34	132	161	85	26	3
28	231	27	136	160	82	25	2
29	225	20	140	158	80	25	1
30	219	13	143	155	77	25	0
	+	+	∓	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum III.

Argumentum angulus $p+q$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	−	−	
0	0	152	187	85	37	68	30
1	7	156	183	79	39	67	29
2	12	160	182	74	42	66	28
3	19	164	180	70	46	65	27
4	25	167	178	65	49	63	26
5	31	169	175	61	50	63	25
6	37	172	172	56	52	61	24
7	43	174	169	52	55	58	23
8	49	177	166	47	57	57	22
9	54	180	164	43	59	55	21
10	59	180	161	38	60	52	20
11	65	182	158	34	62	51	19
12	69	184	154	29	63	47	18
13	76	186	151	25	65	45	17
14	81	187	147	20	67	44	16
15	86	187	145	17	67	41	15
16	91	188	141	12	68	40	14
17	96	189	137	9	69	37	13
18	101	189	133	5	69	35	12
19	107	190	130	+ 1	69	32	11
20	112	190	126	− 4	69	29	10
21	116	190	122	6	70	26	9
22	121	190	116	11	70	23	8
23	125	191	116	15	71	20	7
24	129	191	110	18	71	17	6
25	134	190	106	21	71	14	5
26	138	189	101	23	71	12	4
27	141	189	97	26	70	9	3
28	146	188	92	29	70	6	2
29	150	187	89	33	69	3	1
30	152	187	85	37	68	0	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum IV.

Argumentum angulus $(p - q)$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	+	+	+	−	
0	6108	3096	2940	5994	3054	2898	30
1	6104	2912	3120	5992	2873	3076	29
2	6093	2724	3298	5983	2688	3251	28
3	6076	2534	3471	5968	2500	3421	27
4	6050	2340	3640	5944	2309	3587	26
5	6017	2144	3805	5913	2116	3749	25
6	5976	1944	3965	5875	1919	3907	24
7	5929	1743	4119	5830	1721	4059	23
8	5874	1540	4269	5777	1520	4206	22
9	5812	1334	4414	5718	1318	4349	21
10	5744	1128	4554	5652	1114	4486	20
11	5669	920	4686	5579	909	4615	19
12	5587	712	4814	5500	703	4741	18
13	5498	502	4936	5412	496	4860	17
14	5402	292	5052	5320	289	4984	16
15	5301	−81	5162	5220	+81	5081	15
16	5192	+129	5264	5114	−126	5182	14
17	5078	340	5361	5002	334	5276	13
18	4957	551	5452	4884	542	5366	12
19	4831	759	5535	4760	748	5445	11
20	4700	968	5612	4632	954	5520	10
21	4561	1174	5682	4496	1158	5588	9
22	4418	1380	5745	4355	1360	5648	8
23	4269	1583	5802	4209	1561	5703	7
24	4115	1785	5851	4057	1760	5750	6
25	3957	1984	5893	3901	1956	5789	5
26	3793	2181	5928	3740	2150	5822	4
27	3625	2376	5956	3575	2342	5848	3
28	3453	2566	5975	3406	2530	5865	2
29	3276	2755	5988	3232	2716	5876	1
30	3096	2940	5994	3054	2898	5880	0
	−	+	+	+	−	−	
	XI S.	X S.	IX S.	VIII S.	VII S.	VI S.	

Tabula

Tabula Aequationum IV.

Argumentum angulus $(p - q)$.

Pro coordinata y.

	o. Sig	I. Sig.	II. Sig.	III. S.	IV. Sig	V. S.	
	+	+	+	±	−	−	
0	0	9277	10200	2523	5830	6753	30
1	367	9472	10057	2199	6009	6626	29
2	734	9655	9902	1875	6178	6489	28
3	1099	9828	9735	1552	6338	6342	27
4	1464	9992	9561	1228	6489	6187	26
5	1827	10144	9377	906	6630	6024	25
6	2188	10285	9183	585	6761	5852	24
7	2547	10416	8982	±264	6881	5672	23
8	2903	10534	8769	∓ 53	6991	5484	22
9	3255	10641	8548	368	7092	5289	21
10	3604	10736	8320	680	7181	5086	20
11	3949	10819	8083	990	7259	4876	19
12	4289	10892	7839	1296	7329	4660	18
13	4625	10953	7588	1599	7386	4437	17
14	4956	11002	7330	1897	7434	4208	16
15	5280	11039	7064	2190	7471	3974	15
16	5600	11064	6793	2478	7496	3734	14
17	5913	11078	6515	2762	7511	3489	13
18	6220	11079	6232	3041	7516	3239	12
19	6520	11069	5944	3313	7509	2985	11
20	6812	11047	5650	3578	7492	2726	10
21	7097	11014	5352	3838	7465	2465	9
22	7374	10969	5049	4089	7426	2199	8
23	7644	10913	4742	4334	7375	1931	7
24	7904	10843	4433	4573	7315	1660	6
25	8156	10754	4120	4803	7250	1387	5
26	8399	10673	3804	5025	7170	1112	4
27	8632	10570	3486	5239	7080	835	3
28	8857	10458	3167	5444	6981	558	2
29	9072	10335	2845	5641	6875	279	1
30	9277	10200	2523	5830	6753	0	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum V.

Argumentum angulus $(2p+q.)$

Pro coordinata x.

	0 S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	1875	1624	940	0	940	1624	30
1	1875	1607	911	33	968	1640	29
2	1874	1590	882	65	996	1656	28
3	1872	1572	852	98	1024	1670	27
4	1870	1554	823	132	1051	1685	26
5	1867	1535	793	165	1077	1700	25
6	1864	1517	763	197	1103	1713	24
7	1860	1497	733	229	1129	1726	23
8	1856	1478	702	262	1156	1739	22
9	1850	1457	672	294	1181	1751	21
10	1847	1437	642	326	1207	1762	20
11	1841	1415	610	358	1232	1773	19
12	1834	1393	579	391	1267	1784	18
13	1827	1371	548	423	1279	1793	17
14	1820	1348	517	455	1302	1802	16
15	1811	1335	486	486	1335	1811	15
16	1802	1302	455	517	1348	1820	14
17	1793	1279	423	548	1371	1827	13
18	1784	1267	391	579	1398	1834	12
19	1773	1232	358	610	1415	1841	11
20	1762	1207	326	642	1437	1847	10
21	1751	1181	294	672	1457	1850	9
22	1739	1156	262	702	1478	1856	8
23	1726	1129	229	733	1497	1860	7
24	1713	1103	197	763	1517	1864	6
25	1700	1077	165	793	1535	1867	5
26	1685	1051	132	823	1554	1870	4
27	1670	1024	98	852	1572	1872	3
28	1656	996	65	882	1590	1874	2
29	1640	968	33	911	1607	1875	1
30	1624	940	0	940	1624	1875	0
	−	−	−	+	+	+	
	XI S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum V.

Argumentum angulus $(2p+q)$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	−	−	−	
0	0	885	1533	1769	1533	885	30
1	31	912	1549	1769	1516	857	29
2	62	938	1564	1767	1500	830	28
3	93	964	1587	1765	1483	802	27
4	124	990	1591	1763	1467	773	26
5	155	1015	1604	1761	1449	745	25
6	186	1040	1616	1759	1432	718	24
7	217	1065	1629	1756	1413	690	23
8	248	1091	1641	1752	1394	662	22
9	279	1114	[illegible]	1748	1375	634	21
10	309	1138	166[illegible]	1742	1356	605	20
11	339	1151	1672	1736	1335	576	19
12	369	1185	1682	1730	1314	546	18
13	399	1207	1691	1723	1293	515	17
14	429	1229	1699	1716	1272	486	16
15	458	1250	1707	1707	1250	458	15
16	486	1272	1716	1699	1229	429	14
17	515	1293	1723	1691	1207	398	13
18	546	1314	1730	1682	1185	368	12
19	576	1335	1736	1672	1151	339	11
20	605	1356	1742	1662	1138	309	10
21	634	1375	1748	1651	1114	279	9
22	662	1394	1752	1641	1091	248	8
23	690	1413	1756	1629	1065	217	7
24	718	1432	1759	1616	1040	186	6
25	745	1449	1761	1604	1015	155	5
26	773	1467	1763	1591	990	124	4
27	802	1483	1765	1587	964	93	3
28	830	1500	1767	1564	938	62	2
29	857	1516	1769	1549	912	31	1
30	885	1553	1769	1533	885	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum VI.

Argumentum angulus $(2p-q)$.

Pro coordinata x.

	o. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	+	+	±	−	−	−	
0	102352	88391	50500	677	51276	87615	30
1	102336	87471	48935	2451	52685	88569	29
2	102288	86522	47356	4223	54176	89396	28
3	102209	85547	45763	5994	55651	90196	27
4	102097	84545	44155	7762	57109	90969	26
5	101955	83517	42585	9528	58548	91713	25
6	101782	82467	40901	11289	59972	92431	24
7	101574	81387	39257	13048	61375	93122	23
8	101337	80284	37601	14801	62760	93785	22
9	101067	79157	35934	165[illegible]	64127	94419	21
10	100771	78004	34256	18[illegible]01	65472	95024	20
11	100434	76829	32568	20027	66798	95601	19
12	100071	75630	30871	21757	68105	96150	18
13	99677	74409	29165	23480	69890	96671	17
14	99251	73163	27451	25194	70653	97162	16
15	98796	71895	25729	26901	71895	97624	15
16	98310	70605	24000	28599	73115	98057	14
17	97703	69294	22264	30287	74313	98461	13
18	97246	67963	20521	31967	75488	98835	12
19	96669	66610	18773	33636	76641	99180	11
20	96062	65238	17019	35294	77770	99499	10
21	95425	63845	15261	36940	78875	99779	9
22	94759	62434	13499	38575	79958	100035	8
23	94062	61003	11634	40197	81015	100260	7
24	93337	59564	9955	41807	82049	100458	6
25	92[illegible]83	58086	8194	43405	83055	100621	5
26	91801	56603	6422	44987	84039	100757	4
27	90990	55101	4648	46557	84997	100863	3
28	90152	53582	2873	48112	85928	100938	2
29	89285	52049	+ 1097	49651	86835	100982	1
30	88391	50500	− 677	51276	87615	100998	0
	+	+	±	−	−	−	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. S.	VII. Sig.	VI. Sig.	

Tabula

Tabula Aequationum VI.

Argumentum angulus $(2p-q)$.

Pro coordinata y.

	o. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	—	—	—	—	—	—	
0	0000	110997	192474	222601	193082	111605	30
1	3873	114338	194393	222579	191116	108217	29
2	7744	117646	196254	222481	189091	104796	28
3	11613	120917	198055	222333	187009	101343	27
4	15479	124152	199795	222108	184870	97859	26
5	19340	127349	201476	221815	182674	94344	25
6	23195	130508	203095	221455	180422	90801	24
7	27043	133627	204652	221027	178115	87230	23
8	30883	136706	206148	220532	175753	83632	22
9	34713	139743	207581	219969	173337	80008	21
10	38532	142739	208951	219341	170868	76360	20
11	42341	145691	210257	218644	168347	72688	19
12	46138	148600	211500	217881	165774	68993	18
13	49920	151461	212678	217050	163150	65278	17
14	53687	154281	213791	216154	160477	61543	16
15	57438	157052	214840	215192	157754	57790	15
16	61171	159775	215824	214163	154983	54017	14
17	64886	162450	216742	213070	152161	50228	13
18	68581	165076	217595	211912	149298	46424	12
19	72256	167651	218378	210689	146387	42607	11
20	75908	170176	219097	209403	143431	38776	10
21	79538	172649	219751	208051	140431	34931	9
22	83144	175071	220338	206636	137388	31077	8
23	86724	177439	220857	205158	134303	27213	7
24	90279	179754	221309	203617	131176	23341	6
25	93806	182014	221693	202014	128009	19462	5
26	97305	184220	222010	200349	124802	15577	4
27	100775	186369	222259	198623	121557	11687	3
28	104214	188461	222441	196836	118276	7794	2
29	107621	190496	222555	194989	114958	3897	1
30	110997	192474	222601	193082	111605	0000	0
	+	+	+	+	+	+	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. S.	VII. S.	VI. Sig.	

Tabula Aequationum VII.

Argumentum angulus $(4p-q)$.

Pro coordinata x.

	o.Sig	I.Sig	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	358	310	179	0	179	310	30
1	358	307	174	7	185	313	29
2	358	304	169	13	190	316	28
3	358	300	163	19	195	319	27
4	357	297	158	26	200	322	26
5	357	294	152	32	205	325	25
6	356	290	146	38	210	327	24
7	356	286	141	44	215	330	23
8	355	282	135	50	220	332	22
9	354	278	128	56	225	334	21
10	353	274	123	63	230	337	20
11	352	270	117	69	235	339	19
12	350	266	111	75	240	341	18
13	349	262	105	81	245	343	17
14	348	258	99	87	249	345	16
15	346	253	93	93	253	346	15
16	345	249	87	99	258	348	14
17	343	245	81	105	262	349	13
18	341	240	75	111	266	350	12
19	339	235	69	117	270	352	11
20	337	230	63	123	274	353	10
21	334	225	56	128	278	354	9
22	332	220	50	135	282	355	8
23	330	215	44	141	286	356	7
24	327	210	38	146	290	356	6
25	325	205	32	152	294	357	5
26	322	200	26	158	297	357	4
27	319	195	19	163	300	358	3
28	316	190	13	169	304	358	2
29	313	185	7	174	307	358	1
30	310	179	0	179	310	358	0
	−	−	−	+	+	+	
	XI.S	X. S.	IX.S	VIII.S	VII.S.	VI.S.	

Tabula Aequationum VII.

Argumentum angulus $(4p-q)$.

Pro coordinata y.

	o. Sig	I. Sig.	II. S.	III. Sig	IV. Sig	V. S.	
	−	−	−	−	−	−	
0	0	228	400	455	400	228	30
1	8	236	403	455	394	220	29
2	17	243	405	455	390	215	28
3	25	251	408	454	384	208	27
4	31	258	411	454	380	202	26
5	40	264	414	453	375	194	25
6	48	268	417	452	369	187	24
7	55	276	420	451	364	179	23
8	63	284	423	450	359	172	22
9	71	291	426	449	354	164	21
10	78	296	429	448	350	156	20
11	86	301	432	447	345	148	19
12	93	306	434	446	340	139	18
13	101	312	436	445	334	131	17
14	109	317	438	443	329	124	16
15	116	322	440	440	322	116	15
16	124	329	443	438	317	109	14
17	131	334	445	436	312	101	13
18	139	340	446	434	306	93	12
19	148	345	447	432	301	86	11
20	156	350	448	429	296	78	10
21	164	354	449	426	291	71	9
22	172	359	450	423	284	63	8
23	179	364	451	420	276	55	7
24	187	369	452	417	268	48	6
25	194	375	453	414	264	40	5
26	202	380	454	411	258	31	4
27	208	384	454	408	251	25	3
28	215	390	455	405	243	17	2
29	220	394	455	403	236	8	1
30	228	400	455	400	228	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S	VII. S.	VI. S.	

Tabula Aequationum VIII.

Argumentum angulus $(2p - 3q)$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. Sig	V. Sig	
	+	+	+	−	−	−	
0	422	370	211	0	211	370	30
1	422	366	205	8	218	374	29
2	422	362	199	16	225	377	28
3	422	358	192	23	231	380	27
4	421	354	186	31	237	383	26
5	421	349	180	38	242	386	25
6	420	344	173	45	248	390	24
7	419	339	167	53	254	393	23
8	418	334	160	60	260	396	22
9	416	329	153	67	266	399	21
10	415	324	146	75	272	402	20
11	414	319	139	82	278	404	19
12	413	315	132	89	282	406	18
13	412	310	125	97	288	407	17
14	411	304	118	104	294	408	16
15	409	299	111	111	299	409	15
16	408	294	104	118	304	411	14
17	407	288	97	125	310	412	13
18	406	282	89	132	315	413	12
19	404	278	82	139	319	414	11
20	402	271	75	146	324	415	10
21	399	266	67	153	329	416	9
22	396	260	60	160	334	418	8
23	393	254	53	167	339	419	7
24	390	248	45	173	344	420	6
25	386	242	38	180	349	421	5
26	383	237	31	186	354	421	4
27	380	231	23	192	358	422	3
28	377	225	16	199	362	422	2
29	374	218	8	205	366	422	1
30	370	211	0	211	370	422	0
	+	+	+	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S	VII. S.	VI. S.	

Tabula

Tabula Aequationum VIII.

Argumentum angulus $(2p - 3q)$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	−	−	−	
0	0	178	309	356	309	178	30
1	6	183	312	356	306	173	29
2	12	188	315	356	303	167	28
3	18	193	318	355	299	161	27
4	24	198	321	355	295	155	26
5	30	203	324	354	292	149	25
6	36	209	326	354	288	143	24
7	42	214	328	353	284	137	23
8	48	219	330	353	281	132	22
9	54	224	333	352	277	127	21
10	61	230	335	351	273	122	20
11	67	234	337	350	269	115	19
12	73	239	338	349	265	109	18
13	79	244	340	348	260	103	17
14	85	248	342	346	256	97	16
15	91	252	344	344	252	91	15
16	97	256	345	342	248	85	14
17	103	260	346	340	244	79	13
18	109	265	348	338	239	73	12
19	115	269	350	337	234	67	11
20	122	273	351	335	230	61	10
21	127	277	352	333	224	54	9
22	132	281	353	330	219	48	8
23	137	284	353	328	214	42	7
24	143	288	354	326	209	36	6
25	149	292	354	324	203	30	5
26	156	295	355	321	198	24	4
27	161	299	356	318	193	18	3
28	167	303	356	315	188	12	2
29	173	306	356	312	183	6	1
30	178	309	356	309	178	0	0
	+	+	+	+	+	+	
	XI. S	X. S.	IX. S	VIII. S	VII. S.	VI. S.	

Tabula Aequationum IX.

Argumentum angulus *t*.

Pro coordinata *x*.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	1075	929	537	0	537	929	30
1	1075	920	521	19	554	939	29
2	1074	910	505	39	569	948	28
3	1073	900	488	57	585	956	27
4	1071	890	472	75	600	965	26
5	1069	880	455	93	616	973	25
6	1068	869	437	112	631	981	24
7	1067	858	420	130	646	989	23
8	1065	847	403	149	660	996	22
9	1062	835	385	167	675	1003	21
10	1059	823	367	186	689	1009	20
11	1055	811	349	205	704	1016	19
12	1051	798	332	223	718	1021	18
13	1047	785	314	242	732	1025	17
14	1042	772	297	260	746	1030	16
15	1037	759	278	278	759	1037	15
16	1030	746	260	297	772	1042	14
17	1025	732	242	315	785	1047	13
18	1021	718	223	332	798	1051	12
19	1016	704	205	349	811	1055	11
20	1009	689	186	367	823	1059	10
21	1003	675	167	385	835	1062	9
22	996	660	149	403	847	1065	8
23	989	646	130	420	858	1067	7
24	981	631	112	437	869	1068	6
25	973	616	93	455	880	1069	5
26	965	600	75	472	890	1071	4
27	956	585	57	488	900	1073	3
28	948	569	39	505	910	1074	2
29	939	554	19	521	920	1075	1
30	929	537	0	537	929	1075	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum IX.

Argumentum angulus t.

Pro coordinata y.

	o. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	+	+	+	+	+	+	
0	000	15767	27349	31643	27457	15875	30
1	550	16242	27623	31640	27179	15394	29
2	1100	16712	27887	31628	26891	14907	28
3	1650	17177	28143	31606	26595	14416	27
4	2198	17637	28390	31575	26290	13921	26
5	2746	18092	28630	31534	25979	13421	25
6	3295	18540	28861	31483	25659	12916	24
7	3841	18984	29082	31422	25330	12409	23
8	4387	19421	29296	31352	24995	11898	22
9	4931	19854	29500	31272	24652	11381	21
10	5474	20279	29696	31183	24301	10862	20
11	6015	20699	29881	31085	23942	10340	19
12	6554	21111	30057	30977	23577	9815	18
13	7091	21518	30225	30859	23204	9287	17
14	7626	21919	30384	30732	22824	8755	16
15	8159	22313	30524	30596	22437	8221	15
16	8689	22700	30674	30450	22043	7684	14
17	9217	23080	30805	30295	21642	7145	13
18	9741	23453	30927	30131	21235	6604	12
19	10264	23820	31039	29957	20821	6061	11
20	10784	24179	31141	29774	20401	5516	10
21	11299	24532	31234	29582	19974	4969	9
22	11811	24875	31318	29382	19541	4421	8
23	12319	25212	31382	29172	19102	3871	7
24	12824	25541	31457	28953	18658	3321	6
25	13325	25863	31512	28726	18208	2768	5
26	13821	26176	31557	28490	17751	2216	4
27	14314	26481	31594	28245	17291	1662	3
28	14803	26779	31620	27991	16824	1108	2
29	15288	27069	31636	27729	16352	554	1
30	15767	27349	31643	27457	15875	.000	0
	—	—	—	—	—	—	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. S.	VII. Sig.	VI. S.	

Tabula

Tabula Aequationum X.

Argumentum angulus $2p+q$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	543	470	271	0	271	470	30
1	543	466	263	10	280	475	29
2	543	460	254	19	289	480	28
3	543	454	246	28	297	485	27
4	542	448	238	37	305	489	26
5	542	442	230	47	312	493	25
6	541	437	221	56	319	497	24
7	540	431	212	66	327	501	23
8	539	425	204	75	334	504	22
9	537	420	195	85	342	508	21
10	535	414	187	94	349	511	20
11	533	408	177	104	357	514	19
12	532	402	167	113	364	517	18
13	530	396	159	122	370	519	17
14	528	389	151	132	376	522	16
15	525	383	141	141	383	525	15
16	522	376	132	151	389	528	14
17	519	370	122	159	396	530	13
18	517	364	113	167	402	532	12
19	514	357	104	177	408	533	11
20	511	349	94	187	414	535	10
21	508	342	85	195	420	537	9
22	504	334	75	204	425	539	8
23	501	327	66	212	431	540	7
24	497	319	56	221	437	541	6
25	493	312	47	230	442	542	5
26	489	305	37	238	448	542	4
27	485	297	28	246	454	543	3
28	480	289	19	254	460	543	2
29	475	280	10	263	466	543	1
30	470	271	0	271	470	543	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum X.

Argumentum angulus $2p+t$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	447	775	895	775	447	30
1	16	459	783	895	768	433	29
2	31	473	791	894	759	419	28
3	47	487	798	894	750	406	27
4	63	500	804	893	742	392	26
5	78	514	811	892	733	378	25
6	94	526	817	890	724	364	24
7	110	539	823	888	714	350	23
8	126	551	829	886	703	335	22
9	140	563	835	884	694	321	21
10	156	575	841	881	685	306	20
11	172	587	846	878	675	291	19
12	186	598	851	875	664	277	18
13	202	610	855	872	653	262	17
14	217	621	860	868	643	247	16
15	231	632	864	864	632	231	15
16	247	643	868	860	621	217	14
17	262	653	872	855	610	202	13
18	277	664	875	851	598	186	12
19	291	675	878	846	587	172	11
20	306	685	881	841	575	156	10
21	321	694	884	835	563	140	9
22	335	703	886	829	551	126	8
23	350	714	888	823	539	110	7
24	364	724	890	817	526	94	6
25	378	733	892	811	514	78	5
26	392	742	893	804	500	63	4
27	406	750	894	798	487	47	3
28	419	759	894	791	473	31	2
29	433	768	895	783	459	16	1
30	447	775	895	775	447	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XI.

Argumentum angulus $2p - h$

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	−	−	−	
0	4853	4203	2427	0	2427	4203	30
1	4852	4160	2352	86	2501	4244	29
2	4850	4115	2277	170	2573	4284	28
3	4846	4069	2202	243	2644	4324	27
4	4841	4022	2127	328	2714	4361	26
5	4834	3974	2052	409	2783	4398	25
6	4827	3921	1973	505	2852	4433	24
7	4818	3875	1896	590	2923	4468	23
8	4807	3824	1818	674	2988	4560	22
9	4793	3771	1739	758	3054	4531	21
10	4778	3717	1659	842	3120	4560	20
11	4763	3661	1579	925	3184	4589	19
12	4747	3605	1498	1007	3247	4616	18
13	4719	3548	1417	1090	3308	4641	17
14	4699	3490	1337	1173	3370	4665	16
15	4688	3431	1255	1255	3431	4688	15
16	4665	3370	1173	1337	3490	4699	14
17	4641	3308	1090	1417	3548	4719	13
18	4616	3247	1007	1498	3605	4747	12
19	4589	3184	925	1579	3661	4763	11
20	4560	3120	842	1659	3717	4778	10
21	4531	3054	758	1739	3771	4793	9
22	4500	2988	674	1818	3824	4807	8
23	4468	2923	590	1896	3875	4818	7
24	4433	2852	505	1973	3921	4827	6
25	4398	2783	409	2052	3974	4834	5
26	4361	2714	328	2127	4022	4841	4
27	4324	2644	243	2202	4069	4846	3
28	4284	2573	170	2277	4115	4850	2
29	4244	2501	86	2352	4160	4852	1
30	4203	2427	0	2427	4203	4853	0
	+	+	+	−	−	−	
	IX.S.	X. S.	IX.S.	VIII.S.	VII.S.	VI.S.	

Tabula

Tabula Aequationum XI.

Argumentum angulus $2p - t$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	−	−	−	
0	0	3586	6210	7171	6210	3586	30
1	125	3694	6271	7169	6146	3476	29
2	249	3801	6330	7166	6080	4366	28
3	375	3907	6389	7161	6013	3255	27
4	499	4010	6444	7153	5943	3144	26
5	624	4112	6498	7143	5873	3031	25
6	749	4215	6550	7132	5801	2918	24
7	873	4315	6600	7120	5725	2803	23
8	997	4414	6647	7102	5649	2688	22
9	1122	4513	6694	7085	5572	2571	21
10	1245	4609	6737	7061	5493	2453	20
11	1368	4704	6779	7039	5411	2335	19
12	1491	4799	6820	7014	5328	2216	18
13	1613	4892	6857	6986	5244	2098	17
14	1735	4982	6893	6957	5158	1978	16
15	1857	5070	6927	6927	5070	1857	15
16	1978	5158	6957	6893	4982	1735	14
17	2098	5244	6986	6857	4892	1613	13
18	2216	5328	7014	6820	4799	1491	12
19	2335	5411	7039	6779	4704	1368	11
20	2453	5493	7061	6737	4609	1245	10
21	2571	5572	7085	6694	4513	1122	9
22	2688	5649	7102	6647	4414	997	8
23	2803	5725	7120	6600	4315	873	7
24	2918	5801	7132	6550	4215	749	6
25	3031	5873	7143	6498	4112	624	5
26	3144	5943	7153	6444	4010	449	4
27	3255	6013	7161	6389	3907	375	3
28	3366	6080	7166	6330	3801	249	2
29	3476	6146	7169	6271	3694	125	1
30	3586	6210	7171	6210	3586	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XII.

Argumentum angulus $q + t$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	−	−	
0	769	665	385	0	385	665	30
1	769	659	373	14	395	672	29
2	768	651	361	27	407	679	28
3	768	643	350	40	419	686	27
4	767	636	338	54	430	691	26
5	767	629	326	67	442	697	25
6	765	621	314	81	453	703	24
7	763	612	302	95	464	708	23
8	761	603	289	109	475	713	22
9	760	596	277	121	485	718	21
10	758	589	264	135	495	723	20
11	755	580	251	149	506	727	19
12	752	571	239	161	515	730	18
13	749	562	226	174	526	735	17
14	746	554	213	187	535	739	16
15	743	544	199	199	544	743	15
16	739	535	187	213	554	746	14
17	735	526	174	226	562	749	13
18	730	515	161	239	571	752	12
19	727	506	149	251	580	755	11
20	723	495	135	264	589	758	10
21	718	485	121	277	596	760	9
22	713	475	109	289	603	761	8
23	708	464	95	302	612	763	7
24	703	453	81	314	621	765	6
25	697	442	67	326	629	767	5
26	691	430	54	338	636	767	4
27	686	419	40	350	643	768	3
28	679	407	27	361	651	768	2
29	672	395	14	373	659	769	1
30	665	385	0	385	665	769	0
	+	+	+	−	−	−	
	IX. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XIII.

Argumentum angulus $q + t$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	−	−	−	−	−	
0	0	2223	3852	4447	3852	2223	30
1	79	2291	3888	4447	3811	2155	29
2	156	2357	3926	4445	3771	2097	28
3	234	2422	3962	4441	3729	2017	27
4	313	2492	3996	4437	3686	1951	26
5	392	2551	4029	4431	3643	1882	25
6	471	2614	4064	4423	3598	1809	24
7	547	2676	4096	4415	3552	1739	23
8	622	2739	4126	4404	3506	1667	22
9	697	2799	4155	4391	3457	1595	21
10	774	2857	4181	4378	3417	1502	20
11	851	2917	4207	4365	3356	1449	19
12	926	2974	4232	4350	3304	1376	18
13	1002	3031	4254	4325	3252	1302	17
14	1078	3088	4275	4306	3198	1228	16
15	1155	3144	4294	4294	3144	1155	15
16	1228	3198	4300	4275	3088	1078	14
17	1302	3252	4325	4254	3031	1002	13
18	1376	3304	4350	4232	2974	926	12
19	1449	3356	4365	4207	2917	851	11
20	1522	3417	4378	4181	2857	774	10
21	1595	3457	4391	4155	2799	697	9
22	1667	3506	4404	4126	2739	622	8
23	1739	3552	4415	4096	2676	547	7
24	1809	3598	4423	4064	2614	471	6
25	1882	3643	4431	4029	2551	392	5
26	1951	3686	4437	3996	2492	313	4
27	2017	3729	4441	3962	2422	234	3
28	2097	3771	4445	3926	2357	156	2
29	2155	3811	4447	3888	2291	79	1
30	2223	3852	4447	3852	2223	[illegible]	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

 Tabula

Tabula Aequationum XIIII.

Argumentum angulus $q - t$.

Pro coordinata x.

	0 S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	1656	1436	828	0	828	1436	30
1	1656	1421	803	29	854	1450	29
2	1655	1405	778	58	879	1463	28
3	1654	1390	753	87	903	1477	27
4	1652	1374	727	113	926	1493	26
5	1650	1357	701	141	951	1504	25
6	1647	1341	674	173	974	1515	24
7	1645	1324	648	202	998	1526	23
8	1641	1306	621	231	1020	1537	22
9	1636	1288	594	259	1043	1548	21
10	1632	1269	567	288	1065	1558	20
11	1628	1250	540	314	1078	1567	19
12	1624	1232	512	344	1110	1576	18
13	1611	1212	484	372	1130	1585	17
14	1605	1193	456	401	1151	1593	16
15	1598	1172	429	429	1172	1601	15
16	1593	1151	401	456	1193	1605	14
17	1585	1130	372	484	1212	1611	13
18	1576	1110	344	512	1232	1617	12
19	1567	1078	314	540	1250	1626	11
20	1558	1065	288	567	1269	1631	10
21	1548	1043	259	594	1288	1636	9
22	1537	1020	231	621	1306	1641	8
23	1526	998	202	648	1324	1645	7
24	1515	974	173	674	1341	1647	6
25	1504	951	141	701	1357	1650	5
26	1493	926	113	727	1374	1652	4
27	1477	903	87	753	1390	1654	3
28	1463	879	58	778	1405	1655	2
29	1450	854	29	803	1421	1656	1
30	1436	828	0	828	1436	1656	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XIII.

Argumentum angulus $q - \epsilon$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	3021	5232	6041	5232	3021	30
1	105	3111	5284	6040	5178	2929	29
2	211	3201	5334	6037	5123	2836	28
3	316	3290	5383	6033	5066	2743	27
4	421	3378	5430	6026	5008	2648	26
5	526	3465	5475	6018	4949	2553	25
6	631	3551	5519	6008	4887	2457	24
7	736	3636	5561	5996	4825	2360	23
8	841	3719	5601	5982	4760	2263	22
9	945	3802	5640	5967	4695	2165	21
10	1049	3883	5677	5949	4628	2066	20
11	1153	3963	5712	5930	4559	1967	19
12	1256	4042	5745	5909	4489	1867	18
13	1359	4120	5777	5886	4418	1766	17
14	1461	4197	5807	5862	4346	1665	16
15	1563	4272	5835	5835	4272	1563	15
16	1665	4346	5862	5807	4197	1461	14
17	1766	4418	5886	5777	4120	1359	13
18	1867	4489	5909	5745	4042	1256	12
19	1967	4559	5930	5712	3963	1153	11
20	2066	4628	5949	5677	3883	1049	10
21	2165	4695	5967	5640	3802	945	9
22	2263	4760	5982	5601	3719	841	8
23	2360	4825	5996	5561	3636	736	7
24	2457	4887	6008	5519	3551	631	6
25	2553	4949	6018	5475	3465	526	5
26	2648	5008	6026	5430	3378	421	4
27	2743	5066	6033	5383	3290	316	3
28	2836	5123	6037	5334	3201	211	2
29	2929	5178	6040	5284	3111	105	1
30	3021	5232	6041	5232	3021	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XIV.

Argumentum angulus $2p - q - t$.

Pro coordinata x

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	−	−	+	+	+	
0	3721	3222	1861	0	1861	3222	30
1	3720	3190	1804	65	1917	3254	29
2	3718	3156	1747	130	1972	3285	28
3	3716	3121	1689	195	2027	3315	27
4	3713	3085	1631	260	2081	3344	26
5	3708	3048	1572	325	2134	3372	25
6	3701	3010	1513	389	2187	3399	24
7	3693	2971	1454	454	2239	3425	23
8	3684	2932	1394	518	2291	3450	22
9	3675	2891	1334	582	2342	3475	21
10	3664	2850	1273	646	2392	3496	20
11	3653	2808	1212	710	2441	3518	19
12	3640	2765	1150	774	2490	3539	18
13	3626	2721	1088	837	2538	3558	17
14	3610	2677	1026	900	2585	3576	16
15	3594	2631	963	963	2631	3594	15
16	3576	2585	900	1026	2677	3610	14
17	3558	2538	837	1088	2721	3626	13
18	3539	2490	774	1150	2765	3640	12
19	3518	2441	710	1212	2808	3653	11
20	3496	2392	646	1273	2850	3664	10
21	3475	2342	582	1334	2891	3675	9
22	3450	2291	518	1394	2932	3684	8
23	3425	2239	454	1454	2971	3693	7
24	3399	2187	389	1513	3010	3701	6
25	3372	2134	325	1572	3048	3708	5
26	3344	2081	260	1631	3085	3713	4
27	3315	2027	195	1689	3121	3716	3
28	3285	1972	130	1747	3156	3718	2
29	3254	1917	65	1804	3190	3720	1
30	3222	1861	0	1861	3222	3721	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XIV.

Argumentum angulus $2p - q - A$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	4939	8556	9886	8556	4939	30
1	171	5082	8630	9883	8467	4783	29
2	343	5229	8714	9876	8376	4637	28
3	516	5376	8794	9867	8283	4479	27
4	688	5520	8872	9854	8187	4328	26
5	861	5663	8949	9838	8088	4270	25
6	1031	5804	9023	9820	7987	4015	24
7	1202	5936	9094	9798	7883	3855	23
8	1372	6086	9160	9774	7777	3698	22
9	1543	6216	9225	9746	7668	3536	21
10	1715	6349	9286	9728	7568	3379	20
11	1882	6473	9333	9699	7456	3223	19
12	2051	6604	9388	9664	7340	3049	18
13	2219	6731	9442	9625	7223	2884	17
14	2386	6858	9492	9584	7103	2729	16
15	2555	6982	9540	9540	6982	2555	15
16	2729	7103	9584	9492	6858	2386	14
17	2884	7223	9625	9442	6731	2219	13
18	3049	7340	9664	9388	6604	2051	12
19	3223	7456	9699	9333	6473	1882	11
20	3379	7568	9728	9286	6349	1715	10
21	3536	7668	9746	9225	6216	1543	9
22	3698	7777	9774	9160	6080	1372	8
23	3855	7883	9798	9093	5936	1202	7
24	4015	7987	9820	9023	5804	1031	6
25	4270	8088	9838	8949	5663	861	5
26	4328	8187	9854	8872	5520	688	4
27	4479	8283	9867	8794	5376	516	3
28	4637	8376	9876	8714	5229	343	2
29	4783	8467	9883	8630	5082	171	1
30	4939	8556	9886	8556	4939	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XV.

Argumentum angulus $2p - q + r$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	−	−	−	
0	281	246	142	0	142	246	30
1	281	244	138	5	146	249	29
2	281	242	134	10	150	252	28
3	281	239	130	15	154	254	27
4	281	236	126	20	158	256	26
5	280	233	121	25	162	258	25
6	280	230	116	30	166	260	24
7	280	227	112	35	170	262	23
8	279	224	108	40	174	264	22
9	279	221	103	45	178	267	21
10	278	218	98	50	182	268	20
11	277	215	93	55	186	269	19
12	276	212	88	60	190	270	18
13	275	209	84	65	194	271	17
14	274	206	80	70	198	272	16
15	273	202	75	75	202	273	15
16	272	198	70	80	206	274	14
17	271	194	65	84	209	275	13
18	270	190	60	88	212	276	12
19	269	186	55	93	215	277	11
20	268	182	50	98	218	278	10
21	267	178	45	103	221	279	9
22	264	174	40	108	224	279	8
23	262	170	35	112	227	280	7
24	260	166	30	116	230	280	6
25	258	162	25	121	233	280	5
26	256	158	20	126	236	281	4
27	254	154	15	130	239	281	3
28	252	150	10	134	242	281	2
29	249	146	5	138	244	281	1
30	246	142	0	142	246	281	0
	+	+	+	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XV.
Argumentum angulus $2p - q + t$.
Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	576	998	1153	998	576	30
1	20	594	1007	1153	989	560	29
2	41	611	1017	1152	978	543	28
3	62	628	1025	1151	967	534	27
4	81	645	1035	1149	956	504	26
5	101	661	1043	1147	945	488	25
6	121	677	1052	1145	933	469	24
7	141	693	1061	1144	921	451	23
8	160	709	1068	1142	909	431	22
9	180	725	1076	1139	896	412	21
10	200	740	1082	1136	883	393	20
11	220	756	1090	1132	869	373	19
12	240	771	1095	1127	856	356	18
13	262	786	1100	1123	842	336	17
14	282	801	1105	1118	829	318	16
15	299	815	1113	1113	815	299	15
16	318	829	1118	1105	801	282	14
17	336	842	1123	1100	786	262	13
18	356	856	1127	1095	771	240	12
19	373	869	1132	1090	756	220	11
20	393	883	1136	1082	740	200	10
21	412	896	1139	1076	725	180	9
22	431	909	1142	1068	709	160	8
23	451	921	1144	1061	693	141	7
24	469	933	1145	1052	677	121	6
25	488	945	1147	1043	661	101	5
26	504	956	1149	1035	645	81	4
27	534	967	1151	1025	628	62	3
28	543	978	1152	1017	611	41	2
29	560	989	1153	1007	594	20	1
30	576	998	1153	998	576	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XVI.

Argumentum angulus $p + t$.

Pro coordinata m.

	0. S. +	I. S. +	II. S. +	III. S. −	IV. S. −	V. S. −	
0	264	232	133	0	133	232	30
1	264	230	131	5	135	235	29
2	264	228	127	10	141	236	28
3	264	225	123	14	145	238	27
4	264	222	119	20	149	240	26
5	263	219	115	23	153	242	25
6	263	216	110	27	156	244	24
7	263	214	106	32	160	246	23
8	262	211	102	37	164	248	22
9	262	208	97	41	168	251	21
10	261	205	92	46	172	252	20
11	260	203	87	51	176	253	19
12	259	200	82	56	180	254	18
13	258	197	78	60	183	254	17
14	257	194	75	65	187	255	16
15	256	191	70	70	191	256	15
16	255	187	65	75	194	257	14
17	254	183	60	78	197	258	13
18	254	180	56	82	200	259	12
19	253	176	51	87	203	260	11
20	252	172	46	92	205	261	10
21	251	168	41	97	208	262	9
22	248	164	37	102	211	262	8
23	246	160	32	106	214	263	7
24	244	156	27	110	216	263	6
25	242	153	23	115	219	263	5
26	240	149	20	119	222	264	4
27	238	145	14	123	225	264	3
28	236	141	10	127	228	264	2
29	235	135	5	131	230	264	1
30	232	133	0	133	232	264	0
	+ XI. S.	+ X. S.	+ IX. S.	− VIII. S.	− VII. S.	− VI. S.	

Tabula

Tabula Aequationum XVI.

Argumentum angulus $p + t$.

Pro coordinata y.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	271	470	543	470	271	30
1	10	280	475	543	466	263	29
2	19	289	480	543	460	254	28
3	28	297	485	543	454	246	27
4	37	305	489	542	448	238	26
5	47	312	493	542	442	230	25
6	56	319	497	541	437	221	24
7	66	327	501	540	431	212	23
8	75	334	504	539	425	204	22
9	85	342	508	537	420	195	21
10	94	349	511	535	414	187	20
11	104	357	514	533	408	177	19
12	113	364	517	532	402	167	18
13	122	370	519	530	396	159	17
14	132	376	522	528	389	151	16
15	141	383	525	525	383	141	15
16	151	389	528	522	376	132	14
17	159	396	530	519	370	122	13
18	167	402	532	517	364	113	12
19	177	408	533	514	357	104	11
20	187	414	535	511	349	94	10
21	195	420	537	508	342	85	9
22	204	425	539	504	334	75	8
23	212	431	540	501	327	66	7
24	221	437	541	497	319	56	6
25	230	442	542	493	312	47	5
26	238	448	542	489	305	37	4
27	246	454	543	485	297	28	3
28	254	460	543	480	289	19	2
29	263	466	543	475	280	10	1
30	271	470	543	470	271	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XVII.

Argumentum angulus $2r$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	−	−	−	
0	19870	17212	9940	0	9940	17212	30
1	19867	17037	9638	346	10239	17383	29
2	19858	16856	9333	693	10535	17549	28
3	19843	16669	9025	1040	10827	17710	27
4	19822	16478	8715	1387	11116	17864	26
5	19796	16283	8410	1732	11402	18014	25
6	19762	16081	8087	2078	11686	18158	24
7	19723	15875	7768	2424	11966	18296	23
8	19679	15665	7448	2768	12238	18428	22
9	19627	15448	7125	3111	12509	18555	21
10	19571	15227	6799	3454	12777	18676	20
11	19508	15003	6473	3794	13041	18792	19
12	19438	14773	6144	4134	13301	18901	18
13	19363	14538	5812	4472	13557	19006	17
14	19282	14300	5480	4809	13808	19103	16
15	19196	14057	5146	5146	14057	19196	15
16	19103	13808	4809	5480	14300	19282	14
17	19006	13557	4472	5812	14538	19363	13
18	18901	13301	4134	6144	14773	19438	12
19	18792	13041	3794	6473	15003	19508	11
20	18676	12777	3454	6799	15227	19571	10
21	18555	12509	3111	7125	15448	19627	9
22	18428	12238	2768	7448	15665	19679	8
23	18296	11966	2424	7768	15875	19723	7
24	18158	11686	2078	8087	16081	19762	6
25	18014	11402	1732	8410	16283	19796	5
26	17864	11116	1387	8715	16478	19822	4
27	17710	10827	1040	9025	16669	19843	3
28	17549	10535	693	9333	16856	19858	2
29	17383	10239	346	9638	17037	19867	1
30	17212	9940	0	9940	17212	19870	0
	+	+	+	−	−	−	
	XI. S.	X. S.	IX. S.	VIII S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XVII.

Argumentum angulus $2r$.

Pro coordinata y.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	9907	17156	19804	17156	9907	30
1	345	10206	17327	19802	16981	9607	29
2	691	10500	17492	19793	16800	9301	28
3	1037	10792	17653	19778	16615	8995	27
4	1383	11079	17806	19757	16424	8687	26
5	1728	11365	17954	19731	16230	8375	25
6	2072	11649	18099	19697	16030	8061	24
7	2417	11927	18238	19659	15824	7744	23
8	2760	12202	18368	19615	15614	7422	22
9	3102	12472	18495	19563	15399	7102	21
10	3443	12737	18616	19508	15178	6778	20
11	3782	13000	18730	19444	14954	6452	19
12	4120	13259	18839	19374	14724	6124	18
13	4457	13514	18944	19300	14491	5794	17
14	4793	13764	19042	19219	14253	5462	16
15	5129	14011	19132	19132	14011	5129	15
16	5462	14253	19219	19042	13764	4793	14
17	5794	14491	19300	18944	13514	4457	13
18	6124	14724	19374	18839	13259	4120	12
19	6452	14954	19444	18730	13000	3782	11
20	6778	15178	19508	18616	12737	3443	10
21	7102	15399	19563	18495	12472	3102	9
22	7422	15614	19615	18368	12202	2760	8
23	7744	15824	19659	18238	11927	2417	7
24	8061	16030	19697	18099	11649	2072	6
25	8375	16230	19731	17954	11365	1728	5
26	8687	16424	19757	17806	11079	1383	4
27	8995	16615	19778	17653	10792	1037	3
28	9301	16800	19793	17492	10500	691	2
29	9607	16981	19802	17327	10206	345	1
30	9907	17156	19804	17156	9907	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XVIII.

Argumentum angulus $(2p-2r)$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
+	−	−	−	+	+	+	−
0	999	866	500	0	500	866	30
1	999	857	485	17	515	875	29
2	998	848	469	34	530	883	28
3	998	839	454	52	545	891	27
4	997	829	438	70	560	899	26
5	996	819	423	87	574	906	25
6	994	809	407	104	588	913	24
7	993	799	391	122	602	920	23
8	990	788	375	139	616	927	22
9	988	777	358	156	629	934	21
10	985	766	342	174	643	940	20
11	982	755	326	191	656	946	19
12	978	743	309	208	669	951	18
13	974	731	292	225	682	956	17
14	970	719	276	242	695	961	16
15	966	707	259	259	707	966	15
16	961	695	242	276	719	970	14
17	956	682	225	292	731	974	13
18	951	669	208	309	743	978	12
19	946	656	191	326	755	982	11
20	948	643	174	342	766	985	10
21	934	629	156	358	777	988	9
22	927	616	139	375	788	990	8
23	920	602	122	391	799	993	7
24	913	588	104	407	809	994	6
25	906	574	87	423	819	996	5
26	899	560	70	438	829	997	4
27	891	545	52	454	839	998	3
28	883	530	34	469	848	998	2
29	875	515	17	485	857	998	1
30	866	500	0	500	866	998	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XVIII.

Argumentum angulus $(2p - 2r.)$

Pro coordinata y.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	1370	2375	2741	2375	1370	30
1	48	1413	2399	2740	2350	1328	29
2	96	1453	2421	2739	2325	1280	28
3	143	1493	2442	2737	2299	1245	27
4	191	1533	2465	2734	2274	1202	26
5	239	1573	2484	2730	2246	1159	25
6	286	1612	2505	2725	2218	1116	24
7	334	1650	2523	2720	2189	1072	23
8	381	1689	2542	2715	2160	1027	22
9	429	1728	2559	2708	2131	982	21
10	476	1761	2577	2699	2104	938	20
11	524	1798	2593	2691	2078	892	19
12	572	1835	2607	2681	2038	847	18
13	617	1869	2622	2670	2005	802	17
14	663	1905	2635	2658	1973	756	16
15	710	1940	2647	2647	1940	710	15
16	756	1973	2658	2635	1905	663	14
17	802	2005	2670	2622	1869	617	13
18	847	2038	2681	2607	1835	572	12
19	892	2078	2691	2593	1798	524	11
20	938	2104	2699	2577	1761	476	10
21	982	2131	2708	2559	1728	429	9
22	1027	2160	2715	2542	1689	381	8
23	1072	2189	2720	2523	1650	334	7
24	1116	2218	2725	2505	1612	286	6
25	1159	2246	2730	2484	1573	239	5
26	1202	2274	2734	2465	1533	191	4
27	1245	2299	2737	2442	1493	143	3
28	1286	2325	2739	2421	1453	96	2
29	1328	2350	2740	2399	1413	48	1
30	1370	2375	2741	2375	1370	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XIX.

Argumentum angulus $q + 2r$.

Pro coordinata x.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	552	478	276	0	276	478	30
1	552	474	267	10	285	483	29
2	552	468	258	19	294	488	28
3	552	462	250	28	302	493	27
4	551	456	242	38	310	497	26
5	551	449	234	48	317	501	25
6	550	444	225	57	324	505	24
7	549	438	216	67	332	509	23
8	548	432	207	76	340	512	22
9	546	427	198	86	348	516	21
10	544	421	190	96	355	520	20
11	542	415	180	106	363	523	19
12	541	409	170	115	370	526	18
13	539	403	160	124	376	528	17
14	537	395	150	134	382	531	16
15	534	389	142	142	389	534	15
16	531	382	134	150	395	537	14
17	528	376	124	160	403	539	13
18	526	370	115	170	409	541	12
19	523	363	106	180	415	542	11
20	520	355	96	190	421	544	10
21	516	348	86	198	427	546	9
22	512	340	76	207	432	548	8
23	509	332	67	216	438	549	7
24	505	324	57	225	444	550	6
25	501	317	48	234	449	551	5
26	497	310	38	242	456	551	4
27	493	302	28	250	462	552	3
28	488	294	19	258	468	552	2
29	483	285	10	267	474	552	1
30	478	276	0	276	478	552	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XIX.

Argumentum angulus $q + 2r$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	273	473	546	473	273	30
1	10	282	478	546	469	264	29
2	19	291	483	546	463	255	28
3	28	299	488	546	457	247	27
4	37	307	492	545	450	239	26
5	47	314	495	545	446	231	25
6	56	321	500	544	439	222	24
7	66	329	504	543	433	213	23
8	75	336	507	542	427	204	22
9	85	344	511	540	422	196	21
10	95	351	514	538	416	188	20
11	105	359	517	536	410	178	19
12	114	362	520	535	404	168	18
13	123	372	522	533	398	160	17
14	133	378	525	531	391	152	16
15	142	385	528	528	385	142	15
16	152	391	531	525	378	133	14
17	160	398	533	522	371	123	13
18	168	404	535	520	366	114	12
19	178	410	536	517	359	105	11
20	188	416	538	514	351	95	10
21	196	422	540	511	344	85	9
22	205	427	542	507	336	75	8
23	213	433	543	504	329	66	7
24	222	439	544	500	321	56	6
25	231	446	545	495	314	47	5
26	239	450	545	492	307	37	4
27	247	457	546	488	299	28	3
28	255	463	546	483	291	19	2
29	264	469	546	478	282	10	1
30	273	473	546	473	273	0	0
	–	–	–	–	–	–	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XX.

Argumentum angulus $q - 2r$.

Pro coordinata x.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	+	+	+	
0	267	231	134	0	134	231	30
1	267	229	130	5	138	234	29
2	267	227	126	10	142	236	28
3	267	225	122	15	146	238	27
4	266	222	118	20	150	240	26
5	266	219	113	24	154	242	25
6	266	216	109	29	158	244	24
7	265	213	105	34	162	246	23
8	265	210	101	39	166	248	22
9	264	207	97	43	169	250	21
10	263	204	92	46	172	251	20
11	262	201	88	51	176	253	19
12	261	199	84	56	179	255	18
13	260	196	79	61	183	256	17
14	259	193	74	65	186	257	16
15	258	189	69	69	189	258	15
16	257	186	65	74	193	259	14
17	256	183	61	79	196	260	13
18	255	179	56	84	199	261	12
19	253	176	51	88	201	262	11
20	251	172	46	92	204	263	10
21	250	169	43	97	207	264	9
22	248	166	39	101	210	265	8
23	246	162	34	105	213	265	7
24	244	158	29	109	216	266	6
25	242	154	24	113	219	266	5
26	240	150	20	118	222	266	4
27	238	146	15	122	225	267	3
28	236	142	10	126	227	267	2
29	234	138	5	130	229	267	1
30	231	134	0	134	231	267	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S	VII. S.	VI. S.	

Tabula

Tabula Aequationum XX.

Argumentum angulus $q - 2r$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	−	−	−	−	−	−	
0	0	1087	1881	2173	1881	1087	30
1	39	1120	1900	2173	1863	1054	29
2	77	1153	1919	2173	1842	1019	28
3	116	1185	1936	2171	1822	985	27
4	155	1217	1953	2169	1802	951	26
5	193	1248	1969	2166	1781	918	25
6	230	1278	1986	2162	1759	884	24
7	267	1309	2002	2156	1737	850	23
8	304	1339	2017	2151	1714	815	22
9	340	1368	2032	2145	1690	780	21
10	377	1397	2045	2139	1666	745	20
11	414	1425	2057	2132	1641	709	19
12	452	1453	2068	2125	1615	673	18
13	488	1481	2079	2116	1590	636	17
14	525	1509	2089	2106	1564	599	16
15	562	1537	2097	2097	1537	562	15
16	599	1564	2106	2089	1509	525	14
17	636	1590	2116	2079	1481	488	13
18	673	1615	2125	2068	1453	452	12
19	709	1641	2132	2057	1425	414	11
20	745	1666	2139	2045	1397	377	10
21	780	1690	2145	2032	1368	340	9
22	815	1714	2151	2017	1339	304	8
23	850	1737	2156	2002	1309	267	7
24	884	1759	2162	1986	1278	230	6
25	918	1781	2166	1969	1248	193	5
26	951	1802	2169	1953	1217	155	4
27	985	1822	2171	1936	1185	116	3
28	1019	1843	2173	1919	1153	77	2
29	1054	1863	2173	1900	1120	39	1
30	1087	1881	2173	1881	1087	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XXI.
Argumentum angulus $2p + q - 2r$.
Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. Sig	V. Sig	
	−	−	−	+	+	+	
0	457	402	229	0	229	402	30
1	457	396	221	8	238	405	29
2	457	392	216	17	245	408	28
3	456	386	209	25	253	410	27
4	456	382	203	31	260	413	26
5	455	377	195	39	266	416	25
6	454	371	188	48	270	419	24
7	452	366	180	56	278	422	23
8	451	361	173	64	286	425	22
9	450	356	165	72	293	428	21
10	449	352	157	79	298	431	20
11	448	347	149	87	303	434	19
12	447	342	140	94	308	436	18
13	446	336	132	102	314	438	17
14	444	331	125	110	319	440	16
15	442	324	117	117	324	442	15
16	440	319	110	125	331	444	14
17	438	314	102	132	336	446	13
18	436	308	94	140	342	447	12
19	434	303	87	149	347	448	11
20	431	298	79	157	352	449	10
21	428	293	72	165	356	450	9
22	425	286	64	173	361	451	8
23	422	278	56	180	366	452	7
24	419	270	49	188	371	454	6
25	416	266	40	195	377	455	5
26	413	260	31	203	382	456	4
27	410	253	25	209	386	456	3
28	407	245	17	216	392	457	2
29	405	238	8	222	396	457	1
30	402	229	0	229	402	457	0
	−	−	−	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S	VII. S.	VI. S.	

Tabula

Tabula Aequationum XXI.

Argumentum angulus $2p + q - 2r$.

Pro coordinata y.

	0. S.	I. S.	II. S.	III S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	395	685	790	685	395	30
1	14	406	692	790	678	383	29
2	28	418	698	789	671	371	28
3	42	430	704	788	663	359	27
4	56	442	710	787	656	346	26
5	70	453	716	786	647	334	25
6	83	464	722	786	639	321	24
7	97	475	728	785	631	308	23
8	110	486	733	783	622	296	22
9	123	496	738	781	613	283	21
10	137	507	743	779	604	270	20
11	151	518	747	776	595	257	19
12	165	528	751	773	587	244	18
13	179	538	755	770	578	231	17
14	193	548	758	767	568	218	16
15	205	558	763	763	558	205	15
16	218	568	767	758	548	193	14
17	231	578	770	755	538	179	13
18	244	587	773	751	528	165	12
19	257	595	776	747	518	151	11
20	270	604	779	743	507	137	10
21	283	613	781	738	496	123	9
22	296	622	783	733	486	110	8
23	308	631	785	728	475	97	7
24	321	639	786	722	464	83	6
25	334	647	786	716	453	70	5
26	346	656	787	710	442	56	4
27	359	663	788	704	430	42	3
28	371	671	789	698	418	28	2
29	383	678	790	692	406	14	1
30	395	685	790	685	395	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum I.

Argumentum angulus r.

Pro coordinata z.

	o. S. +	I. S. +	II. S. +	III. S. +	IV. S. +	V. S. +	
0	00000	448200	776306	896400	776306	448200	30
1	15644	461680	784009	896284	768365	434583	29
2	31284	475022	791474	895854	760190	420834	28
3	46914	488215	798697	895172	751785	406957	27
4	62528	501260	805678	894237	743148	392955	26
5	78126	514152	812414	892990	734287	378834	25
6	93699	526890	818902	891490	725202	364598	24
7	109243	539465	825141	889718	715895	350251	23
8	124754	551871	831128	887676	706370	335795	22
9	140227	564122	836862	885365	696634	321240	21
10	155657	576194	842340	882782	686682	306586	20
11	171040	588090	847562	879931	676522	291840	19
12	186371	599799	852526	876811	666155	277003	18
13	201646	611343	857231	873424	655585	262072	17
14	216855	622692	861675	869772	644816	247082	16
15	232006	633851	865754	865754	633851	232006	15
16	247082	644816	869772	861675	622692	216855	14
17	262072	655585	873424	857231	611343	201646	13
18	277003	666155	876811	852526	599799	186371	12
19	291840	676522	879931	847562	588090	171040	11
20	306587	686682	882782	842340	576194	155657	10
21	321240	696634	885365	836862	564122	140227	9
22	335795	706370	887676	831128	551871	124754	8
23	350251	715895	889718	825141	539465	109243	7
24	364598	725202	891490	818902	526890	93699	6
25	378834	734287	892990	812414	514152	78126	5
26	392955	743148	894237	805678	501260	62528	4
27	406957	751784	895172	798697	488215	46914	3
28	420834	760190	895854	791474	475022	31284	2
29	434583	768365	896284	784009	461680	15644	1
30	448200	776306	896400	776306	448200	00000	0
	− XI. S.	− X. S.	− IX. S.	− VIII. S.	− VII. S.	− VI. S.	

Tabula Aequationum II.

Argumentum angulus $2p + r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	889	1541	1779	1541	889	30
1	32	916	1557	1779	1525	863	29
2	64	942	1571	1778	1509	836	28
3	94	969	1586	1776	1492	808	27
4	125	995	1599	1774	1475	780	26
5	155	1021	1612	1771	1458	752	25
6	186	1046	1625	1769	1439	722	24
7	218	1071	1636	1766	1422	695	23
8	248	1096	1648	1762	1402	666	22
9	278	1119	1660	1758	1383	637	21
10	308	1143	1671	1752	1363	607	20
11	339	1167	1682	1746	1343	577	19
12	368	1190	1692	1740	1322	547	18
13	399	1213	1701	1734	1301	518	17
14	429	1235	1710	1726	1279	488	16
15	458	1257	1718	1718	1257	458	15
16	488	1279	1726	1710	1235	429	14
17	518	1301	1734	1701	1213	399	13
18	547	1322	1740	1692	1190	368	12
19	577	1343	1746	1682	1167	339	11
20	607	1363	1752	1671	1143	308	10
21	637	1383	1758	1660	1119	278	9
22	666	1402	1762	1648	1096	248	8
23	695	1422	1766	1636	1071	218	7
24	722	1439	1769	1625	1046	186	6
25	752	1458	1771	1612	1021	155	5
26	780	1475	1774	1599	995	125	4
27	808	1492	1776	1586	969	94	3
28	836	1509	1778	1571	942	64	2
29	863	1525	1779	1557	916	32	1
30	889	1541	1779	1541	889	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum III.

Argumentum angulus $2\phi - \pi$.

Pro coordinata x.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	000	16544	28829	33299	28829	16644	30
1	581	17346	29115	33283	28535	16138	29
2	1162	17642	29391	33268	28231	15628	28
3	1743	18131	29660	33243	27918	15113	27
4	2323	18616	29919	33206	27598	14593	26
5	2902	19[illegible]94	30170	33161	27269	14068	25
6	3480	19566	30410	33105	26933	13546	24
7	4057	20[illegible]33	30641	33040	26586	13006	23
8	4633	20494	30864	32965	26232	12470	22
9	5208	20940	31077	32879	25870	11930	21
10	5781	21397	31280	32783	25501	11384	20
11	6353	21839	31474	32677	25124	10837	19
12	6922	22274	31659	32561	24738	10287	18
13	7489	22703	31833	32435	24345	9734	17
14	8054	23124	31998	32300	23946	9178	16
15	8617	23539	32155	32155	23539	8617	15
16	9178	23945	32300	31998	23124	8054	14
17	9734	24345	32435	31833	22703	7489	13
18	10287	24738	32561	31659	22274	6922	12
19	10837	25124	32677	31474	21839	6353	11
20	11384	25501	32783	31280	21397	5781	10
21	11930	25870	32879	31077	20949	5208	9
22	12470	26232	32965	30864	20494	4633	8
23	13006	26586	33040	30641	20033	4057	7
24	13546	26933	33105	30410	19566	3480	6
25	14068	27269	33161	30170	19094	2902	5
26	14593	27598	33206	29919	18616	2323	4
27	15113	27918	33243	29660	18131	1743	3
28	15628	28231	33268	29391	17642	1162	2
29	16138	28535	33283	29115	17346	581	1
30	16644	28829	33299	28829	16644	000	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum IV.

Argumentum angulus $q + r$.

Pro coordinata z.

	o. Sig.	I. Sig.	II. Sig.	III. Sig.	IV. Sig.	V. Sig.	
	—	—	—	—	—	—	
0	0	12330	21360	24669	21360	12330	30
1	431	12703	21573	24665	21143	11952	29
2	860	13069	21778	24654	20918	11572	28
3	1291	13431	21977	24636	20686	11196	27
4	1721	13790	22169	24608	20458	10819	26
5	2151	14145	22355	24574	20204	10430	25
6	2580	14496	22532	24532	19955	10038	24
7	3007	14841	22705	24483	19697	9643	23
8	3434	15183	22879	24427	19435	9246	22
9	3862	15521	23027	24363	19165	8843	21
10	4287	15853	23159	24283	18893	8441	20
11	4709	16180	23323	24214	18614	8035	19
12	5131	16502	23459	24128	18329	7628	18
13	5552	16820	23588	24036	18038	7216	17
14	5972	17132	23710	23934	17742	6804	16
15	6390	17439	23826	23826	17439	6390	15
16	6804	17742	23934	23710	17132	5972	14
17	7216	18038	24036	23588	16820	5552	13
18	7628	18329	24128	23459	16502	5131	12
19	8035	18614	24214	23323	16180	4709	11
20	8441	18893	24283	23159	15853	4287	10
21	8843	19165	24363	23027	15521	3862	9
22	9246	19435	24427	22879	15183	3434	8
23	9643	19697	24483	22705	14841	3007	7
24	10038	19955	24532	22532	14496	2580	6
25	10430	20204	24574	22355	14145	2151	5
26	10819	20458	24608	22169	13790	1721	4
27	11196	20686	24636	21977	13431	1291	3
28	11572	20918	24654	21778	13069	860	2
29	11952	21143	24665	21573	12703	431	1
30	12330	21360	24669	21360	12330	0	0
	+	+	+	+	+	+	
	XI. Sig.	X. Sig.	IX. Sig.	VIII. S.	VII. Sig	VI. S.	

Tabula Aequationum V.

Argumentum angulus $q - r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	36230	62752	72460	62752	36230	30
1	1265	37320	63375	72449	62110	35129	29
2	2529	38398	63978	72416	61449	34018	28
3	3792	39465	64562	72361	60770	32896	27
4	5055	40519	65127	72283	60072	31765	26
5	6315	41561	65671	72184	59356	30623	25
6	7574	42591	66195	72063	58621	29472	24
7	8831	43608	66700	71920	57869	28312	23
8	10085	44611	67184	71755	57099	27144	22
9	11335	45601	67648	71568	56312	25967	21
10	12582	46576	68090	71359	55508	24783	20
11	13826	47538	68512	71129	54686	23591	19
12	15065	48485	68913	70877	53848	22391	18
13	16300	49418	69294	70603	52994	21185	17
14	17530	50335	69653	70308	52123	19972	16
15	18754	51237	69991	69991	51237	18754	15
16	19972	52123	70308	69653	50335	17530	14
17	21185	52994	70603	69294	49418	16300	13
18	22391	53848	70877	68913	48485	15065	12
19	23591	54686	71129	68512	47538	13826	11
20	24783	55508	71359	68090	46576	12582	10
21	25967	56312	71568	67648	45601	11335	9
22	27144	57099	71755	67184	44611	10085	8
23	28312	57869	71920	66700	43608	8831	7
24	29472	58621	72063	66195	42591	7574	6
25	30623	59356	72184	65671	41561	6315	5
26	31765	60072	72283	65127	40519	5055	4
27	32896	60770	72361	64562	39465	3792	3
28	34018	61449	72416	63978	38398	2529	2
29	35129	62110	72449	63375	37320	1265	1
30	36230	62752	72460	62752	36230	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum VI.

Argumentum angulus $2p - q - r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	5914	10242	11818	10242	5914	30
1	206	6092	10342	11817	10136	5733	29
2	413	6269	10441	11815	10029	5552	28
3	619	6443	10537	11809	9919	5367	27
4	826	6615	10629	11797	9807	5182	26
5	1032	6786	10717	11781	9691	4996	25
6	1238	6954	10803	11761	9569	4808	24
7	1440	7119	10886	11736	9445	4619	23
8	1644	7282	10967	11709	9320	4429	22
9	1849	7443	11042	11679	9191	4238	21
10	2052	7605	11113	11645	9061	4044	20
11	2255	7754	11180	11607	8928	3850	19
12	2458	7920	11245	11567	8791	3656	18
13	2660	8066	11307	11521	8648	3457	17
14	2861	8211	11365	11473	8505	3260	16
15	3062	8358	11421	11421	8358	3062	15
16	3260	8505	11473	11365	8211	2861	14
17	3457	8648	11521	11307	8066	2660	13
18	3656	8791	11567	11245	7920	2458	12
19	3850	8928	11607	11180	7754	2255	11
20	4044	9061	11645	11113	7605	2052	10
21	4238	9191	11679	11042	7443	1849	9
22	4429	9320	11709	10967	7282	1644	8
23	4619	9445	11736	10886	7119	1440	7
24	4808	9569	11761	10803	6954	1238	6
25	4996	9691	11781	10717	6786	1032	5
26	5182	9807	11797	10629	6615	826	4
27	5367	9919	11809	10537	6443	619	3
28	5552	10029	11815	10441	6269	413	2
29	5733	10136	11817	10342	6092	206	1
30	5914	10242	11818	10242	5914	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum VII.

Argumentum angulus $2p+q-n$

Pro coordinata x.

	0 S.	I S.	II S.	III S.	IV S.	V S.	
	—	—	—	—	—	—	
0	0	398	691	797	691	398	30
1	14	410	698	797	684	386	29
2	28	422	705	797	677	374	28
3	42	434	711	796	669	362	27
4	56	446	717	795	661	350	26
5	70	457	723	794	653	337	25
6	83	468	729	793	645	324	24
7	97	479	735	792	637	311	23
8	111	490	740	790	628	298	22
9	124	501	745	788	619	285	21
10	138	512	750	786	610	262	20
11	152	523	754	783	601	259	19
12	166	533	758	780	592	246	18
13	180	543	762	777	583	233	17
14	194	553	764	774	573	220	16
15	207	563	770	770	563	207	15
16	220	573	774	766	553	194	14
17	233	583	777	762	543	180	13
18	246	592	780	758	533	166	12
19	259	601	783	754	523	152	11
20	262	610	786	750	512	138	10
21	285	619	788	745	501	124	9
22	298	628	790	740	490	111	8
23	311	637	792	735	479	97	7
24	324	645	793	729	468	83	6
25	337	653	794	723	457	70	5
26	350	661	795	717	446	56	4
27	362	669	796	711	434	42	3
28	374	677	797	705	422	28	2
29	386	684	797	698	410	14	1
30	398	691	797	691	398	0	0
	+	+	+	+	+	+	
	XI S.	X S.	IX S.	VIII S.	VII S.	VI S.	

Tabula Aequationum VIII.

Argumentum angulus a p + q + r.

Pro coordinata z.

	0 Sig.	I Sig.	II S.	III Sig.	IV Sig.	V S.	
	−	−	−	+	−	+	
0	0	2720	4711	5440	4711	2720	30
1	95	2802	4757	5439	4662	2637	29
2	190	2883	4802	5436	4612	2553	28
3	284	2962	4846	5432	4562	2469	27
4	379	3042	4888	5426	4509	2384	26
5	474	3120	4919	5419	4455	2299	25
6	568	3197	4969	5410	4400	2212	24
7	663	3273	5006	5399	4344	2125	23
8	757	3349	5043	5387	4286	2037	22
9	850	3423	5078	5373	4227	1949	21
10	944	3496	5111	5357	4167	1860	20
11	1037	3569	5143	5340	4105	1771	19
12	1130	3639	5173	5321	4042	1680	18
13	1224	3709	5202	5300	3978	1590	17
14	1316	3778	5229	5278	3913	1499	16
15	1407	3846	5254	5254	3846	1407	15
16	1499	3913	5278	5229	3778	1316	14
17	1590	3978	5300	5202	3709	1224	13
18	1680	4042	5321	5173	3639	1130	12
19	1771	4105	5340	5143	3569	1037	11
20	1860	4167	5357	5111	3496	944	10
21	1949	4227	5373	5078	3423	850	9
22	2037	4286	5387	5043	3349	757	8
23	2125	4344	5399	5006	3273	663	7
24	2212	4400	5410	4969	3197	568	6
25	2299	4455	5419	4919	3120	474	5
26	2384	4509	5426	4888	3042	379	4
27	2469	4562	5432	4846	2962	284	3
28	2553	4612	5436	4802	2883	190	2
29	2637	4662	5439	4757	2802	95	1
30	2720	4711	5440	4711	2720	0	0
	+	+	+	+	+	+	
	XI S.	X S.	IX S.	VIII S.	VII S.	VI S.	

Tabula

Tabula Aequationum IX.

Argumentum angulus $2q+r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	504	873	1008	873	504	30
1	17	519	883	1007	864	488	29
2	35	533	890	1007	855	472	28
3	52	548	898	1006	844	457	27
4	70	563	906	1005	836	441	26
5	87	577	914	1004	826	426	25
6	105	592	921	1002	816	410	24
7	123	606	928	1000	805	394	23
8	140	620	935	998	794	377	22
9	157	634	941	996	783	360	21
10	175	648	947	994	772	344	20
11	192	661	953	989	761	327	19
12	209	674	959	986	749	311	18
13	226	687	964	982	737	294	17
14	243	699	969	978	725	277	16
15	261	713	974	974	713	261	15
16	277	725	978	969	699	243	14
17	294	737	982	964	687	226	13
18	311	749	986	959	674	209	12
19	327	761	989	953	661	192	11
20	344	772	994	947	648	175	10
21	360	783	996	941	634	157	9
22	377	794	998	935	620	140	8
23	394	805	1000	928	606	123	7
24	410	816	1002	921	592	105	6
25	426	826	1004	914	577	87	5
26	441	836	1005	906	563	70	4
27	457	845	1006	898	548	52	3
28	472	855	1007	890	533	35	2
29	488	864	1007	883	519	17	1
30	504	873	1008	873	504	0	0
	—	—	—	—	—	—	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum X.

Argumentum angulus $2q-r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	456	790	911	790	456	30
1	16	469	799	910	782	441	29
2	32	482	805	910	773	427	28
3	47	495	812	909	764	413	27
4	63	508	819	908	756	399	26
5	78	521	827	908	747	385	25
6	95	535	833	906	738	371	24
7	111	548	839	904	729	356	23
8	126	560	845	902	718	340	22
9	141	574	851	900	708	325	21
10	158	587	856	898	698	311	20
11	173	598	861	894	688	296	19
12	189	610	866	891	677	281	18
13	204	621	871	888	666	266	17
14	220	632	876	884	656	250	16
15	236	645	880	880	645	236	15
16	250	656	884	876	632	220	14
17	266	666	888	871	621	204	13
18	281	677	891	866	610	189	12
19	296	688	894	861	598	173	11
20	311	698	898	856	587	158	10
21	325	708	900	851	574	141	9
22	340	718	902	845	560	126	8
23	356	729	904	839	548	111	7
24	371	738	906	833	535	95	6
25	385	747	908	827	521	78	5
26	399	756	908	819	508	63	4
27	413	764	909	812	495	47	3
28	427	773	910	805	482	32	2
29	441	782	910	799	469	16	1
30	456	790	911	790	456	0	0
	—	—	—	—	—	—	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XI.

Argumentum angulus $2p - 2q - r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	226	391	450	391	226	30
1	8	231	394	450	387	218	29
2	16	239	397	450	381	211	28
3	23	244	401	449	378	204	27
4	31	251	404	449	374	197	26
5	39	258	408	449	369	190	25
6	47	264	411	448	365	183	24
7	55	271	414	447	360	176	23
8	63	277	417	446	355	168	22
9	70	284	420	445	350	161	21
10	79	290	423	444	345	154	20
11	85	296	425	442	340	147	19
12	93	302	428	440	334	139	18
13	101	306	430	439	329	132	17
14	109	312	433	437	324	124	16
15	117	318	435	435	318	117	15
16	124	324	437	433	312	109	14
17	132	329	439	430	306	101	13
18	139	334	440	428	302	93	12
19	147	340	442	425	296	85	11
20	154	345	444	423	290	79	10
21	161	350	445	420	284	70	9
22	168	355	446	417	277	63	8
23	176	360	447	414	271	55	7
24	183	365	448	411	264	47	6
25	190	369	449	408	258	39	5
26	197	374	449	404	251	31	4
27	204	378	449	401	244	23	3
28	211	381	450	397	239	16	2
29	218	387	450	394	231	8	1
30	226	391	450	391	226	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XII.

Argumentum angulus $r + t$.

Pro coordinata z.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	126	218	252	218	126	30
1	4	130	221	252	216	122	29
2	8	133	223	252	214	118	28
3	13	137	225	252	211	114	27
4	17	140	226	251	209	110	26
5	22	144	228	251	206	106	25
6	26	147	230	251	204	102	24
7	30	151	232	250	201	98	23
8	34	155	234	250	199	94	22
9	39	158	235	249	196	90	21
10	43	161	236	248	193	86	20
11	47	165	238	247	190	82	19
12	52	168	240	246	187	78	18
13	56	171	241	245	184	74	17
14	60	174	242	244	180	70	16
15	65	177	243	243	177	65	15
16	70	180	244	242	174	60	14
17	74	184	245	241	171	56	13
18	78	187	246	240	168	52	12
19	82	190	247	238	165	47	11
20	86	193	248	236	161	43	10
21	90	196	249	235	158	39	9
22	94	199	250	234	155	34	8
23	98	201	250	232	151	30	7
24	102	204	251	230	147	26	6
25	106	206	251	228	144	22	5
26	110	209	251	226	140	17	4
27	114	211	252	225	137	13	3
28	118	214	252	223	133	8	2
29	122	216	252	221	130	4	1
30	126	218	252	218	126	0	0
	—	—	—	—	—	—	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XIII.

Argumentum angulus $r - t$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	151	261	302	261	151	30
1	5	155	264	302	259	146	29
2	10	160	267	302	256	142	28
3	15	164	270	302	253	137	27
4	21	169	272	301	251	133	26
5	26	173	274	301	248	128	25
6	31	177	276	300	245	123	24
7	37	182	278	300	242	118	23
8	42	186	280	299	238	113	22
9	47	190	282	298	235	108	21
10	52	194	284	297	232	103	20
11	57	198	286	296	228	97	19
12	62	202	288	295	225	92	18
13	67	206	289	294	222	87	17
14	72	210	290	293	218	82	16
15	78	214	292	292	214	78	15
16	82	218	293	290	210	72	14
17	87	222	294	289	206	67	13
18	92	225	295	288	202	62	12
19	97	228	296	286	198	57	11
20	103	232	297	284	194	52	10
21	108	235	298	282	190	47	9
22	113	238	299	280	186	42	8
23	118	242	300	278	182	37	7
24	123	245	300	276	177	31	6
25	128	248	301	274	173	26	5
26	133	251	301	272	169	21	4
27	137	253	302	270	164	15	3
28	142	256	302	267	160	10	2
29	146	259	302	264	155	5	1
30	151	261	302	261	151	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula Aequationum XIV.

Argumentum angulus $2p-r-t$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	9	827	1434	1653	1434	827	30
1	29	853	1448	1653	1420	791	29
2	58	877	1461	1652	1404	777	28
3	87	882	1475	1651	1388	752	27
4	12	924	1491	1648	1373	726	26
5	140	949	1501	1646	1357	700	25
6	171	973	1513	1644	1342	673	24
7	201	996	1524	1642	1322	647	23
8	230	1018	1535	1638	1305	620	22
9	258	1042	1545	1633	1286	592	21
10	287	1063	1556	1628	1268	566	20
11	317	1078	1560	1623	1249	538	19
12	343	1108	1573	1614	1221	510	18
13	370	1127	1582	1608	1211	483	17
14	399	1149	1590	1602	1190	455	16
15	427	1170	1596	1596	1170	427	15
16	455	1190	1602	1590	1149	399	14
17	483	1211	1608	1582	1127	370	13
18	510	1221	1614	1573	1108	343	12
19	538	1249	1623	1560	1078	317	11
20	566	1268	1628	1556	1063	287	10
21	592	1286	1633	1545	1042	258	9
22	620	1305	1638	1535	1018	230	8
23	647	1322	1642	1524	996	201	7
24	673	1342	1644	1513	973	171	6
25	700	1357	1646	1501	949	140	5
26	726	1373	1648	1491	924	112	4
27	752	1388	1651	1475	882	87	3
28	777	1404	1652	1461	877	58	2
29	791	1420	1653	1448	853	29	1
30	827	1434	1653	1434	827	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

 Tabula

Tabula Aequationum XV.

Argumentum angulus $2p - r + t$.

Pro coordinata z.

	o. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	+	+	+	+	+	+	
0	0	299	518	601	518	299	30
1	10	309	524	601	512	290	29
2	20	317	528	601	507	282	28
3	31	326	533	600	501	272	27
4	42	335	538	600	496	263	26
5	52	343	543	599	490	253	25
6	64	351	547	598	484	244	24
7	74	360	551	596	479	234	23
8	84	368	555	594	473	224	22
9	94	376	558	592	466	215	21
10	104	384	562	591	458	204	20
11	116	394	566	589	452	194	19
12	126	402	570	586	445	185	18
13	137	409	573	584	438	175	17
14	146	416	576	581	431	165	16
15	155	423	579	579	423	155	15
16	165	431	581	576	416	146	14
17	175	438	584	573	409	137	13
18	185	445	586	570	402	126	12
19	194	452	589	566	394	116	11
20	204	458	591	562	384	104	10
21	215	466	592	558	376	94	9
22	224	473	594	555	368	84	8
23	234	479	596	551	360	74	7
24	244	484	598	547	351	64	6
25	253	490	599	543	343	52	5
26	263	496	600	538	335	42	4
27	272	501	600	533	326	31	3
28	282	507	601	528	317	20	2
29	290	512	601	524	309	10	1
30	299	518	601	518	299	0	0
	−	−	−	−	−	−	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

Tabula

Tabula Aequationum XVI.

Argumentum angulus $p-r$.

Pro coordinata z.

	0. S.	I. S.	II. S.	III. S.	IV. S.	V. S.	
	—	—	—	—	—	—	
0	0	180	312	360	312	180	30
1	7	186	316	360	309	175	29
2	14	191	319	360	305	170	28
3	19	196	322	360	302	164	27
4	26	202	324	360	299	158	26
5	32	208	327	360	296	152	25
6	38	212	330	359	292	147	24
7	44	218	333	359	288	141	23
8	50	223	335	358	285	134	22
9	56	228	337	357	281	128	21
10	62	232	339	356	277	122	20
11	69	237	341	355	272	116	19
12	75	242	342	353	268	111	17
13	81	247	344	352	264	105	18
14	87	251	346	350	260	99	16
15	93	256	348	348	256	93	15
16	99	260	350	346	251	87	14
17	105	264	352	344	247	81	13
18	111	268	353	342	242	75	12
19	116	272	355	341	237	69	11
20	122	277	356	339	232	62	10
21	128	281	357	337	228	56	9
22	134	285	358	335	223	50	8
23	141	288	359	333	218	44	7
24	147	292	359	330	212	38	6
25	152	296	360	327	208	32	5
26	158	299	360	324	202	26	4
27	164	302	360	322	196	19	3
28	170	305	360	318	191	14	2
29	175	309	360	316	186	7	1
30	180	312	360	312	180	0	0
	+	+	+	+	+	+	
	XI. S.	X. S.	IX. S.	VIII. S.	VII. S.	VI. S.	

(o)

CAPUT III.

CAPVT III.

CONSIDERATIONES SVPER GRADV PRAECISIONIS, QVEM A TABVLIS LVNARIBVS EXSPECTARE LICET.

§. 647.

Quamuis in calculo theoretico nihil amplius desiderandum relinqueretur; neutiquam tamen perfectissimum consensum cum obseruationibus, quae quidem ab astronomis plurimae reperiuntur publicatae, exspectare licet, propterea quod in his obseruationibus ipsis haud leues aberrationes sunt pertimescendae, quas aliquanto adcuratius expendere plurimum intererit, ne theoriae vitio vertatur, quod obseruationibus esset tribuendum.

§. 648.

Primum autem hae obseruationes plerumque ex culminatione Lunae sunt deductae, ideoque ascensio recta Lunae ad ascensionem rectam Solis ita refertur, vt vt quantum in hac fuerit aberratum, tantundem in illam redundet; at Clarissimus Abbas de la Caille, qui optimas tabulas solares condidisse censetur, ipse fatetur, in longitudine Solis errorem quandoque vsque ad 30″ assurgere posse, quamobrem etiam vtique euenire posset, vt in longitudine Lunae tantundem fere aberraretur. Deinde huiusmodi obseruationes eo nituntur temporis momento, quo limbus Lunae meridianum attingere videtur; in quo momento tam propter ipsam appulsus incertitudinem, quam ob vitium etsi leuissimum horologii (siquidem gradus caloris a meridie fuerit immutatus) error fortasse vnum minutum secundum superans inesse posset; vnde in longitudinem Lunae error quindecim minuta secunda superans influeret. Tertio vero hinc ascensio recta centri Lunae deduci nequit, nisi diameter Lunae apparens exactissime fuerit cognita, in qua certe aliquot minutorum secundorum error vix euitari posse videtur; vt taceamus errorem, qui hinc in altitudinem Lunae obseruatam redundare potest, etiamsi circa refractionem ne minimum quidem dubium relinqueretur. Quarto denique determinatio loci Lunae geocentrici necessario requirit adcuratissimam parallaxis Lunae cognitionem, circa quam cum astronomi adhuc fere integro minuto primo dissenserint, facile intelligitur, quanta incertitudo hinc tam in longitudinem quam latitudinem Lunae geocentricam irrepere queat. His rationibus coniunctis nemo certe mirabitur, si obseruationes Lunae quandoque vltra integrum minutum primum a veritate aberrauerint. Quamobrem etiamsi tabulas Lunares omnibus numeris absolutas haberemus; nefas foret postulare, vt calculus inde superstructus cum talibus obseruationibus adcurate conueniret.

§. 649.

Quanquam autem fere incredibili labore calculum theoreticum simus executi; tamen iam fateri fuimus coacti, nonnullos inaequalitatum ordines ob insuperabiles calculi molestias penitus a nobis expediri non potuisse; interim tamen facile intelligere licet, errores hinc oriundos non nisi quam minimos esse posse. Sed multo magis peregrinas caussas motum Lunae adficientes pertimescere debemus, inter quas sine dubio primum locum occupat aberratio a figura sphaerica, cui tam Terra quam ipsa Luna est obnoxia, postquam inprimis ex motu libratorio Lunae manifesto concludere licuit, eius figuram vehementer a sphaerica recedere. Iam autem abunde extra dubium est positum, ob hanc caussam vim, qua Luna ad Terram vrgetur, non amplius quadrato distantiae reciproce esse proportionalem, sed ad hanc legem insuper adiungi oportere exiguam quandam particulam biquadrato distantiae reciproce proportionalem. Euidens autem est ob talem accessionem omnes inaequalitates Lunae aliquantillam variationem esse subituras, quas quidem maxime operae pretium foret omni studio inuestigare; id id quod iisdem veſtigiis, quibus hic ſumus vſi, inſiſtendo praeſtari poſſet. Praeterea vero quoniam valores litterarum K et *i* ex eiusmodi obſeruationibus concluſimus, quae fortaſſe magis minusue ſunt erroneae, mirum ſane non eſſet, ſi veri earum valores aliquantum ab his aſſumtis diſcreparent. Tum vero nulla prorſus ratione ſumus conuicti, in tabulis Majerianis loca media tam apogei, quam nodi recte asſignari, quin potius probabile videtur, horum elementorum errores integrum fortaſſe minutum primum ſuperare poſſe. Hanc autem emendationem ob memoratam ipſarum obſeruationum incertitudinem nemo facile ſuſcipere valebit. Primo autem intuitu motus Lunae maxime ab actione tam planetarum quam cometarum pati videbatur; verum re adcuratius perpenſa agnouimus, hinc vix vllam perturbationem in motum Lunae redundare poſſe, propterea quod ob Lunae propinquitatem omnes vires tam planetarum quam cometarum aequaliter agunt in Terram atque in Lunam; conſtat autem, quatenus Terra et Luna ab aequalibus viribus ſollicitantur, inde motum Lunae reſpectu Terrae nullam plane mutationem perpeti. Interim tamen effectus harum virium in ipſam Terram exſerti ſimul etiam Lunam adficiunt, propterea quod quantum ipſa terra in motu ſuo circa ſolem perturbatur, tantas praeciſe perturbationes ipſa Luna ſubire debeat. Quare eaedem correctiones, quae ob actionem planetarum Soli tribuuntur, etiam in motum Lunae inferri debebunt, ac ſi eueniret, vt a Cometa quopiam Terra in motu ſuo turbaretur, eundem plane effectum in Luna eſſemus deprehenſuri, etiamſi ipſae tabulae lunares inde nullam mutationem eſſent accepturae; ratio ſcilicet manifeſto in eo eſt ſita, quod locus Lunae ex loco Solis ſuppoſito plerumque definiri ſolet.

§. 650.

Denique coronidis loco obſeruaſſe iuuabit, tum demum adcuratam motuum lunarium cognitionem ſperari poſſe, quando in pluribus terrae locis omnes occultationes ſtellarum fixarum a Luna factae ſummo ſtudio obſeruarentur; namque ex huiusmodi obſeruationibus locum Lunae tam exacte in coelo definire licebit, vt error fortaſſe nunquam decem minuta ſecunda ſit ſuperaturus; quodſi autem ſufficientem talium obſeruationum numerum in poteſtate habuerimus, tum demum theoriae innixi adcuratiſſimas tabulas pro motu Lunae condere licebit.

CAPVT IV.

CAPVT IV.

METHODVS PRAECEDENTES TABVLAS VLTERIVS PERFICIENDI.

§. 651.

Quoniam elementa harum tabularum ex eiusmodi obseruationibus sunt conclusa, quarum errores facile integrum minutum primum superare possunt; nullum est dubium, quin istae tabulae vlteriori correctione indigeant, quam autem non ante suscipere vel saltem sperare licet, quam ingens numerus adcuratissimarum obseruationum fuerit in promtu; quales potissimum tam eclipses solares, quam occultationes stellarum fixarum in pluribus locis factae suppeditare poterunt.

§. 652.

Correctiones autem harum tabularum duplicis sunt generis; primum enim necesse est, errores, qui forte in valoribus litterarum K et *i* assumtis insunt, inuestigari; deinde etiam loca media tam ipsius Lunae,

nae, quam Apogei et Nodi, quae ex tabulis Majerianis hic desumimus, correctione quapiam indigebunt. Quocirca operae pretium erit methodum hic subiungere, cuius ope istas correctiones ex obseruationibus exquisitissimis inuenire liceat.

§. 653.

Quoniam in his tabulis litteris K et i istos valores assignauimus $K = 0{,}0544500$ et $i = 0{,}0896400$, ponamus reuera esse debere $K = 0{,}0544500 + dK$; $i = 0{,}0896400 + di$. Deinde vero ponamus ad quoduis tempus longitudinem Lunae mediam L ex tabulis Majerianis desumtam elemento dL; longitudinem vero apogei P elemento dP; at longitudinem nodi N elemento dN augeri oportere; quae quidem elementa non in minutis primis vel secundis, sed in partibus radii exprimi sumamus. Hinc igitur angulus p incrementum accipiet $dp = dL$; angulus vero q incrementum $dq = dL - dP$; atque angulus r incrementum $dr = dL - dN$; vnde vicissim ex incrementis angularibus dp, dq, dr ipsae tabularum correctiones colliguntur $dL = dp$; $dP = dp - dq$ et $dN = dp - dr$.

§. 654.

Ex his autem correctionibus ipsae ternae coordinatae x, y et z certa accipient incrementa, quae sequenti

sequenti modo repraesentari conueniet:

$$dx = dK\left(\frac{dx}{dK}\right) + di\left(\frac{dx}{di}\right) + dp\left(\frac{dx}{dp}\right) + dq\left(\frac{dx}{dq}\right) + dr\left(\frac{dx}{dr}\right)$$

$$dy = dK\left(\frac{dy}{dK}\right) + di\left(\frac{dy}{di}\right) + dp\left(\frac{dy}{dp}\right) + dq\left(\frac{dy}{dq}\right) + dr\left(\frac{dy}{dr}\right)$$

$$dz = dK\left(\frac{dz}{dK}\right) + di\left(\frac{dz}{di}\right) + dp\left(\frac{dz}{dp}\right) + dq\left(\frac{dz}{dq}\right) + dr\left(\frac{dz}{dr}\right)$$

quia autem satis sumus certi, omnes has correctiones per se esse quam minimas, ac praecipue quidem correctionem longitudinis mediae dp; sufficit harum formularum partes tantum potiores considerasse. Quare elemento dp penitus reiecto, quoniam coordinatarum priorum x et y correctiones potissimum ab elementis dK et dq, tertiae vero z ab elementis di et dr pendent, sufficiet posuisse:

$$dx = dK\left(\frac{dx}{dK}\right) + dq\left(\frac{dx}{dq}\right)$$

$$dy = dK\left(\frac{dy}{dK}\right) + dq\left(\frac{dy}{dq}\right)$$

$$dz = di\left(\frac{dz}{di}\right) + dr\left(\frac{dz}{dr}\right)$$

§. 655.

Cum igitur posuissemus

$$x = \mathfrak{O} + K\,\mathfrak{P} + K^2.\,\mathfrak{Q} \text{ etc.}$$

$$y = O + K\,P + K^2.\,Q \text{ etc.}$$

$$z = i\,\mathfrak{p} + i\,K\,\mathfrak{q} \text{ etc.}$$

hinc

hinc formularum illarum valores sequenti modo definiemus:

$$1^\circ.\ \left(\frac{dx}{dK}\right) = \mathfrak{P} + 2\,K\,\mathfrak{Q}$$

vnde sumtis tantum terminis potioribus fiet

$$\left(\frac{dx}{dK}\right) = +\,\text{cos.}\,q + 0{,}188\ \text{cos.}\,2p - q.$$

$$2^\circ.\ \left(\frac{dy}{dK}\right) = P + 2\,K\,Q$$

vnde sumtis tantum partibus potioribus fit

$$\left(\frac{dy}{dK}\right) = -\,2{,}013.\ \text{sin.}\,q - 0{,}411.\ \text{sin.}\,2p - q.$$

$$3^\circ.\ \left(\frac{dx}{dq}\right) = K\left(\frac{d\mathfrak{P}}{dq}\right) = K(-\,\text{sin.}\,q + 0{,}188.\ \text{sin.}\,2p - q)$$

$$= -\,0{,}054.\ \text{sin.}\,q + 0{,}010.\ \text{sin.}\,2p - q.$$

$$4^\circ.\ \left(\frac{dy}{dq}\right) = K\left(\frac{dP}{dq}\right) = K(-\,2{,}013.\ \text{cos.}\,q + 0{,}411.\ \text{cos.}\,2p - q)$$

siue $\left(\frac{dy}{dq}\right) = -\,0{,}109.\ \text{cos.}\,q + 0{,}022.\ \text{cos.}\,2p - q.$

$$5^\circ.\ \left(\frac{dz}{di}\right) = \mathfrak{p} + K\,\mathfrak{q}.$$

sumtis ergo tantum terminis potioribus erit

$$\frac{dz}{di} = +\,\text{sin.}\,r.$$

$$6^\circ.\ \left(\frac{dz}{dr}\right) = i\left(\frac{d\mathfrak{p}}{dr}\right)$$

sumtaque tantum parte principali fit

$$\left(\frac{dz}{dr}\right) = i\left(\frac{d\mathfrak{p}}{dr}\right) = i.\ \text{cos.}\,r$$

siue $\left(\frac{dz}{dr}\right) = 0{,}089.\ \text{cos.}\,r.$

§. 656.

§. 656.

Cum introductis his correctionibus ipsae coordinatae x, y et z abeant in $x+dx$; $y+dy$; $z+dz$; aequatio, quam pro longitudine inuenimus,

$$\text{tang.}\,\Phi = \frac{y}{1+x};\ \text{siue}\ (1+x)\,\text{tang.}\,\Phi - y = 0.$$

sequentem induet formam

$$(1+x)\,\text{tang.}\,\Phi - y = dy - dx.\,\text{tang.}\,\Phi$$

siue valoribus substitutis

$$(1+x)\,\text{tg.}\,\Phi - y = dK \begin{cases} -2{,}013\ \text{sin.}\,q - 0{,}411.\,\text{sin.}\,(2p-q) \\ -\text{cos.}\,q.\,\text{tang.}\,\Phi\ \ 0{,}188.\,\text{cos.}\,(2p-q)\,\text{tang.}\,\Phi \end{cases}$$

verum quatenus ipsa longitudo media tabularis L incrementum accipit dp; quoniam angulus Φ denotat excessum longitudinis verae supra mediam, angulus Φ abire censendus est in $\Phi - dp$; ita, vt iam loco tang. Φ habeamus $\frac{\text{tang.}\,\Phi - dp}{\text{cos.}\,\Phi^2}$, vbi loco cos. Φ^2 tuto scribere licebit vnitatem; tum vero etiam hic loco $1+x$ scribere licebit 1; ita, vt iam aequatio finalis ex longitudine cognita Lunae deducta futura sit

$$(1+x)\,\text{tang.}\,\Phi - y = dp + dy - dx.\,\text{tang.}\,\Phi$$

cuius membrum prius $(1+x)\,\text{tang.}\,\Phi - y$, quod comparatio obseruationis cum nostris tabulis praebet, appellari conueniet errorem longitudinis, cui ergo alterum membrum $dp + dy - dx.\,\text{tang.}\,\Phi$ aequale esse oportet.

§. 657.

§. 657.

Simili prorſus modo circa latitudinem Lunae obſeruatam, quam ſigno ψ indicauimus, erit procedendum; cum enim ſit

$$\text{tang. } \psi = \frac{z. \text{ cof. } \Phi}{1+x} \text{ ſiue } (1+x) \text{ tang. } \psi. \text{ ſec. } \Phi - z = 0.$$

introductis correctionibus habebimus

$$(1+x) \text{ tang. } \psi. \text{ ſec. } \Phi - z = dz - dx \text{ tang. } \psi. \text{ ſec. } \Phi$$

vbi iterum membrum prius $(1+x)$ tang. ψ. ſec. $\Phi - z$ appelletur error latitudinis, quem comparatio obſeruationis cum noſtris tabulis declarat, cui ergo alterum membrum $dz - dx$. tang. ψ. ſec. Φ aequale eſſe debet.

§. 658.

Singulae igitur obſeruationes Lunae, quibus tam longitudo, quam latitudo adcurate eſt explorata, geminam nobis aequationem ſuppeditabunt, alteram ſcilicet pro errore longitudinis, alteram vero latitudinis, vnde, ſi plures huiusmodi obſeruationes praeſto fuerint, totidem quoque huiusmodi aequationes geminatae inde deriuabuntur, ex quibus facile elementa incognita dK, di, dp, dq et dr ita definire licebit, vt omnes errores quantum fieri poteſt ſiue tollantur ſiue minimi reddantur. Huiusmodi igitur laborem, ſi quis ſuſcipere voluerit, noſtris tabulis facile maiorem praeciſionis gradum inducere valebit.

CON-

CONSPECTVS OPERIS.

Praefatio.

LIBER PRIMVS.

Continens ipsam Lunae theoriam.

PARS PRIMA.

Inuestigatio aequationum differentialium, motum Lunae continentium.

PARS SECVNDA.

Euolutio numerica aequationum pro binis coordinatis x et y, in praecedente parte constitutarum.

PARS TERTIA.

Euolutio numerica tertiae aequationis pro coordinata z.

CONTINVATIO PARTIS SECVNDAE.

Euolutio numerica aequationum pro iis membris coordinatarum x et y, quorum characteres inuolvunt inclinationem orbitae lunaris ad eclipticam.

LIBER SECVNDVS.

Continens adplicationem theoriae Lunae ad calculum astronomicum.

PARS PRIMA.

Comparatio formularum inuentarum cum tabulis Celeberrimi de Clairault.

PARS SECVNDA.

Constructio tabularum Astronomicarum ex formulis supra inuentis.

Caput II.

Ad Euleri novam theoriam Lunae.

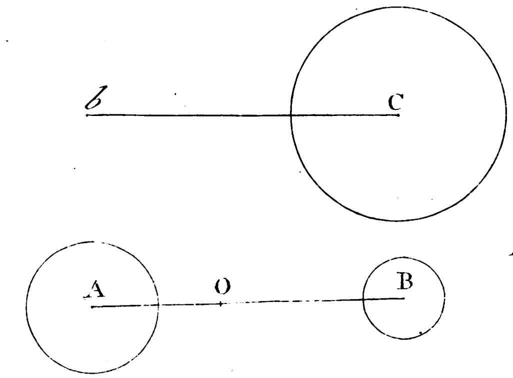

Fig. I.

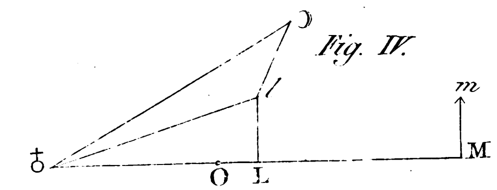

Fig. IV.

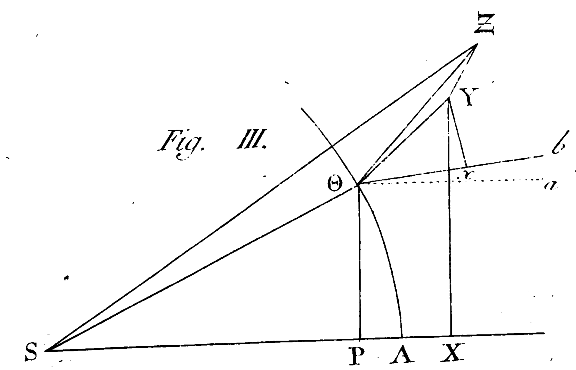

Fig. III.

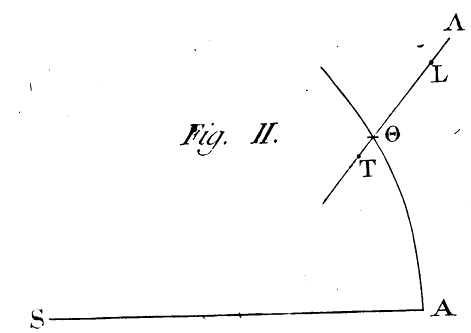

Fig. II.